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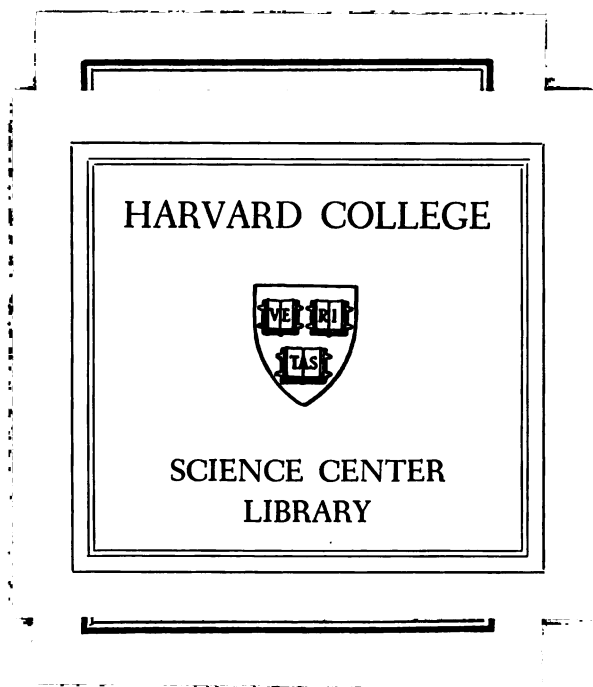
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**EXPERIMENTS ON LOSS OF HEAD IN VALVES AND
PIPES OF ONE-HALF TO TWELVE INCHES
DIAMETER**

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EXPERIMENTS ON LOSS OF HEAD IN VALVES AND PIPES OF ONE-HALF TO TWELVE INCHES DIAMETER

INTRODUCTION

There exists a definite need for experimental information concerning loss of head due to flow of water through valves. Valves are a part of practically every hydraulic installation and as a consequence knowledge of the losses occasioned by them is essential to intelligent and accurate design. Not only will experimental data supply this need but they will also contribute to a better understanding of the probable losses occasioned by other obstructions or restrictions which in effect operate as do valve openings.

This bulletin presents the results of 2,200 tests on 48 different gate and globe valves. Results of 425 tests to determine pipe friction are also included.

The loss of head due to gate valves $\frac{1}{2}$ -inch to 12-inch diameter, was measured for various openings, namely, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, and fully open. The loss of head due to globe valves, $\frac{1}{2}$ -inch to 2-inch diameter, was determined under fully open conditions. As a part of the valve experiments the loss of head in pipes of $\frac{1}{2}$ -inch to 12-inch diameter was determined, and these results are also included in this bulletin.

Acknowledgment.—All the experimental work for the data presented was performed in the Hydraulic Laboratory of the University of Wisconsin under the direction of the writer. In the school year of 1913-1914 experiments on 2-inch, 4-inch, and 6-inch flange connection gate valves were performed as student thesis work by M. J. Kline and E. F. Tanghe. These three valves were retested in the summer of 1914 by H. N. Lendall, instructor of civil engineering, Rutgers College. In the school year of 1914-1915 experiments on $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, and 2-inch screw-connection gate valves were performed by C. A. Olson as student thesis work.

In the summer and fall of 1918 most of the 6-inch valves were tested by Mr. C. D. Dietrich. C. P. Kidder, C. A. Wiepking, and other student assistants performed the tests on the remainder of the valves in the spring and summer of 1919. Especial acknowledgment is made to Mr. L. R. Balch, engineer in the office of Mead and Seastone, Consulting Engineers, for many valuable suggestions.

Formulas.—Pipe frictional losses were studied by using the formula $H = mv^n$ where H is the loss of head in feet in 100 feet of straight pipe, v is the velocity of flow in the pipe in feet per second, m is an empirical coefficient and n is the exponent of v .

For valves, the formula $H = \frac{kv^2}{2g}$ was used, where H is the loss of head in feet occasioned by the valve, k is an empirical coefficient, $g = 32.2$ and v is as before the velocity of flow in the pipe in feet per second. An exact study of valve data will show that the loss of head varies by some power of the velocity which differs slightly from two but it is considered desirable to keep the simpler form since other factors will produce more variation in results than the assumption that the power of v is two.

TABLE OF NOMENCLATURE

d = Diameter of pipe in inches.

d₁ = Diameter of opening through valve seat in inches.

d' = Opening of valve in inches.

g = 32.2

H = Loss of head in feet due to valve or to 100 feet of straight pipe.

k = Constant in valve loss equation ($H = \frac{kv^2}{2g}$)

m = Constant in equation for loss of head in 100 feet of pipe ($H = mv^n$)

n = Exponent of the velocity (*v*) in the equation ($H = mv^n$) for pipe.

v = Velocity of flow in the pipe in feet per second.

APPARATUS

General.—The loss of head due to a valve occurs not only in the valve itself but also in the pipe as far downstream as the velocity distribution is distorted. J. G. Davis, Jr., in *Investigation of Hydraulic Curve Resistance*,¹ concluded the effect of a bend or other disturbing condition would extend not to exceed 40 diameters downstream. Appendix A gives a part of the results of an investigation made subsequent to the testing of the valves which indicates more specifically the effect of placing the downstream piezometer at various distances below the valve.

In order to include all valve loss, 30 to 40 diameters of pipe below the valve was always included in the gage length. Figures 1 to 5, pages 13 to 17, show the arrangement of apparatus for the different valves tested.

Water Supply.—The source of water supply for the earlier experiments was from the pressure tanks of the University Pumping Plant, with the exception that a centrifugal pump in the Hydraulic Laboratory supplied the water for the 6-inch valve tests. For the 1918-1919 experiments water was drawn through a 10-inch pipe line from the laboratory reservoir. This reservoir, situated on a bluff directly in front of the laboratory, has a capacity of 220,000 gallons. The pressure head is approximately 58 feet at the level of the main floor of the laboratory.

Two methods were used to insure normal pipe flow as the water entered the valve. Either a length of 40 or more diameters of straight pipe preceded the valve or there was used a baffle drum followed with a considerable length of straight pipe.

Water Measurement.—Following the gage length, the water flowed to the device for measuring discharge which in most cases consisted of a tank mounted on calibrated platform scales. For the 6-inch, 8-inch, and 12-inch sizes, weirs were used. For the 4-inch size small discharges were measured with tank and scales and the larger quantities with a Venturi meter. All water measuring devices were carefully calibrated.

¹ Page 17, Bulletin 403 of University of Wisconsin.

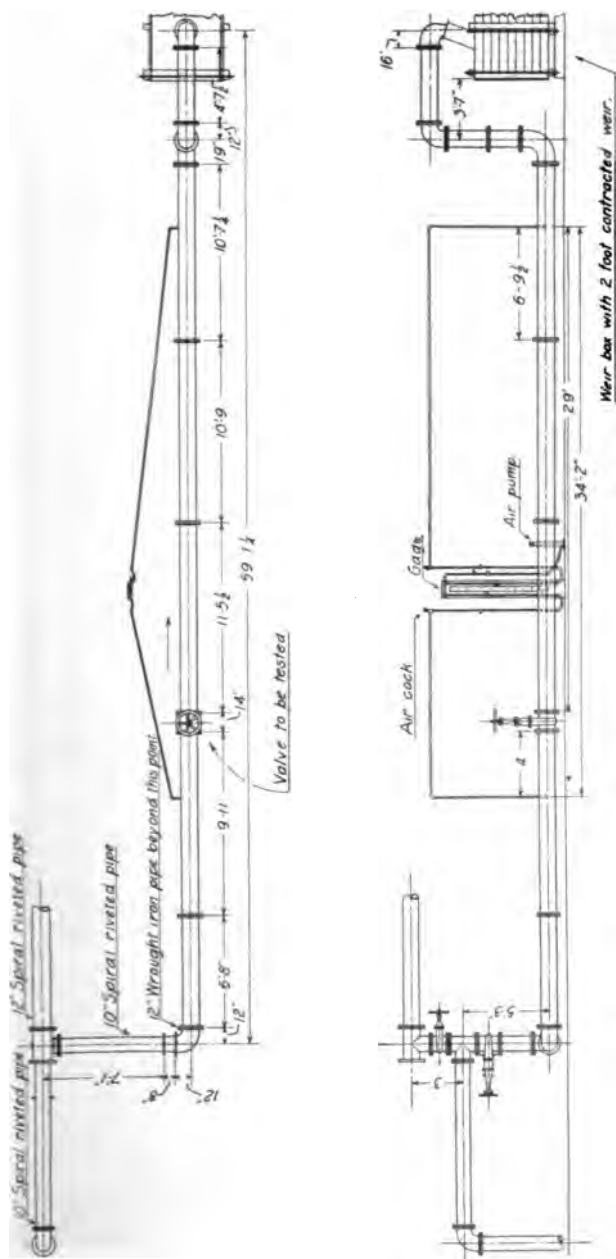


Fig. 1—Arrangement of Piping for Test of 12-inch Chapman Gate Valve. One Valve Tested.

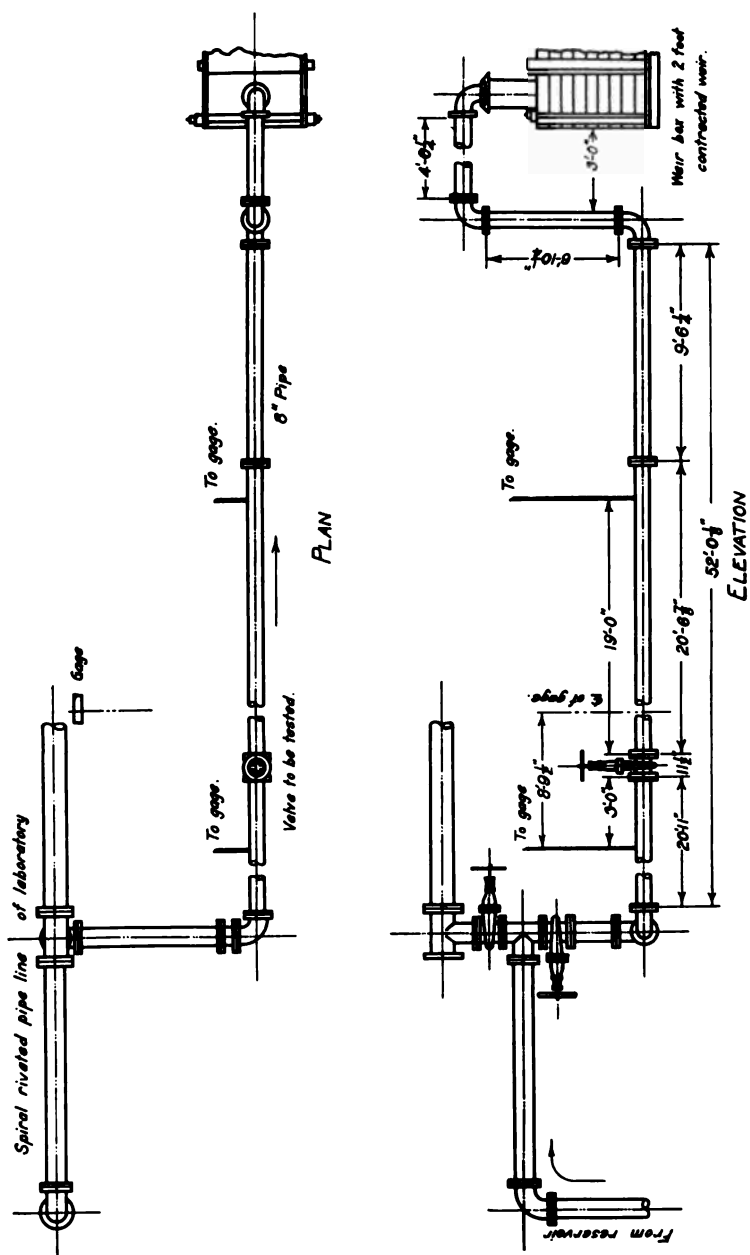


Fig. 2—Arrangement of Piping for Tests of 8-inch Gate Valves. Two Valves Tested.

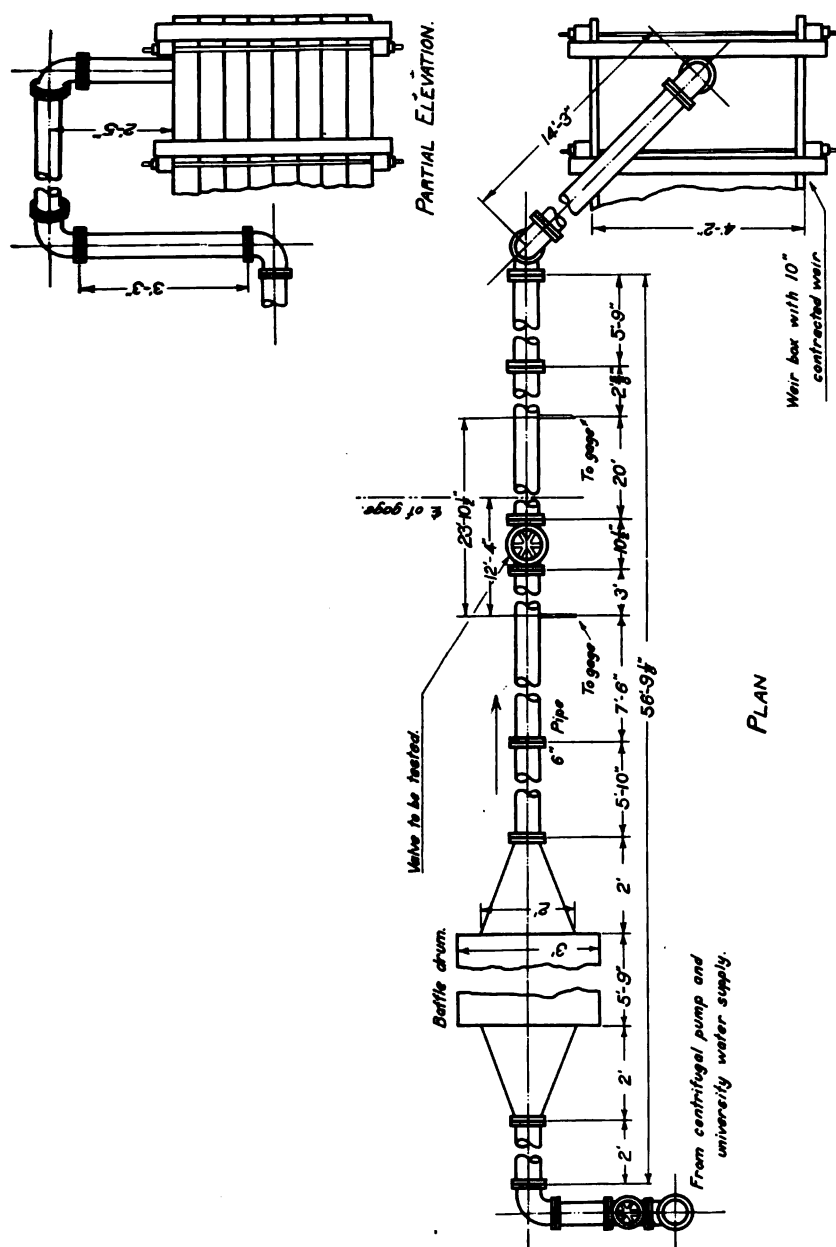
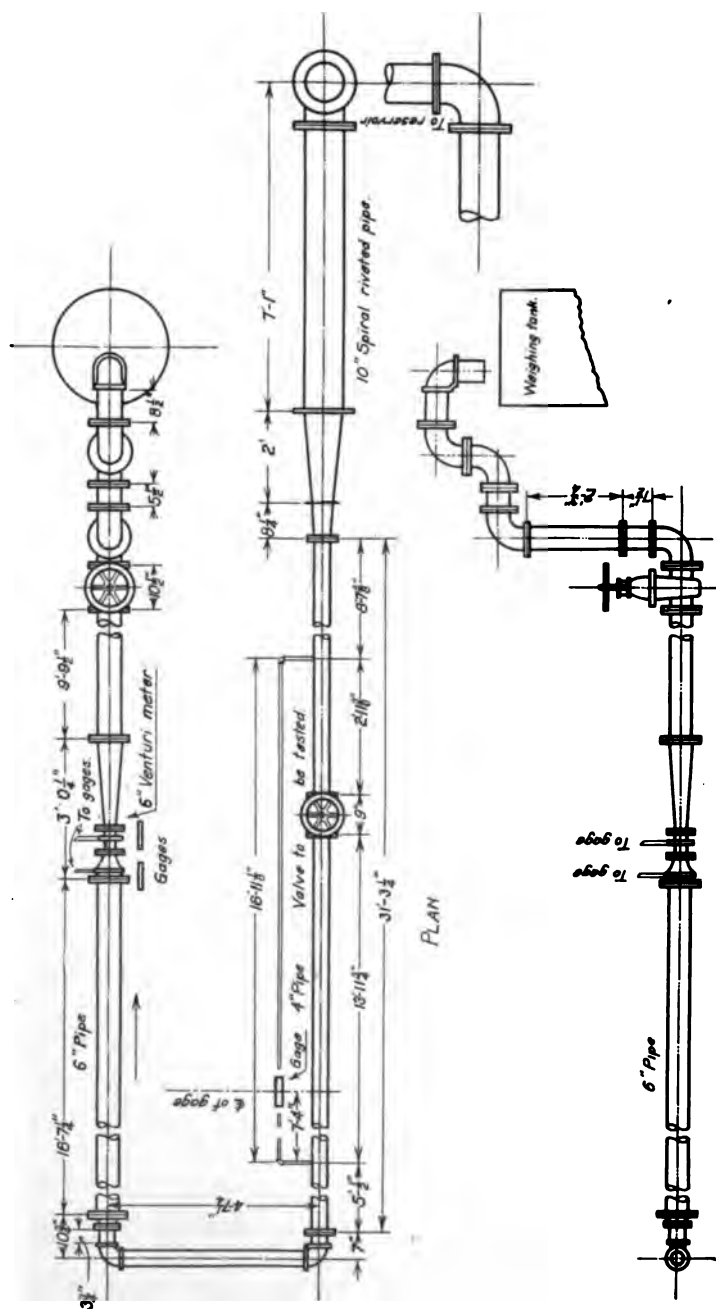


Fig. 3—Arrangement of Piping for Tests of 6-inch Gate Valves. Ten Valves Tested.



PARTIAL ELEVATION

Fig. 4—Arrangement of Piping for Tests of 4-inch Gate Valves. Three Valves Tested.

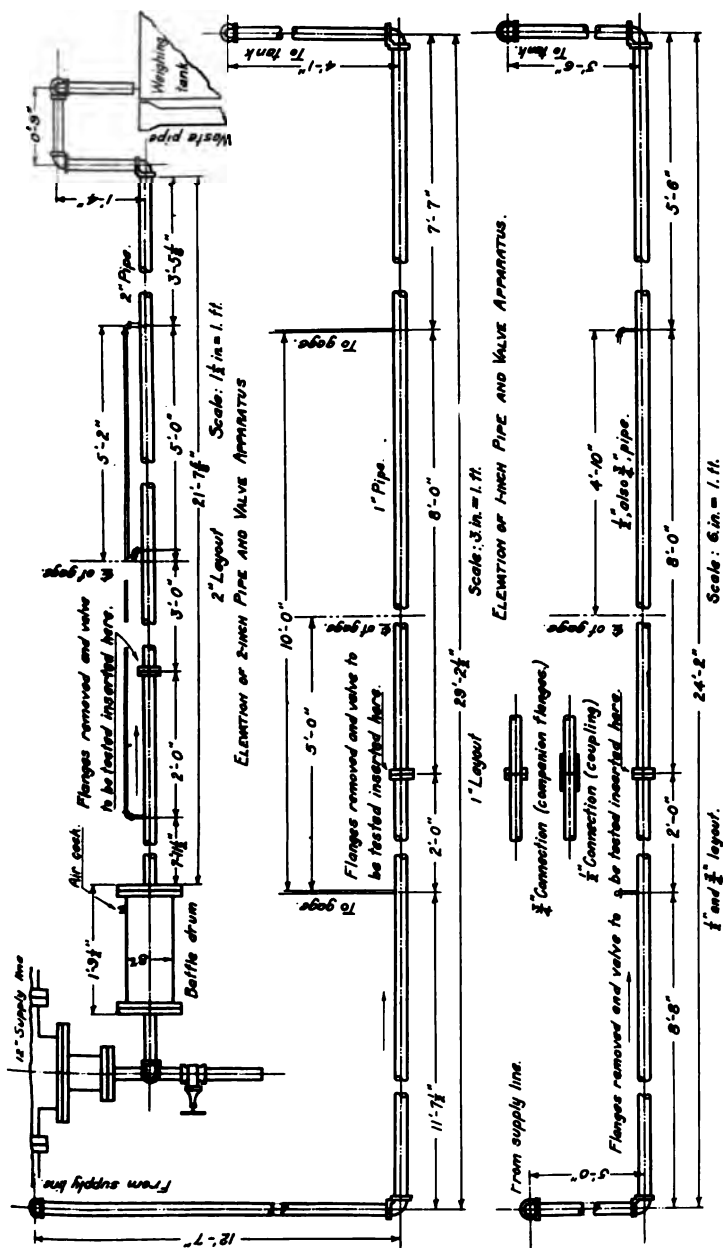


Fig. 5—Arrangement of Piping for Tests of $\frac{1}{2}$, $\frac{3}{4}$, 1, and 2-Inch Gate and Globe Valves.

3- $\frac{1}{4}$ ", 3- $\frac{3}{4}$ ", 3-1", and 4-2" Gate Valves Tested.
 3- $\frac{1}{2}$ ", 3- $\frac{3}{4}$ ", 3-1", and 6-2" Globe Valves Tested.

Gage Connections.—The piezometer openings generally consisted of $\frac{1}{4}$ -inch pipe tapped into the main on a horizontal diameter and with the end carefully made flush with the inner surface of the main pipe. For the $\frac{1}{2}$ -inch and $\frac{3}{4}$ -inch diameter pipe $\frac{1}{8}$ -inch pipe was used as piezometer openings. In several of the earlier experiments special piezometer rings as shown in Fig. 6 were used.

The gage connections from the piezometer openings to the head gage consisted in each case of $\frac{1}{4}$ -inch pipe so installed as to have approximately the same length from both the downstream and upstream piezometer openings to the gage itself. They were also arranged to slope continuously toward high points at which an ample sized air cock was placed to permit thorough flushing of the pipe connections.

Gage.—The head gage consisted of two 6-foot glass tubes placed on either side of a vertical scale, and joined at the top with $\frac{1}{4}$ -inch pipe fittings. See Fig. 7, page 20.

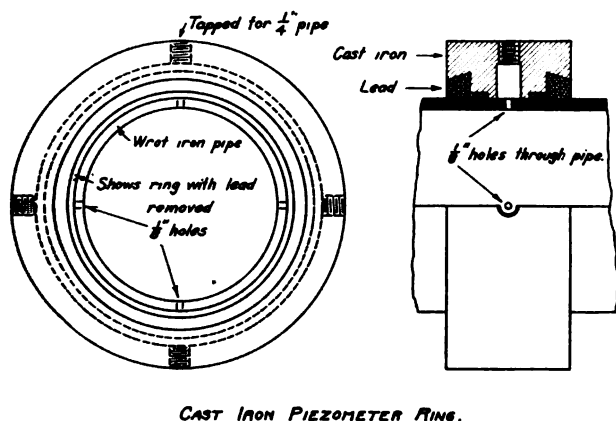


Fig. 6—Special Piezometer Ring.

The lower ends of the glass tubes were connected to the up and down stream piezometer openings. An auxiliary pipe led from the upper connecting link downward to an ordinary pneumatic-tire pump. This connection was provided with a tee and valve opening into the air. The air pump was used to maintain an air pressure on the two water columns so as to depress them to a convenient point for reading. As this pressure was the same on both columns it did not affect their difference. The scale had divisions to one-hundredth of a foot and a venier enabling direct readings to one-thousandth of a foot to be taken.

Pipe Connections.—When testing for pipe friction in the $\frac{1}{2}$ -inch pipe an ordinary pipe coupling was substituted for the test valve. The two pipe ends were carefully faced and threaded so as to butt together at the center of coupling. For all other sizes of pipe, flange unions were used for joints within the gage length.

The 12-inch pipe had two extra joints within the gage length. All other pipes had but the one joint which was provided for the valve to be tested.

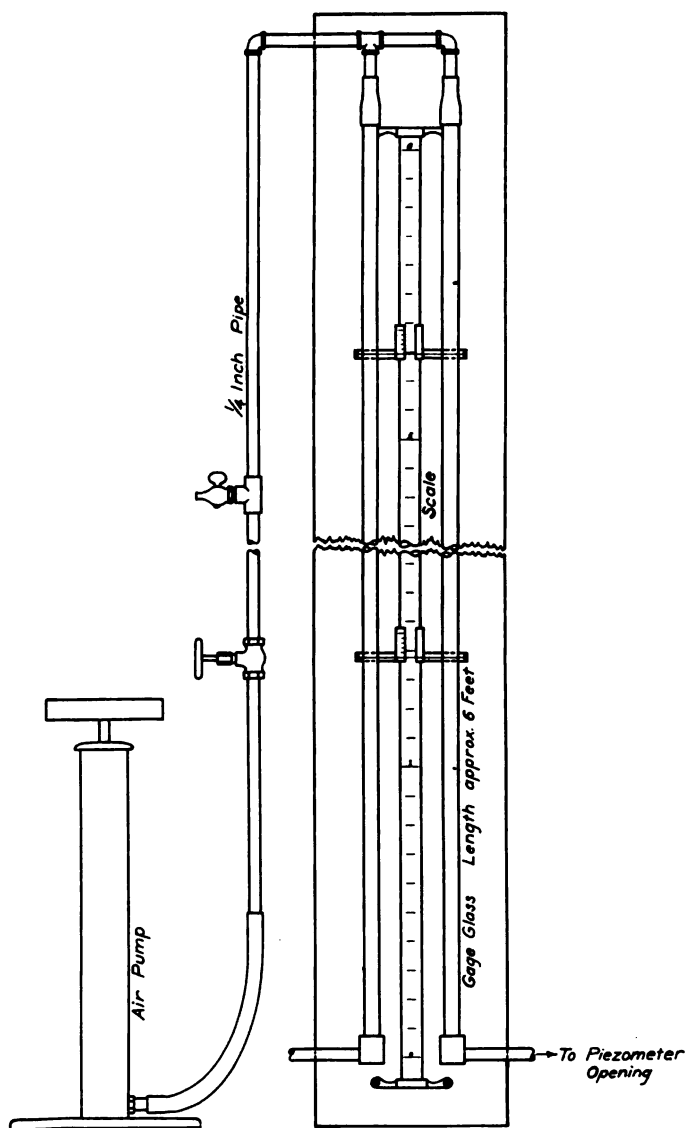


Fig. 7—Drawing of Gage Used to Measure Difference of Heads Between Two Points.

Riser.—A riser of a foot or more in height was placed in the line near the discharge end to insure a full flow through the pipe at all times. A ventilating device was usually installed at the top of this riser to prevent the possible creation of a partial vacuum under the higher rates of flow.

Valves.—Figures 8 to 25 show photographs and drawings of valves tested. All valves and pipes were ordinary stock material, tested as purchased, and new unless otherwise indicated. Valves above 2 inches in diameter had flange connections. Those 2 inches and under had screw connections except for one 2-inch valve with flange connections.

The 12-inch valve and one of the two 8-inch valves were new List 59½ OS Chapman² rising stem gate valves. Figures 8, 9, and 10 show photographs and drawing of them.

The second 8-inch valve tested was manufactured by the Crane Company³ and is their old style No. 461 with non-rising stem. A photograph and a drawing are shown in figures 10 and 11. This valve had been used on the discharge line of a centrifugal pump for nine years but aside from a slight coat of rust was in as good condition as when new.

Of the ten 6-inch gate valves, numbers 0, 2, 3, and 4 were new when tested. Number 5, shown in figure 12, was an old unmarked valve which had been completely submerged for ten to twelve years though little used. It was well rusted all over but unworn, the edges of the seats, disc, etc., being practically as well defined as in a new valve. The other 6-inch valves had

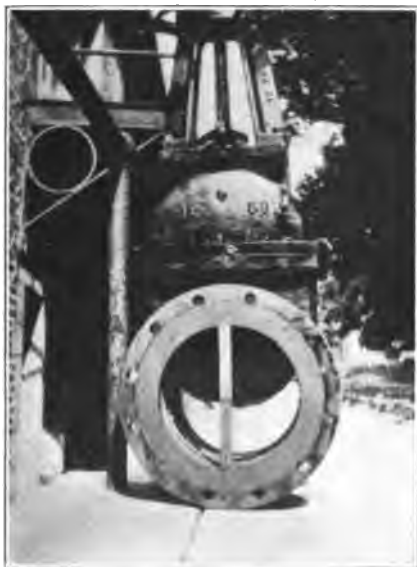


Fig. 8—Photograph of 12-inch Gate Valve Tested. List 59½—OS Chapman Valve Mfg. Co.

²Chapman Valve Mfg. Co., Indian Orchard, Mass.

³Crane Co., Chicago, Ill.

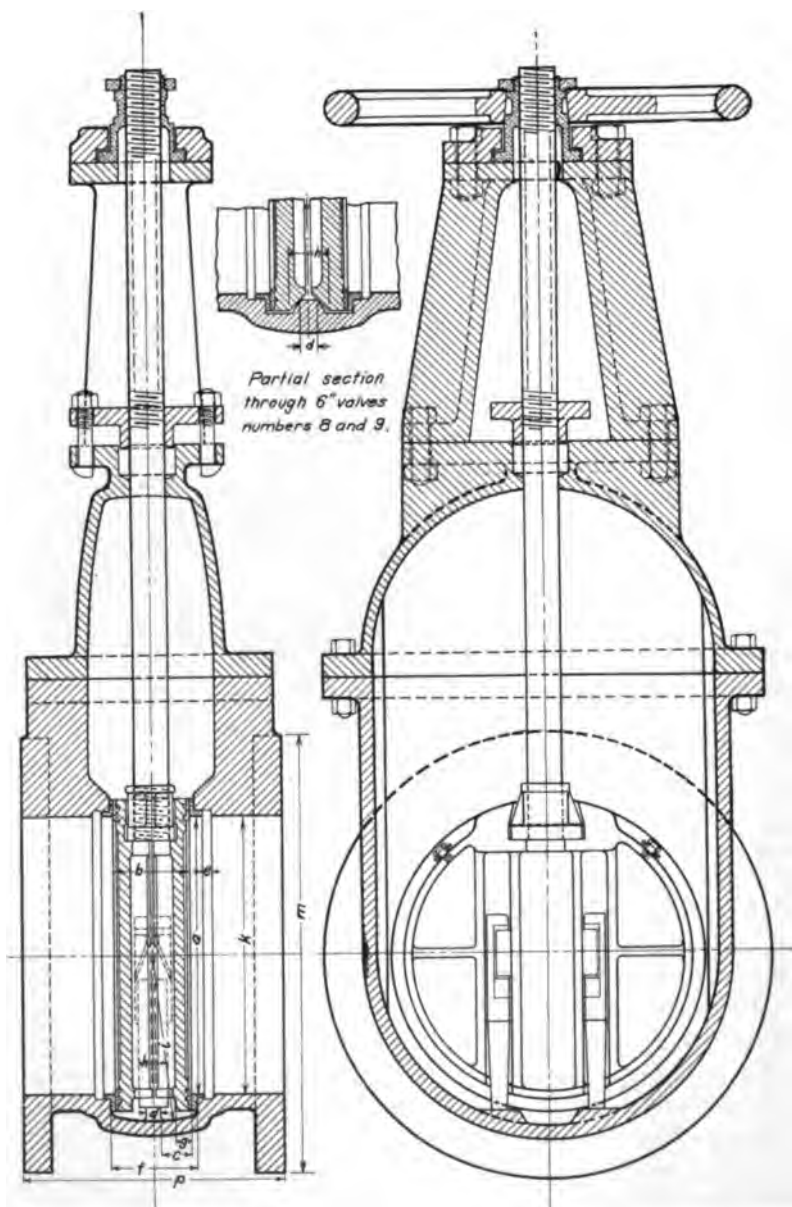


Fig. 9—Drawing of 6, 8, and 12-inch Chapman Gate Valves. Dimensions Given in Table 1, Page 24.

been used but were in good condition and unworn. Numbers 0, 1, 6, and 7 were Crane No. 461 similar to figure 13A. Numbers 2, 3, and 4 were Crane No. 461 similar to figure 13B. Numbers 8 and 9 were Chapman non-rising stem valves similar to figure 13C, page 28.

Dimensions for 6-inch valves in Table 1 refer to figures 9 and 11. The three 4-inch valves tested are shown in figure 14, page 28. Number 0 was a non-rising stem Crane old style No. 461. Numbers 1 and 2 were Crane rising stem new style No. 461. All three were new when tested. Dimensions for the 4-inch valves in Table 1 refer to figure 11, page 24.

All valves tested below 4 inches in diameter were the screw connection type except one 2-inch flanged connection gate valve. This was a Crane old style No. 461, new when tested. The bonnet and gland dimensions were proportionately larger than shown in figure 11, page 24, but this drawing applies for the essential details.

The series of Scott⁴ gate valves tested in 1915 is shown in the photograph of figure 15 and dimensions are given in Table 1, opposite page 24. Letters at the head of dimension columns refer to the sectional drawing of figure 16, page 30.

Figure 17, page 29, is a photograph of part of the 2-inch gate and globe valves tested. Numbers 7 and 8 were Crane gate valves that had been used previous to testing. Letters at the top of dimension columns in Table 1, opposite page 24, refer to the drawing of figure 18, page 30. Number 1, figure 17, was one of two similar Crane 2-inch globe valves tested. Figure 19, page 31, shows a cross-section of this and the smaller Crane globe valves.



Fig. 10—Photograph of 8-inch Gate Valves Tested. List 59½-OS Chapman Valve Mfg. Co. No. 461 (Old Style) Crane Co.

⁴ Scott Valve Mfg. Co., Detroit, Mich.

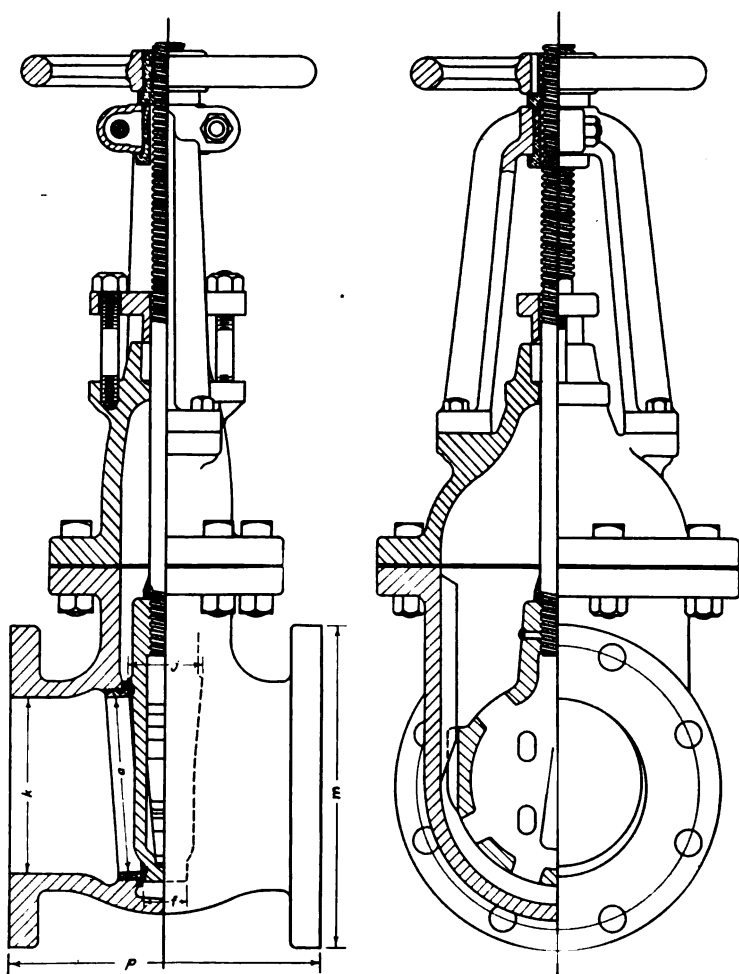


Fig. 11—Drawing of 4, 6 and 8-inch Crane Gate Valves. Dimensions Given in Table 1 Page 24.

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DIMEN

No. of Valve	Size (Inches)	Make and Type	Connection	Condition : New or Used	When Tested	Seats : Parallel or Wedge	Diameter of Disc.	
4	1/2	Crane No. 440	Screw	New	1919, June, July	Parallel	1 1/2	
5	1/2	Crane No. 440	Screw	New	1919, July	Parallel	1 1/2	
6	1/2	Crane No. 440	Screw	New	1919, July	Parallel	1 1/2	
00	1/2	Crane No. 440	Screw	New	1915, May	Wedge	0.84	
4	3/4	Scott	Screw	New	1919, Aug.	Parallel	1.00	
5	3/4	Crane No. 440	Screw	New	1919, Aug.	Parallel	1.00	
6	3/4	Crane No. 440	Screw	New	1919, Aug.	Parallel	1 1/2	
00	3/4	Crane No. 440	Screw	New	1919, Aug.	Parallel	1 1/2	
4	1	Crane No. 440	Screw	New	1915, April	Wedge	1.04	
5	1	Crane No. 440	Screw	New	1919, Aug.	Parallel	1 3/4	
6	1	Crane No. 440	Screw	New	1919, Aug.	Parallel	1 3/4	
00	1	Crane No. 440	Screw	New	1919, Aug.	Parallel	1 3/4	
4	1	Crane No. 440	Screw	New	1919, Aug.	Parallel	1 3/4	
5	1	Crane No. 440	Screw	New	1919, Aug.	Parallel	1 3/4	
6	1	Crane No. 440	Screw	New	1919, Aug.	Parallel	1 3/4	
00	1	Crane No. 440	Screw	New	1915, March	Wedge	1.30	
00	1 1/2	Scott	Screw	New	1915, April	Wedge	1.94	
7	2	Crane	Screw	Used	1919, June	Parallel	2 1/2	
8	2	Crane	Screw	Used	1919, June	Parallel	2 1/2	
00	2	Crane	Screw	Used	1919, June	Wedge	2 3/8	
00	2	Crane	Screw	New	1915, Feb.	Wedge	2 3/8	
0	2	Crane No. 461	Flange	New	1914, July-Aug.	Wedge	5	
0	2	Crane No. 461	Flange	New	1914, May, July	Wedge	5	
1	4	Crane	Flange	New	1919, May	Wedge	4 1/2	
2	4	Crane	Flange	New	1919, July	Wedge	7	
0	6	Crane No. 461*	Flange	New	1914, May, Aug.	Wedge	7	
1	6	Crane No. 461*	Flange	New	1918, July	Wedge	7	
2	6	Crane No. 461†	Flange	New	1918, July	Wedge	7	
3	6	Crane No. 461†	Flange	New	1918, July-Aug.	Wedge	7	
4	6	Crane No. 461†	Flange	New	1918, Aug.	Wedge	7	
5	6	Unknown	Flange	Used	1918, Oct.-Nov.	Parallel	7	
6	6	Crane No. 461*	Flange	Used	1919, Jan.-Feb.	Wedge	7	
7	6	Crane No. 461*	Flange	Used	1919, Feb.-Mar.	Wedge	7	
8	6	Crane No. 461*	Flange	Used	1919, March	Parallel	6 3/8	
9	6	Chapman	Flange	Used	1919, Apr.-May	Parallel	6 3/8	
1	8	Chapman	Flange	New	1919, July	Parallel	9 1/4	
2	8	Crane	Flange	Used	1919, July	Wedge	9 1/4	
1	12	Chapman	Flange	New	1919, June	Parallel	13 1/2	

*Old style.
†New style.

Figure 21, page 31, is an enlarged section of the 2-inch globe valve disc. Figure 22 is an enlarged section of the valve disc of the 1-inch Crane globe valves tested. Figure 20 similarly is for the Crane $\frac{1}{2}$ -inch and $\frac{3}{4}$ -inch globe valves.

Number 3 and number 5, figure 17, are each photographs of one of two similar new 2-inch globe valves that were tested. Number 3 is a Scott valve of which figure 23, page 32, is a sectional drawing. Number 5, the Lunkenheimer⁵ valve is shown in section in figure 24, page 32. The photograph of figure 25, page 33, shows with the exception of one $\frac{1}{2}$ -inch valve, the $\frac{1}{2}$, $\frac{3}{4}$, and 1-inch Crane gate and globe valves that were tested. Letters at the heads of the dimension columns in Table 2 refer to figure 19, page 31.



Fig. 12—Photograph of 6-inch Gate Valve No. 5. Non-rising Stem. Dimensions Given in Table 1, Page 24.

Pipe.—Standard wrought iron or steel gas and water pipe was used for all tests. That within the gage length (from upstream piezometer opening to downstream piezometer opening) was carefully selected. The joints were made with gaskets of rubberoid roofing, proper care being taken to have the gasket slightly large and to have the two companion flanges in the same position relative to each other each time the joint was made. Usually the gasket would be saved when breaking a joint

⁵ The Lunkenheimer Co., Cincinnati, Ohio.

TABLE 2
DIMENSIONS AND DESCRIPTIONS OF GLOBE VALVES
(All dimensions are in inches.)

Size (ins.)	Make or Manufacturer	Conne- tion	When Tested	Condi- tion: New or Used	a	b	c	d	e	f	k	h	p
1	Crane	Screw	1919, May 2-7	Used	2 1/4	1 1/8	1 3/4	1	2 1/8	1 7/8	2 1/2	2 1/2	5 3/4
2	Crane	Screw	1919, May 14-16	Used	2 1/4	1 3/4	1 3/4	1 1/4	2 1/8	1 7/8	2 1/2	2 1/2	5 3/4
3	Scott	Screw	1919, May 19-20	New	1 7/8	1 3/4	1 3/4	5 7/8	1 3/8	1 3/4	2 1/2	2 1/2	4 1/2
4	Scott	Screw	1919, May 21	New	2 1/4	1 1/4	1 3/4	3 3/4	1 1/8	1 3/4	2 1/2	2 1/2	4 1/2
5	Lunkenhimer	Screw	1919, May 21-22	New	2 3/4	1 1/8	1 3/4	3 3/4	1 1/8	1 3/4	2 1/2	2 1/2	5 3/4
6	Lunkenhimer	Screw	1919, May 23	New	2 3/4	1 1/8	1 3/4	3 3/4	1 1/8	1 3/4	2 1/2	2 1/2	5 3/4
1	Crane	Screw	1919, July 21-22	New	1	3/4	3 3/4	1 1/8	1 3/4	6 1/4	1 3/4	1 3/4	3 3/4
1	Crane	Screw	1919, July 21	New	1	3/4	3 3/4	1 1/8	1 3/4	6 1/4	1 3/4	1 3/4	3 3/4
3	Crane	Screw	1919, Aug. 1	New	1	3/4	4 1/4	1 1/8	1 3/4	6 1/4	1 3/4	1 3/4	3 3/4
3/4	Crane No. 12	Screw	1919, Aug. 22	New	3/4	1/2	1 1/8	1 1/8	1 3/8	3 3/4	1 1/2	1 1/2	3 3/4
1	Crane No. 12	Screw	1919, Aug. 22	New	1	1/2	1 1/8	1 1/8	1 3/8	3 3/4	1 1/2	1 1/2	3 3/4
3/4	Crane No. 12	Screw	1919, Aug. 23	New	3/4	1/2	1 1/8	1 1/8	1 3/8	3 3/4	1 1/2	1 1/2	3 3/4
1	Crane	Screw	1919, May 16-17	Used	1	3/4	2 3/4	1 1/8	1 3/4	4 9/8	1 3/4	1 3/4	2 5/8
1/2	Crane	Screw	1919, May 23-June 6	Used	1/2	3 1/4	2 1/4	1 1/4	3 9/8	4 9/8	1 3/4	1 3/4	2 5/8
1/2	Crane	Screw	1919, June 7-19	Used	1/2	3 1/4	2 1/4	1 1/4	3 9/8	4 9/8	1 3/4	1 3/4	2 5/8

TABLE 3
DIMENSIONS AND DESCRIPTION OF PIPES USED IN GAGE LENGTHS

Size (Inches)	Condition	Gage Length	Joints in Gage Length	Cross Sectional Area of Pipe	
				Sq. Feet	How Obtained
$1\frac{1}{2}$	New and clean.....	10'-10"	1	.00206	Filling with water and weighing.
$\frac{3}{4}$	New and clean.....	10'-0"	1	.00358	Filling with water and weighing.
1	New and clean.....	10'-0"	1	.00604	Filling with water and weighing.
$1\frac{1}{2}$	No record.....	20'-0"	1	.01415	Nominal area from handbook.
2	New and clean.....	9'-11 $\frac{3}{4}$ "	1	.02338	Filling with water and weighing.
4	Used pipe but clean and in good condition. Longer section considerably rusted.....	16'-11 $\frac{1}{8}$ "	1	.08855	By caliper pipe ends at joint.
6	Used pipe but clean and in good condition.....	23'-0"	1	.2062	Filling with water and weighing.
8	Old pipe formerly used for steam main. Rusted. Clean on inside.....	22'-0"	1	.358	By caliper pipe ends at joint.
12	New and clean.....	33'-0"	3	.7854	By caliper pipe ends at joint.

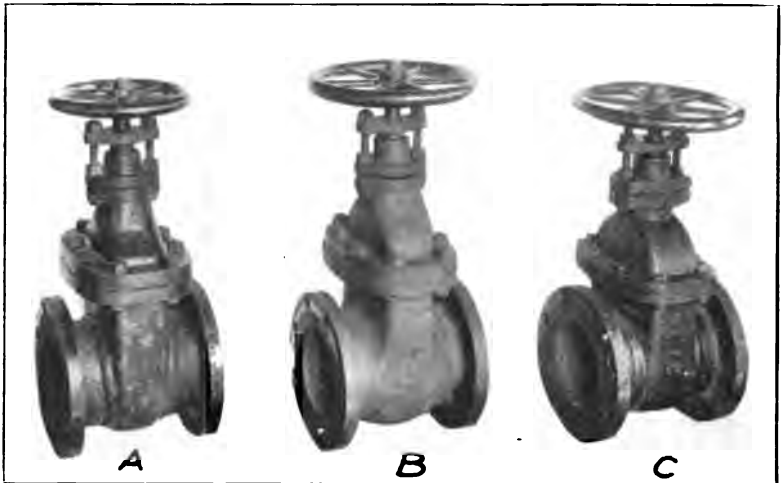


Fig. 13—Photograph of 6-inch Gate Valves. A, Old Style Crane No. 461 Non-rising Stem. B, New Style Crane No. 461 Non-rising Stem. C, Chapman Non-rising Stem.

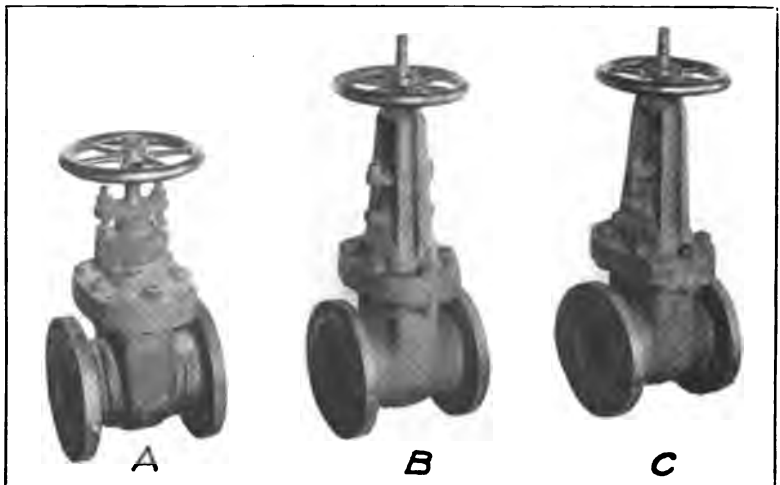


Fig. 14—Photograph of 4-inch Gate Valves Tested. A, Old Style Crane No 461 Non-rising Stem. B and C, Crane Rising Stem.

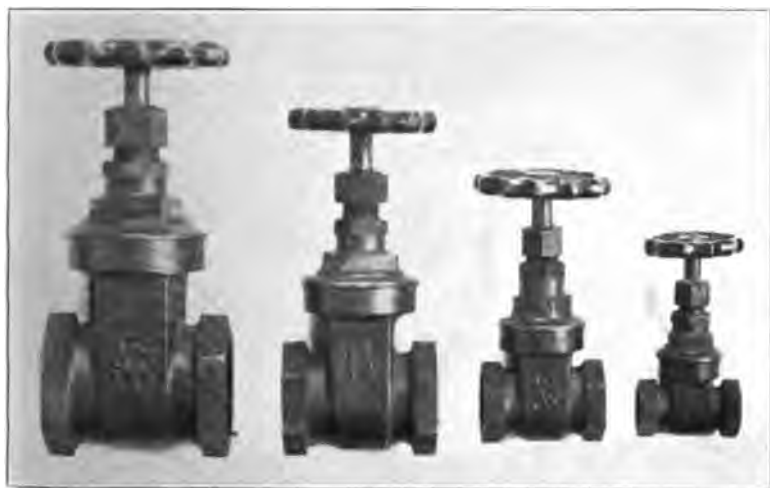


Fig. 15—Photograph of Scott Gate Valves Tested in 1915.



Fig. 17—Photograph of Some of 2-inch Gate and Globe Valves Tested.
No. 1—Crane Globe Valve. No. 5—Lunkenheimer Globe Valve.
No. 3—Scott Globe Valve. No. 7—Crane Gate Valve.
No. 8—Crane Gate Valve.

and remain undisturbed on one companion flange. To aid in this separation the gasket was stuck to one flange in

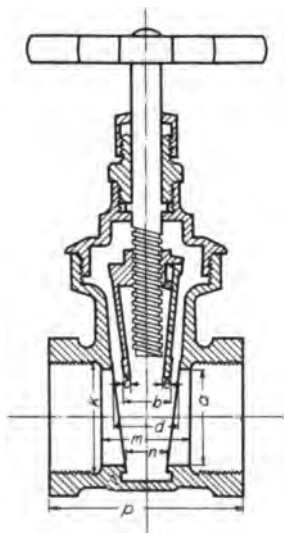


Fig. 16—Section Through Scott Gate Valves. Sizes Tested; $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, and 2-inch Diameter. Dimensions Given in Table 1, Page 24.

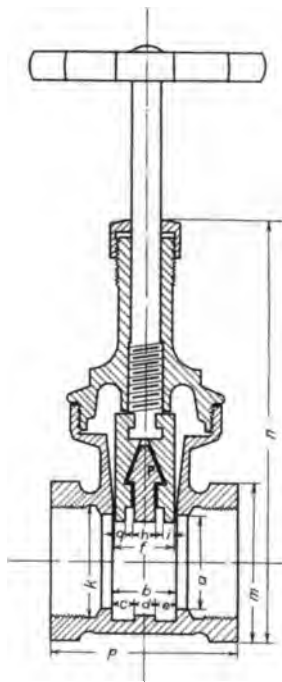


Fig. 18—Section Through Small Crane Gate Valves. Sizes Tested $\frac{1}{2}$, $\frac{3}{4}$, 1, and 2-inch Diameter. Dimensions Given in Table 1, Page 24.

making the joint with asphalt paint and the other side oiled with machine oil. Small dowel pins were put into the smaller flanges to make them come together in exactly the same way each time.

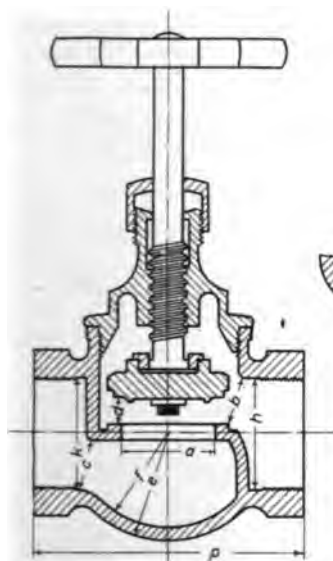


Fig. 19—Section Through Crane Globe Valves. Sizes Tested: $\frac{1}{2}$, $\frac{3}{4}$, 1, and 2-inch Diameter. Enlarged Section of Valve Disc and Seat Shown in Figures 20, 21, and 22, Page 31. Dimensions Given in Table 2, Page 26.

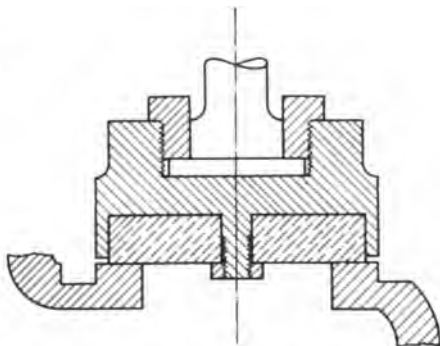


Fig. 20—Enlarged Section Through Valve Disc and Seat for $\frac{1}{2}$ and $\frac{3}{4}$ -inch Crane Globe Valves Tested.

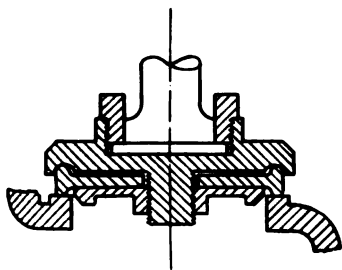


Fig. 21—Enlarged Section Through Valve Disc and Seat for 2-inch Crane Globe Valves Tested. Dimensions Given in Table 2, Page 26.

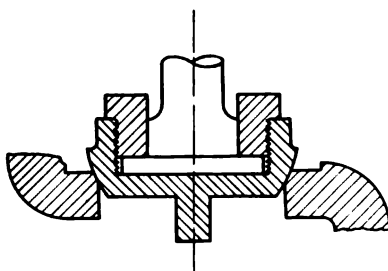


Fig. 22—Enlarged Section Through Valve Disc and Seat for 1-inch Crane Globe Valves Tested. Dimensions Given in Table 2, Page 26.

METHODS OF EXPERIMENTATION

Since normal pipe friction in the gage length had to be deducted from the total loss obtained when a valve was being tested, two tests were necessary to get the net loss caused by the valve. The procedure was the same during the two tests,

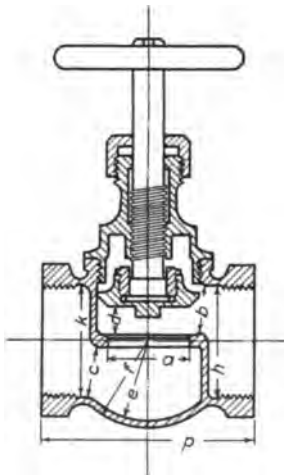


Fig. 23—Section Through 2-inch Scott Globe Valves Tested. Dimensions Given in Table 2, Page 26.

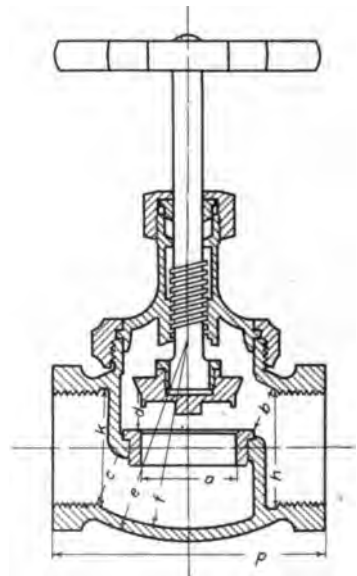


Fig. 24—Section Through 2-inch Lunkenheimer Globe Valves Tested. Dimensions Given in Table 2, Page 26.

the only difference in the apparatus being the presence or absence of the valve, to be tested, in the gage length.

In starting a run, the gage connections were first thoroughly flushed and the gage tested for correctness. The rate of flow through the apparatus was then regulated by the supply valve, upstream from the test valve, until some predetermined differ-

ence of head was shown on the head gage and time given for conditions of flow to become steady. A run was taken when everything was steady, the time of run varying according to circumstances but usually five minutes in length. The observer read the head gage as frequently as possible during the run. Globe valves were tested for one opening. Gate valves were



Fig. 25—Photograph of $\frac{1}{2}$, $\frac{3}{4}$, and 1-inch Crane Gate and Globe Valves Tested. One $\frac{1}{2}$ -inch Globe Valve Missing. See Tables 1 and 2, Pages 24 and 26.

tested for six different openings. For each opening a series of about ten runs was made with the head differences so chosen as to give runs at velocities ranging from two to fifteen feet per second.

Before placing a valve in the line to be tested, a flange union was substituted in its place and the pipe friction loss in the gage length determined for various rates of flow. From these data a

LOSS OF HEAD IN VALVES— $\frac{1}{2}$ -INCH PIPE FRICTION RUNS

Data by D. H. Stiles. June 20, 1919

Area of pipe cross-section (by water volume) = 0.00206 sq. ft.

Run	Time in Min- utes	Water Gage			Weight of Water lbs.	Run	Time in Min- utes	Water Gage			Weight of Water lbs.	
		Left ft.	Right ft.	Diff. ft.				Left ft.	Right ft.	Diff. ft.		
1	8:14	1.731	2.041	0.310	222.5	3	8:50	1.500	2.698	1.198	200.0	
		1.739	2.028	0.289				1.515	2.711	1.196		
		1.747	2.027	0.280				1.495	2.735	1.240		
		1.749	2.023	0.274				1.511	2.755	1.244		
		1.759	2.019	0.260				1.514	2.762	1.248		
		1.749	2.032	0.263				1.511	2.759	1.248		
		1.762	2.022	0.260				1.513	2.769	1.256		
		1.761	2.015	0.254				1.507	2.795	1.288		
		1.757	2.009	0.252				1.508	2.801	1.293		
		1.756	2.010	0.254				1.491	2.698	1.207		
		1.764	2.038	0.274				1.280	2.822	1.542		
		1.764	2.037	0.273								
		1.767	2.037	0.270								
		1.769	2.041	0.272								
		1.765	2.040	0.275	406.2	4	10 min.	1.290	2.822	1.542	200.0	
		1.860	2.792	0.270				1.290	2.786	1.496		
1.861	2.820			0.932	1.245	2.757	1.512	200.0				
				0.935	1.285	2.753	1.468					
					1.263	2.895	1.532					
					1.253	2.821	1.568					
					1.290	2.865	1.575					

LOSS OF HEAD IN ½-INCH GATE VALVE NO. 5. ¾ OPENING

Data by C. D. Dietrich. July 22, 1919

Run	Time in Min- utes	Water Gage			Weight of Water lbs.	Run	Time in Min- utes	Water Gage			Weight of Water lbs.
		Left ft.	Right ft.	Diff. ft.				Left ft.	Right ft.	Diff. ft.	
1	26'-00"	2.319	2.894	0.575	50.17	5	30'-00"	1.507	4.530	3.023	61.48
		2.320	2.878	0.558				1.505	4.543	3.038	
		2.325	2.867	0.542				1.511	4.441	3.030	
		2.326	2.864	0.538				1.507	4.533	3.026	
		2.325	2.864	0.539				1.509	4.536	3.027	
		2.323	2.863	0.540				1.511	4.530	3.019	
2	31'-00"	2.325	2.864	0.539	93.62		35'-00"	1.508	4.532	3.024	168.0
		2.323	2.871	0.548				1.508	4.535	3.027	
		2.128	3.425	1.297				1.409	5.010	3.601	
		2.118	3.449	1.331				1.410	5.012	3.602	
		2.118	3.449	1.331				1.410	5.012	3.602	
		2.117	3.445	1.328				1.412	5.010	3.598	
2	48'-00"	2.117	3.445	1.328	93.62	6	40'-00"	1.413	5.003	3.590	168.
		2.117	3.445	1.328				1.409	5.010	3.601	

Fig. 26.—Sample Data Sheets Showing Manner of Recording Pipe Friction and Valve Loss Runs.

curve, similar to figure 27, page 36, was drawn between the loss of head and the square of the velocity. By plotting in this way the curve takes the form of a broken straight line which is most convenient both for checking the accuracy of the pipe friction data and for using later in reading off the pipe friction to be deducted from the valve runs to find the net valve loss.

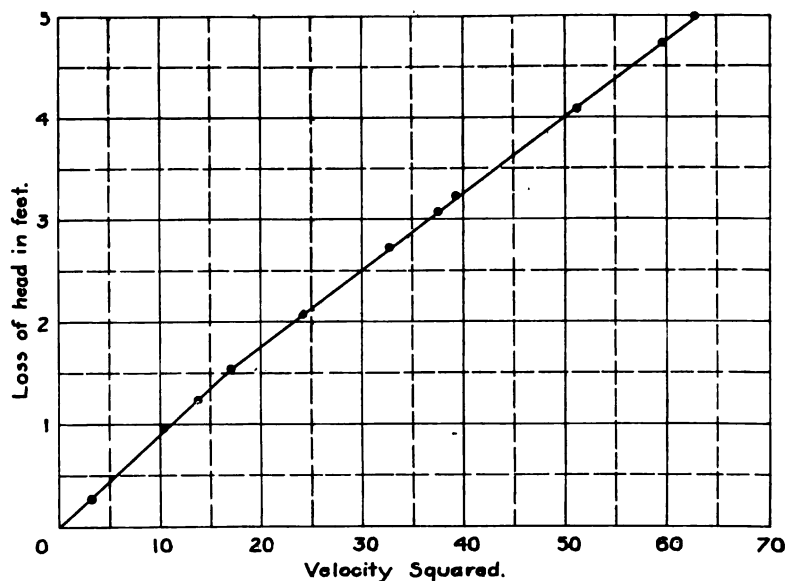


Fig. 27—Curve Showing Pipe Friction Loss in Gage Length for $\frac{1}{2}$ -inch Pipe. Test of June 20, 1919. Illustrates Method of Plotting Data from Which to Obtain Pipe Friction to Deduct from Valve in Line Runs.

When testing with the valve in, the data were plotted in a similar way before changing the valve opening for a new run to see that the data were consistent. Where discrepancies appeared the first run was continued until a satisfactory curve was obtained. Figure 28, page 37, is a sample curve of such a study. The curve for the loss of head due to the valve only has also been drawn in. This was obtained by deducting pipe friction read from curve of figure 27, for each velocity used, when

the valve was in the line, and plotting these differences which are the net valve losses of head. Sample data sheets are shown in figure 26, pages 34 and 35.

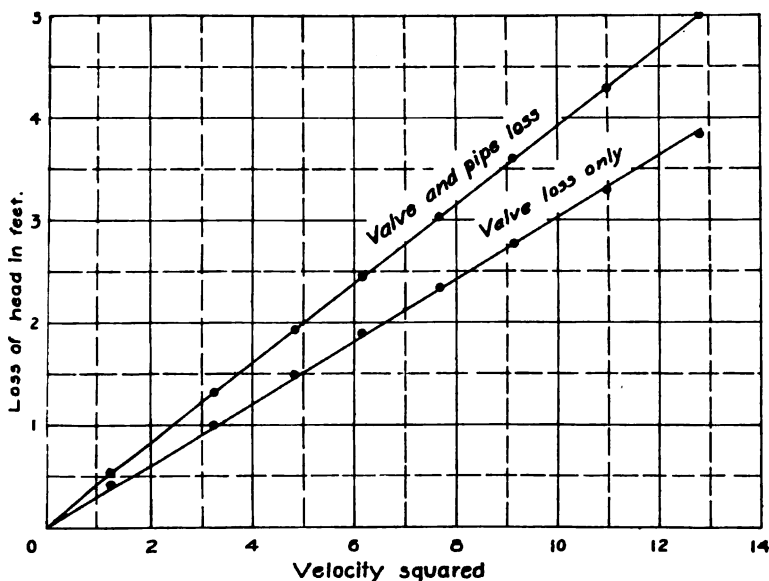


Fig. 28—Sample of Curves Plotted of Valve Runs to Determine Whether Data Were Consistent Before Changing Setting of Valve for New Run. Curve from $\frac{1}{4}$ -inch Gate Valve No. 5, Three-eighths Opening.

RESULTS

Pipes.—As noted in the introduction, it was necessary to obtain the frictional loss in a short length of pipe for each size of valve tested. The lengths varied from 10 to 33 feet and in most cases only one set up of a given size was used.

Table 3, page 27, gives data on the condition of the pipes, how the cross-sections were obtained, gage lengths, and other measurements.

It is recognized that results from single samples of pipe of limited length are but meager data. The very fact that but one joint was in the gage length and this carefully made with a flange union, however, probably accounts for the consistency of the data and justifies some study of them.

The data for the different sizes form a more consistent series than any published data on wrought iron pipe at hand. There being only one joint in the gage length, and this well made, it is probable the series represents more nearly the actual pipe friction than where longer lengths are used with pipe joints as ordinarily made. Undoubtedly disturbances caused by poorly made joints can considerably change losses. The data are given in Tables 13 to 21, pages 83 to 93.

Figures 29 and 30 on pages 39 and 40 are logarithmic plottings of the 1918-1919 data showing the variation of loss of head, per 100 feet of pipe, with velocity. A straight line is drawn for each pipe size averaging the experimental points. Table 4, page 42, gives the equations of these lines and also those obtained for the experiments of earlier years.

Usually two series of runs were made for any given size; one just previous to the testing of the valves and another at the end of the valve test runs.

With the 1918-1919 6-inch valve tests the experimentation continued at intervals for a year. Several pipe friction runs were therefore made on the 6-inch pipe. These showed increased loss, for a given velocity, with the lapse of time, and to bring this out on figure 29 three curves are shown for the three dif-

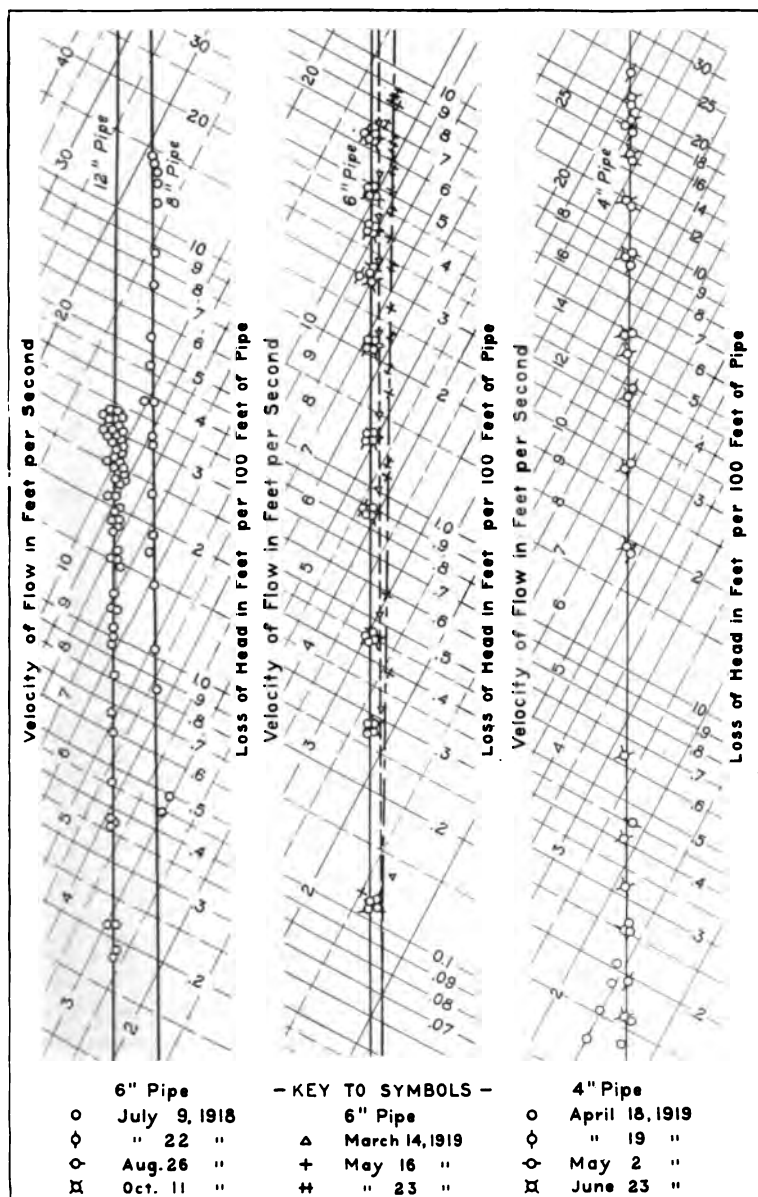


Fig. 29—Logarithmic Plotting of Pipe Friction Loss for Different Velocities of Flow. 4, 6, 8, and 12-inch Pipe.

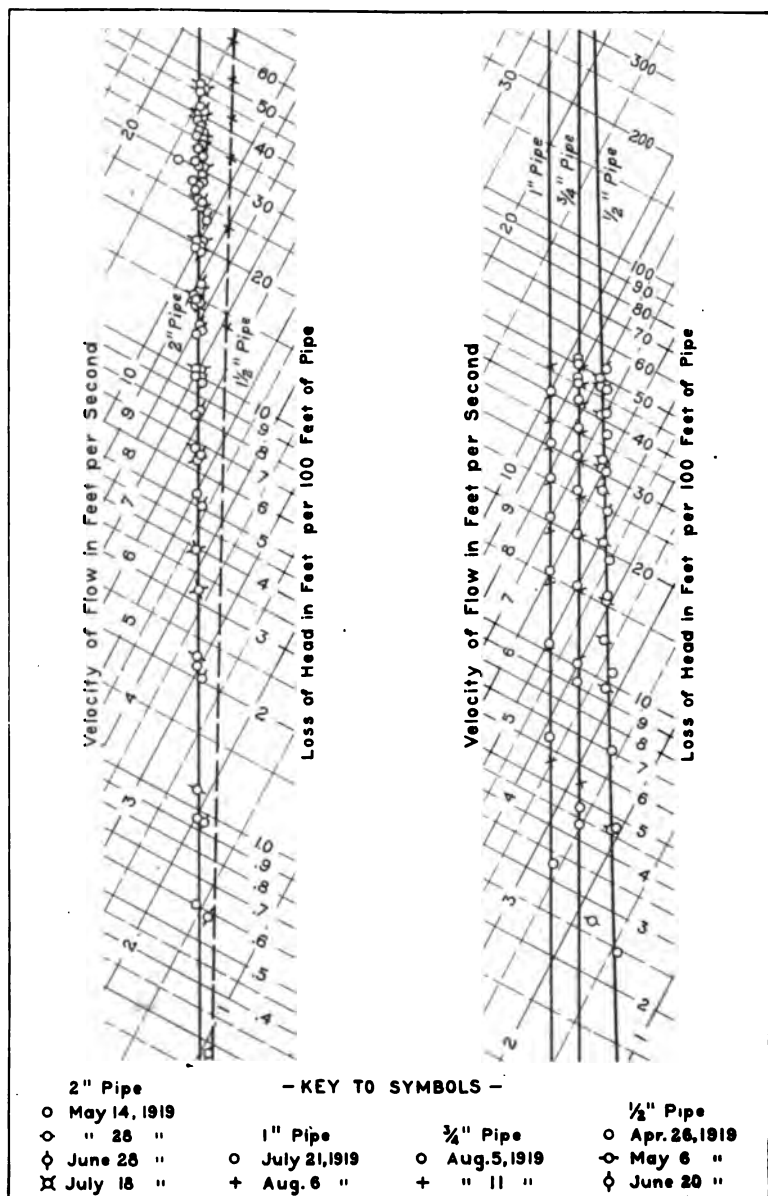


Fig. 30—Logarithmic Plotting of Pipe Friction Loss for Different Velocities of Flow. $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, and 2-inch Pipe.

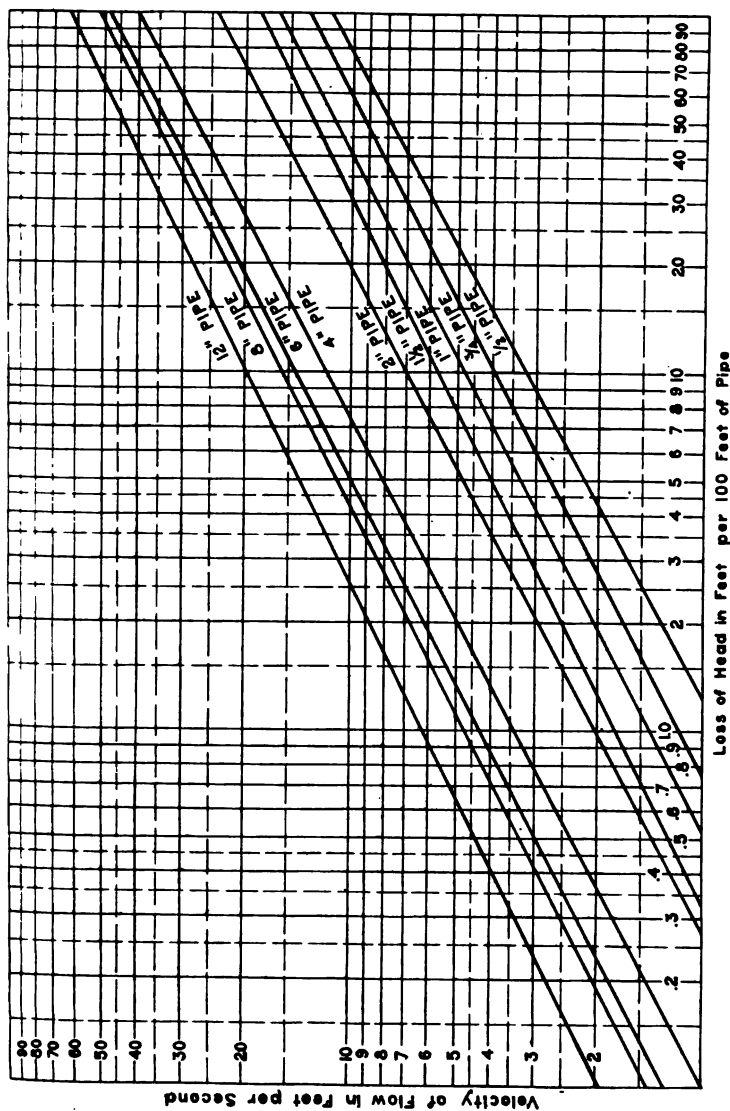


Fig. 31—Logarithmic Curves for All Sizes of Pipe Tested.

TABLE 4

EQUATIONS FOR LOSS OF HEAD IN PIPE FROM LOGARITHMIC PLOTTING OF EXPERIMENTAL RESULTS SHOWN IN FIGURES 29 AND 30, PAGES 39 AND 40.

Size Inches	1913-1914	1914-1915	1918-1919
$\frac{1}{2}$		$H = 3.176 v^{1.818}$	$H = 1.205 v^{1.805}$
$\frac{3}{4}$		$H = .860 v^{1.784}$	$H = .736 v^{1.90}$
1		$H = .670 v^{1.81}$	$H = .5133 v^{1.892}$
$1\frac{1}{2}$		$H = .317 v^{1.978}$	
2	$\left\{ \begin{array}{l} H = .2025 v^{1.918} \\ *H = .2207 v^2 \\ H = .1225 v^{1.90} \end{array} \right.$	$\left\{ \begin{array}{l} \dagger H = .3043 v^{1.869} \\ \dagger H = .2315 v^{2.01} \\ H = .0997 v^{1.952} \end{array} \right.$	$H = .271 v^{1.87}$
4			$H = .0995 v^{1.867}$
6			$H = .0621 v^{1.906}$
8			$H = .0494 v^{1.923}$
12			$H = .0254 v^{1.985}$

* Summer 1914.

† Logarithmic plotting gave break in curve at point $\left\{ \begin{array}{l} v = 7.2 \\ H = 12.18 \end{array} \right\}$ lower part of curve having first equation and upper part second equation.

ferent periods of time that runs were made. It will be noted the aging of the pipe in addition to shifting the straight line curve also gave it different slopes.

On figure 31, page 41, the logarithmic lines for each size of pipe tested have been assembled for better comparison.

No final deductions can be obtained from so limited a series of tests but a reasonable agreement can be noted.

The value of the exponent n of v varies between the limits of $n = 1.805$ and $n = 1.985$, a rough average value being $n = 1.90$.

If the slope of the different straight line curves is changed enough to make $n = 1.90$ in each case, values of m can be determined that will give reasonable agreement between the equations and the experimental points. Such values of m have been determined and are tabulated in Table 5, page 43.

When these values of m are plotted on logarithmic paper against the pipe diameters they fall along a straight line having the equation $m = \frac{.0319}{d^{1.16}}$. See figure 32, page 43.

Substituting this expression for m in the equation $H = mv^{1.9}$ there results $H = \frac{.0319}{d^{1.16}} v^{1.9}$.

It is found that the loss of head per hundred feet of pipe may be determined approximately for the ordinary ranges of velocity

TABLE 5

VALUES OF m IN $H = mv^{1.9}$ FOR A SERIES OF PIPE FRICTION CURVES
HAVING A FIXED SLOPE OF $n=1.9$.

Nominal Diameter of Pipe—Inches	Measured Diameter Feet	m
$\frac{1}{8}$	.05012	1.033
$\frac{3}{8}$	.0875	.7215
1	.0877	.5150
$1\frac{1}{2}$	.1342	.4065
2	.1723	.2510
4	.3346	.1225
6	.5122	.0580
8	.6752	.0516
12	1.0000	.0310

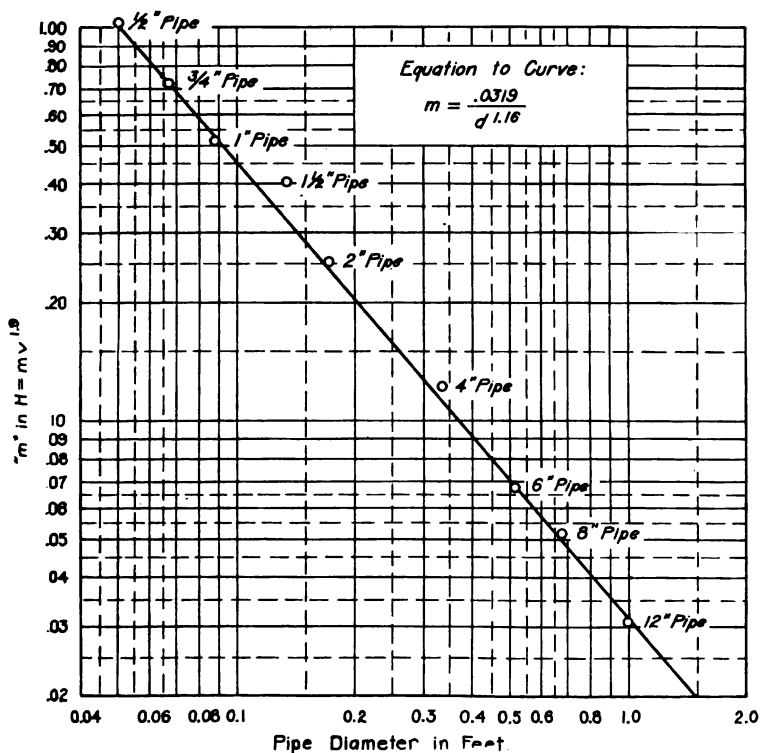


Fig. 32—Logarithmic Curve Between Diameter of Pipe and Coefficient in the Equation $H = mv^{1.9}$.

and for the sizes of pipe tested by the formula $H = \frac{.0319}{d^{1.16}} v^{1.9}$

In order to compare this formula with other experimental data on wrought iron pipe and with some of the formulas used in

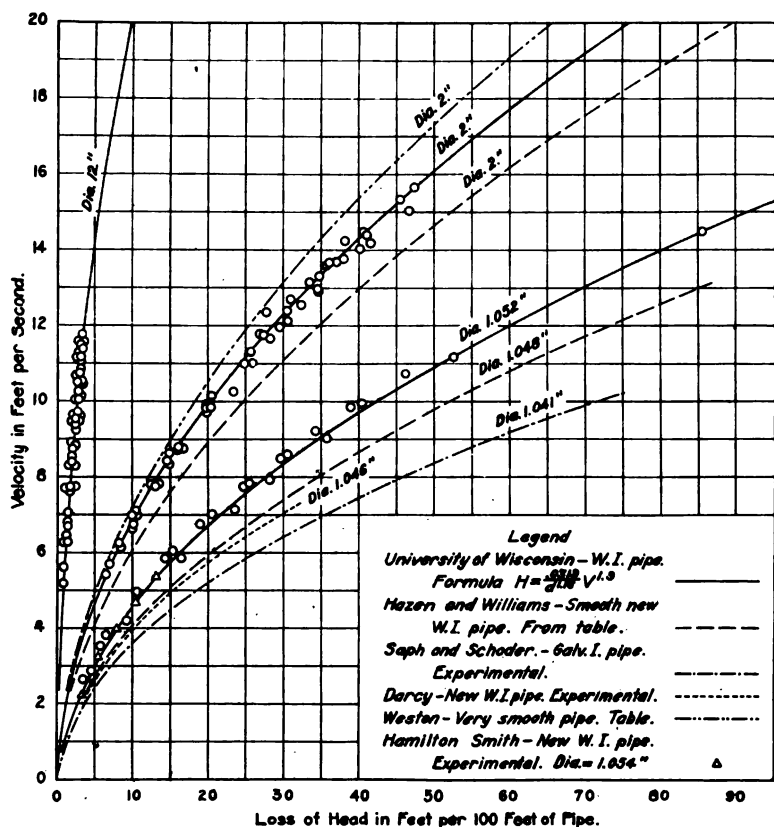


Fig. 33—Comparison of Pipe Results with Common Formulas and Other Experiments.

estimating pipe friction, figure 33, page 44, has been constructed. In figure 33 the circles are experimental points, being part of the results for 1, 2 and 12-inch diameter pipes.

It will be noticed Weston's and Hazen and Williams' curves for 2-inch nominal diameter pipe fall on opposite sides of the

new formula and the experimental points. Hazen and Williams' values show more loss and Weston's values less loss for any given velocity of flow. The group of curves and experimental points for 1-inch pipe are for exact diameters and differences are in part due to size. The curve from Saph and Schoder's 1-inch galvanized iron pipe shows more loss for a given velocity than any of the other experiments. From the pipe friction studies of F. E. Giesecke⁶ at the University of Texas and from data obtained in this laboratory it appears galvanizing slightly reduces the diameter and increases slightly the pipe friction as compared to clean new black pipe. Published results on friction in wrought iron pipe above two inches in diameter are so meager it was not considered of value to complicate the figure by introducing the new formula curves for sizes between 2-inch and 12-inch diameters. The three sizes selected give the degree of agreement between the formula and experimental data.

Gate Valves.—Tables 22 to 30, pages 95 to 134, give the data for the loss of head through the various gate valves. The loss of head through the valve is the total loss occasioned by the valve, namely, the dissipated head within the valve and the excess pipe frictional loss in the gage length over that due to normal pipe flow at the same velocity. Velocities are those in the pipe line and are obtained by dividing the measured discharge by the actual cross-sectional area of the pipe. Valve opening is the ratio of the height the valve disc is lifted, above the position of just opening, to the diameter of the opening through the valve seats. The coefficient k has been obtained by dividing the average loss of head through the valve, for a given opening, by the average velocity squared and multiplying by 64.4, the value of $2g$.

The variations of the loss of head for different valves of the same size and for the various gate openings are shown graphically in figures 34 to 42, pages 46 to 54. The curved lines have been drawn from the formula $H = \frac{kv^2}{2g}$ using the average values of k from table 6, page 56. The experimental points for each valve were then plotted along the appropriate curves. Both

⁶F. E. Giesecke, *The Friction of Water in Pipes and Fittings*, Bulletin No. 1759, University of Texas, October, 1917.

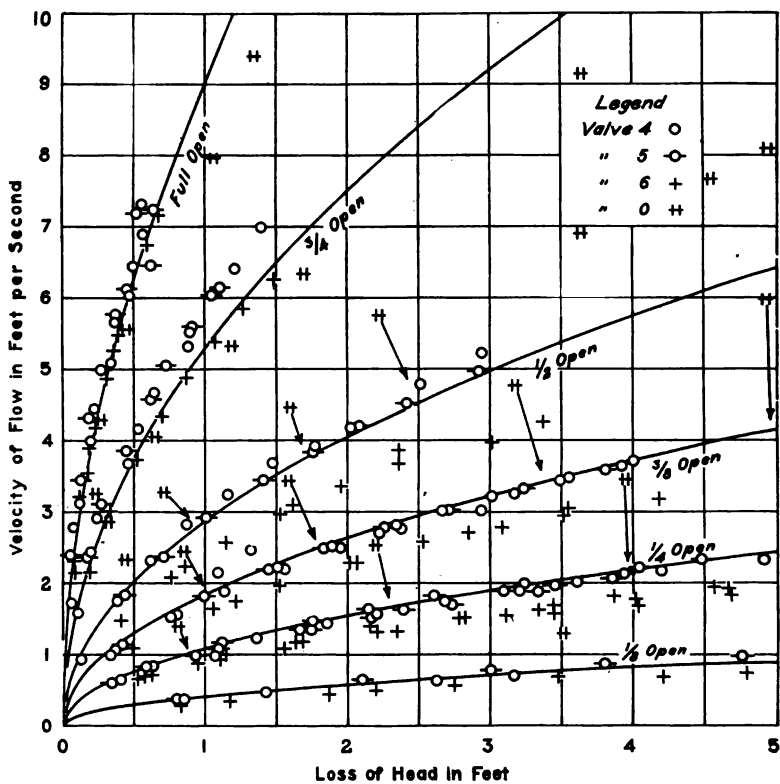


Fig. 34—Curves Showing Net Loss of Head Due to $\frac{1}{2}$ -inch Gate Valves for Different Velocities and at Openings from One-eighth to Fully Open.

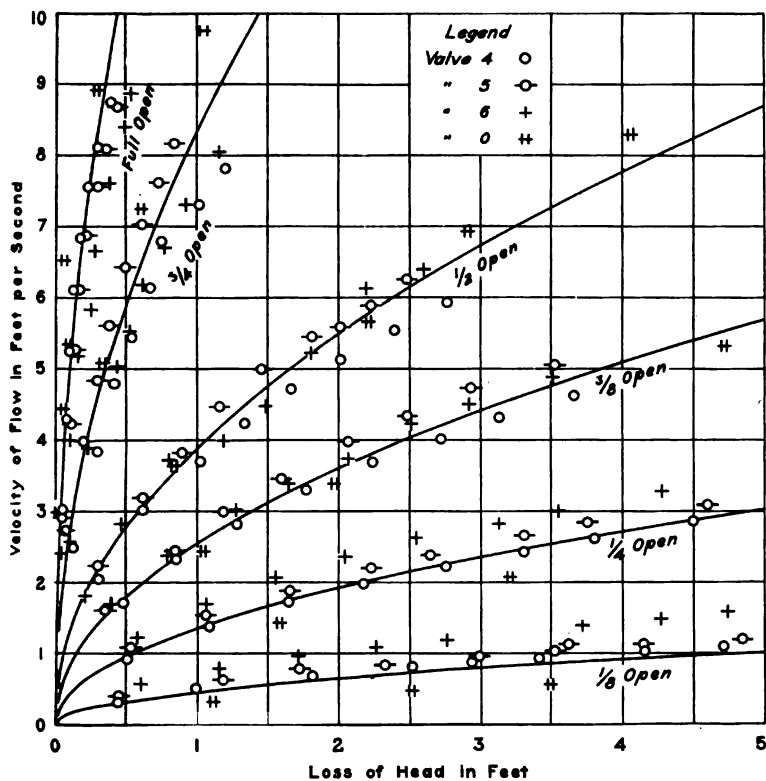


Fig. 35—Curves Showing Net Loss of Head Due to $\frac{3}{4}$ -inch Gate Valves for Different Velocities and at Openings from One-eighth to Fully Open.

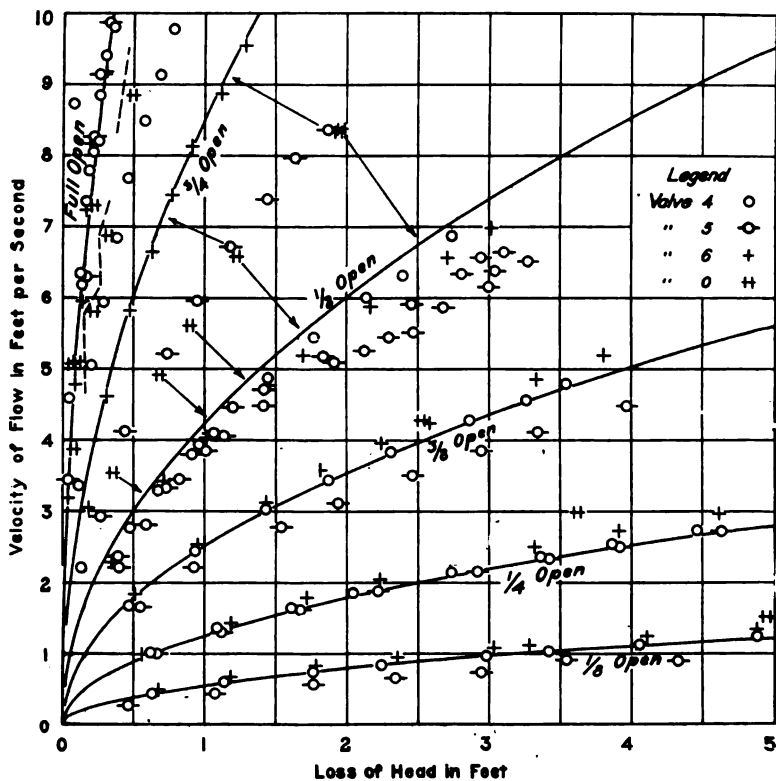


Fig. 36—Curves Showing Net Loss of Head Due to 1-inch Gate Valves for Different Velocities and at Openings from One-eighth to Fully Open

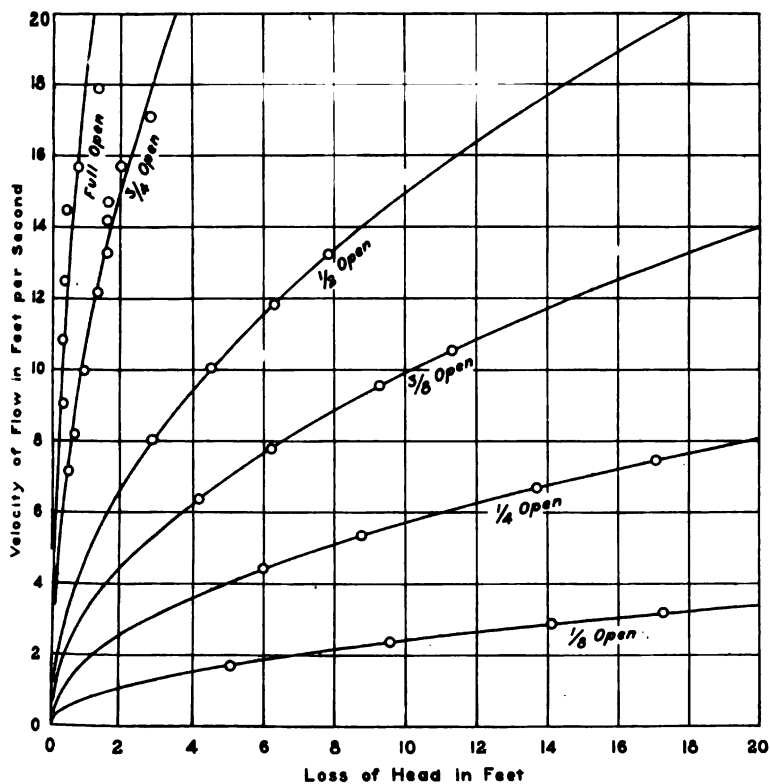


Fig. 37.—Curves Showing Net Loss of Head Due to 1 1/2-inch Gate Valves for Different Velocities and at Openings from One-eighth to Fully Open.

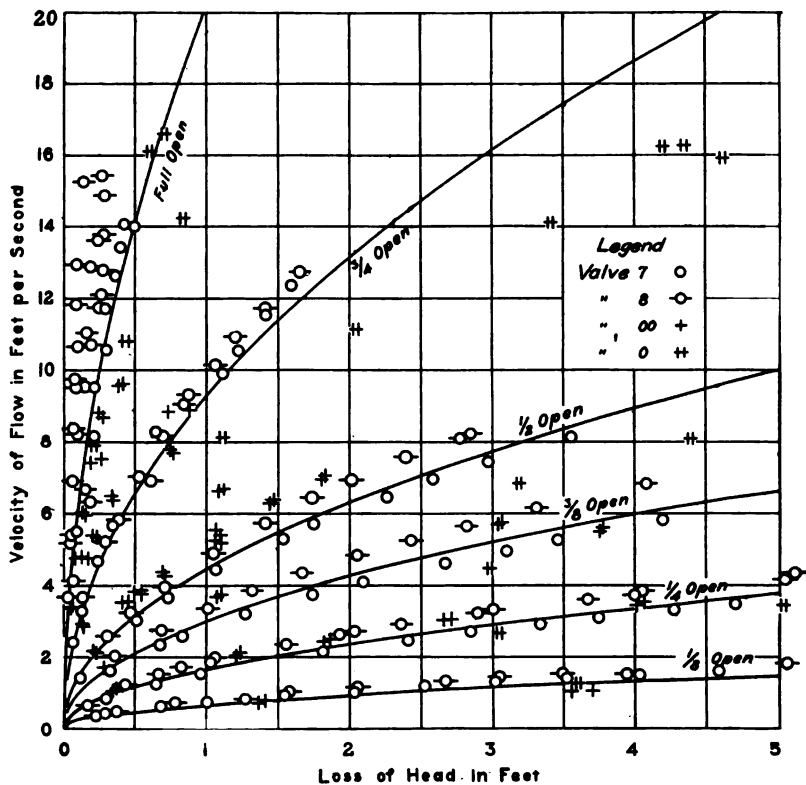


Fig. 38—Curves Showing Net Loss of Head Due to 2-inch Gate Valves for Different Velocities and at Openings from One-eighth to Fully Open.

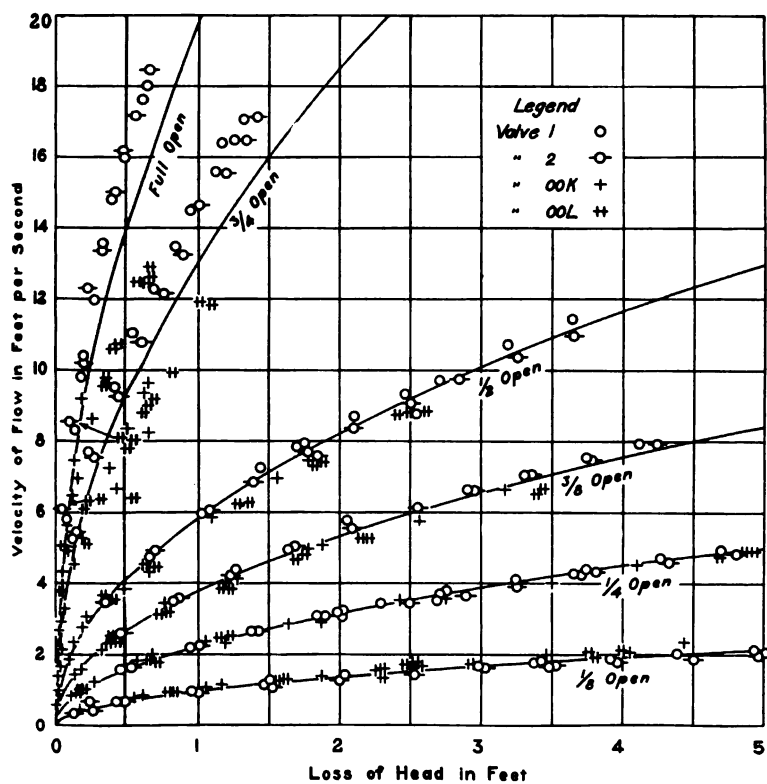


Fig. 39—Curves Showing Net Loss of Head Due to 4-inch Gate Valves for Different Velocities and Openings from One-eighth to Fully Open.

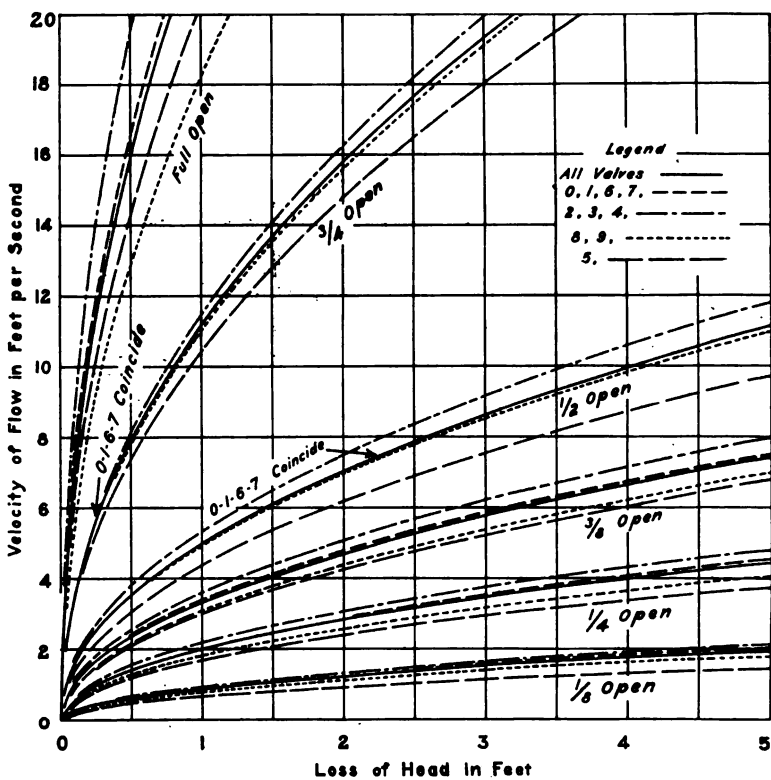


Fig. 40—Curves Showing Net Loss of Head Due to 6-inch Gate Valves for Different Velocities and at Openings from One-eighth to Fully Open.

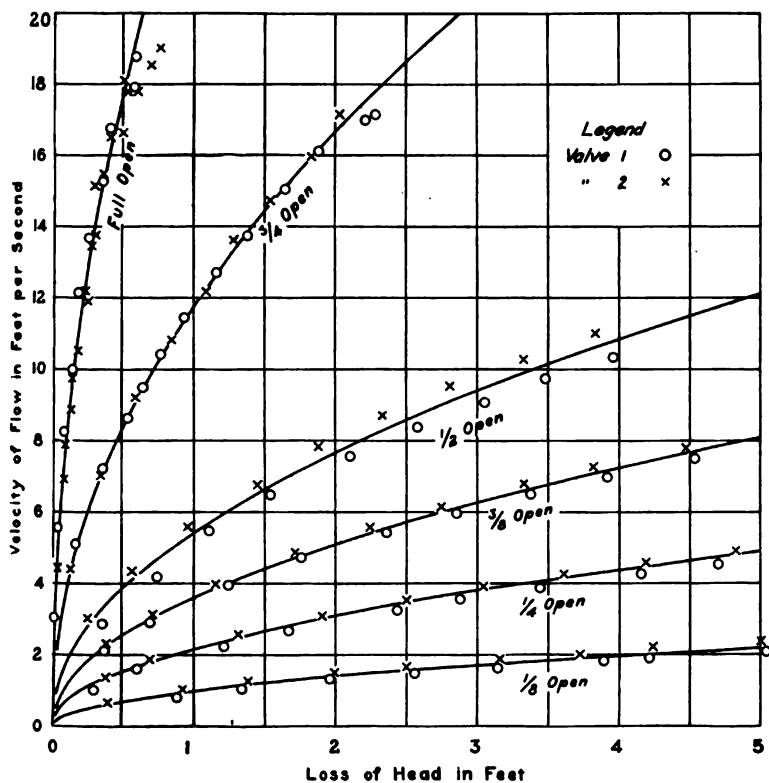


Fig. 41—Curves Showing Net Loss of Head Due to 8-inch Gate Valves for Different Velocities and at Openings from One-eighth to Fully Open.

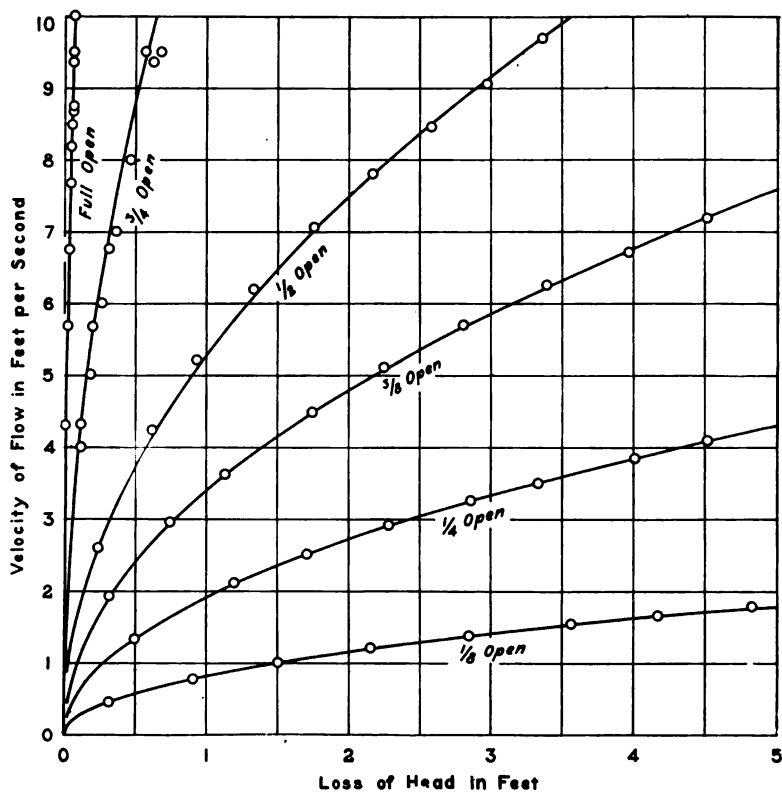


Fig. 42—Curves Showing Net Loss of Head Due to 12-inch Gate Valve for Different Velocities and at Openings from One-eighth to Fully Open.

table 6 and the curves furnish a means of noting the variation of individual valves from the average and from each other. Several factors operate to cause the results of two valves of the same size to differ in the resistance they offer to flow and the consequent loss of head they produce. Obviously, this is especially true of the smaller sizes. The difference in design of different manufacturers is one of these factors. In the 6-inch series a point was made of getting the results of at least two valves from each of two manufacturers. In figure 40, page 52, the results from the ten 6-inch valves tested have been placed in four groups, each group being represented by a special line. Valves 0, 1, 6, and 7 were old style No. 461 Crane valves, 2, 3, and 4 new style No. 461 Crane valves, 8 and 9 Chapman valves, and 5 was an old valve without name which had been used in the laboratory. It was found the experimental points would fall on or near one another in a way to destroy their significance and they were therefore omitted from this figure. The results from the different makes are in general agreement, there being no large deviation from the average line for any opening. The two 8-inch valves shown in figure 41 agree very closely. They were made by different manufacturers.

Generally valves of the same type and manufacture differ enough in their passage dimensions to affect the loss of head they will produce. With large valves this is slight but for the smaller sizes the differences may be considerable. When a valve is fully open the disc will often project slightly into the opening. The pins also often project into the passageway of split disc valves when fully open. This, of course, causes the fully open loss for the smaller valves to be quite variable.

A third influence which may affect the results from a single valve at a given gate opening is the play of the valve disc on its stem. Even though a valve be set at exactly a given opening the play or slack may permit the passage to alter slightly when the water is passing through. It was found for the small valves if the stem projected upward results would often be discordant if a test were repeated without altering the adjustment of the valve. This was overcome by always having the stems point downward so that the weight of the disc would assist in keeping the play or backlash in one direction.

TABLE 6
EXPERIMENTAL VALUES OF COEFFICIENTS FOR LOSS OF HEAD DUE TO
GATE VALVES

$$\text{Values of } k \text{ in } H = \frac{k v^2}{2g}$$

Nominal Diameter of Valve	Valve No.	Ratio of Height of Valve Opening to Diameter of Full Valve Opening					
		1/8	1/4	3/8	1/2	3/4	1
1/2 inch	0	131.3	21.8	8.99	4.84	2.674	0.954
1/2 inch	4	445.0	56.0	19.43	7.19	1.892	0.755
1/2 inch	5	325.8	57.0	19.18	7.78	1.868	0.688
1/2 inch	6	593.0	79.6	25.43	11.16	2.383	0.836
	Ave.	374	53.6	18.26	7.74	2.204	0.808
3/4 inch	0	635.0	47.5	11.28	3.93	0.583	0.152
3/4 inch	4	261.5	35.8	10.75	4.92	1.190	0.281
3/4 inch	5	211.0	31.6	8.64	3.98	0.801	0.340
3/4 inch	6	122.7	24.9	8.95	4.10	1.105	0.346
	Ave.	308	34.9	9.91	4.23	0.920	0.280
1 inch	0	140.2	49.8	8.90	1.72	0.407	0.253
1 inch	4	201.0	38.1	9.97	3.82	0.519	0.224
1 inch	5	334.0	39.6	12.57	4.61	1.690	0.217
1 inch	6	167.0	33.8	9.15	4.00	0.910	0.238
	Ave.	211	40.3	10.15	3.54	0.882	0.233
1 1/2 inch	0	110.7	19.7	6.55	2.88	0.576	0.201
2 inch	00	223.0	19.9	5.25	2.37	0.533	0.256
2 inch	0	147.2	27.4	9.70	4.33	1.081	0.194
2 inch	7	115.3	25.1	8.03	3.47	0.680	0.140
2 inch	8	98.9	17.8	5.62	2.70	0.660	0.100
	Ave.	146	22.5	7.15	3.22	0.739	0.176
4 inch	00	58.0	13.2	4.80	1.80	0.511	0.244*
4 inch	00	61.5	13.1	5.05	2.14	0.527	0.250†
4 inch	1	69.8	12.5	4.27	1.82	0.292	0.125
4 inch	2	79.4	13.3	4.35	1.95	0.319	0.120
	Ave.	67.2	13.0	4.62	1.93	0.412	0.164
6 inch	00	76.4	16.01	6.01	2.67	0.476	0.408*
6 inch	00	71.2	15.44	5.75	2.51	0.549	0.202†
6 inch	1	76.5	16.26	6.12	2.63	0.518	0.074
6 inch	2	70.7	14.05	5.12	2.35	0.464	0.091
6 inch	3	67.7	13.77	5.29	2.39	0.493	0.097
6 inch	4	79.9	14.92	5.35	2.42	0.472	0.083
6 inch	5	146.2	23.78	7.55	3.45	0.644	0.184
6 inch	6	70.3	15.58	5.83	2.54	0.506	0.121
6 inch	7	87.1	16.34	5.93	2.48	0.571	0.152
6 inch	8	87.0	19.47	6.93	2.68	0.526	0.164
6 inch	9	114.0	20.85	7.21	2.87	0.509	0.178
	Ave.	87.3	17.08	6.12	2.64	0.522	0.145
8 inch	1	75.2	14.73	5.18	2.38	0.473	0.100
8 inch	2	56.7	12.78	4.66	2.00	0.454	0.105
	Ave.	66.0	13.50	4.92	2.19	0.464	0.103
12 inch	1	96.2	17.4	5.61	2.29	0.414	0.047

*Test by Kline and Tanghe, Spring, 1914.

† Test by H. N. Lendall, Summer, 1914.

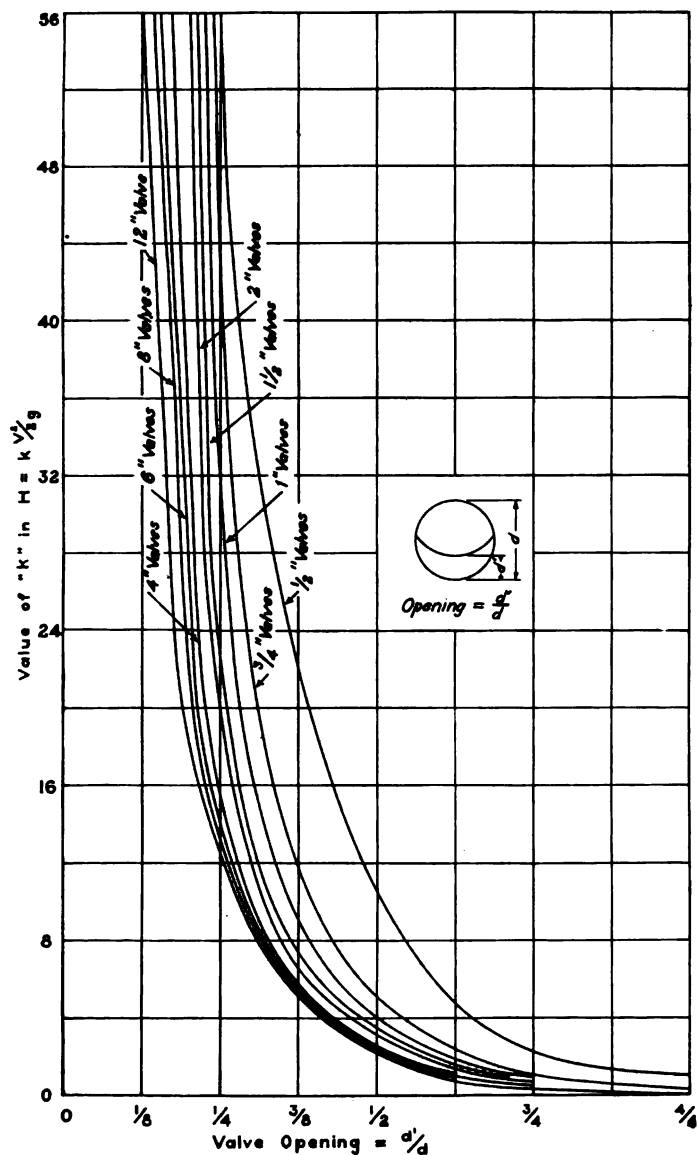


Fig. 43—Curves Showing Variation of Coefficient k in $H = \frac{kv^2}{2g}$ for Valve Openings from One-eighth to Fully Open for All Sizes of Gate Valves Tested.

Note.—Some discrepancies in experimental values have been ironed out to make curves for the different sizes of valves agree in form.

In tables 27 and 28, pages 114 and 121, the data are given for the 4-inch and 6-inch valves No. 00 by two different experimenters. The agreement is probably best studied by comparing the values of k given in table 6, page 56. It is to be expected that loss of head due to valves will vary for a given set of conditions. The amount can best be judged by the results from the ten 6-inch valves.

To obtain figure 43, page 57, the average values of k were plotted against the valve opening and the curves for the different valve sizes then drawn in. The curves do not average strictly the values of k but were altered so as to be in closer agreement with one another. Within its range the curve sheet can also be used by interpolation to determine probable values of k for sizes of valves other than those tested. It was found impractical to extend the curve sheet enough to include the smaller openings. A logarithmic plotting was therefore made, figure 44, page 59, which does cover the entire range of the tests.

The dots and solid curves are from the curves of figure 43. The circles and dotted lines, which average them, are from the average values of k given in table 6, page 56.

Figure 45, page 60, shows the variation of the value of k with diameter for fully open gate valves. Similar curves might be drawn for the other openings but the loss of head through fully open valves is most frequently needed.

It is often useful to express the loss of head due to fully open gate valves as equivalent to so many feet of straight pipe of the same size. This will vary somewhat with velocity, the equivalent length of pipe being greater as the velocity increases. Table 7, p. 61, gives the length of straight pipe which will occasion the same loss of head as will be produced by a valve, for the sizes of valves tested.

The procedure to obtain this table is as follows: The loss of head due to the fully open valve for a given velocity is determined from the formula $H = \frac{kv^2}{2g}$ using the average values of k from table 6, page 56. This loss is divided by the loss of head due to pipe friction read from the curves of figure 31, page 41, giving the desired equivalent in straight pipe of the same diameter as the valve.

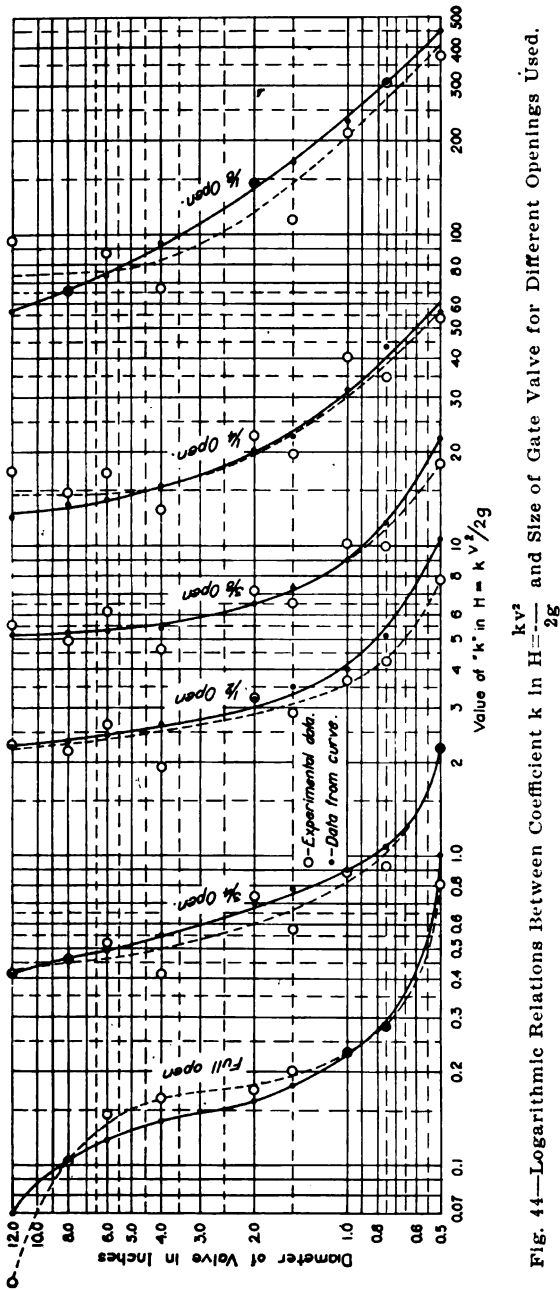


Fig. 44—Logarithmic Relations Between Coefficient k in $H = \frac{kv^2}{2g}$ and Size of Gate Valve for Different Openings Used.

The data of Table 7 have been shown in graphical form in figure 46, page 61.

Figure 47 is a diagram from which may be read the loss of head produced by a valve of any given size for different velocities or rates of discharge. The solid lines are for sizes of valves experimented upon. The dashed lines were obtained by interpolating between the sizes actually tested.

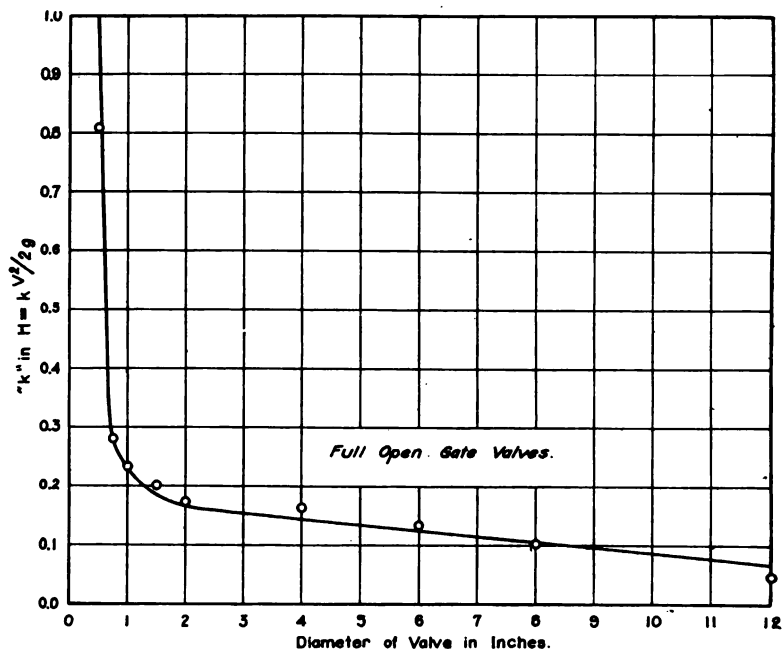


Fig. 45—Curves Showing Variation of Coefficient k in $H = \frac{kv^2}{2g}$ for Gate Valves Fully Open. Sizes $\frac{1}{2}$ -inch to 12-inch.



TABLE 7

LENGTHS OF STRAIGHT PIPE PRODUCING THE SAME LOSS OF HEAD
AS THAT DUE TO FULLY OPEN GATE VALVES

		Diameter of Valve in Inches								
		½	¾	1	1½	2	4	6	8	12
Equivalent Length of Straight Pipe	v = 3 ft./sec.	1.27	0.65	0.79	1.01	1.14	2.97	3.98	3.50	2.82
	v = 10 ft./sec.	1.62	0.78	0.91	1.04	1.34	3.49	4.50	3.84	2.87

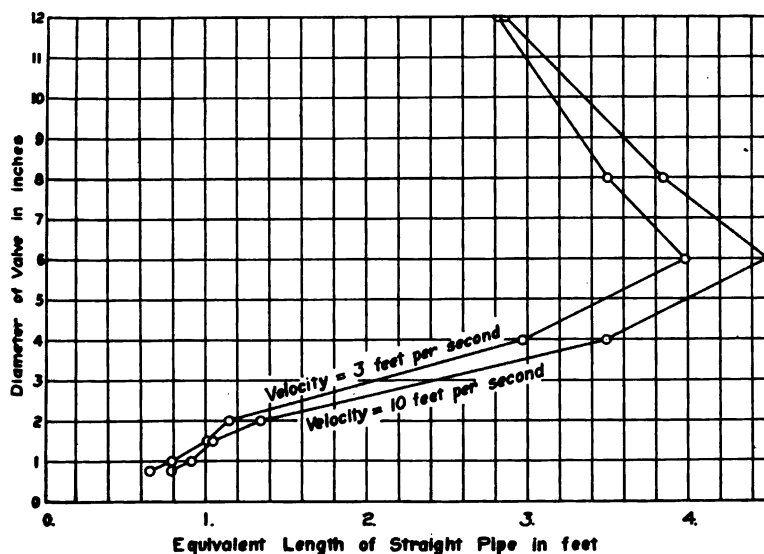


Fig. 46—Curves Showing Lengths of Straight Pipe Producing the Same Loss of Head as Fully Open Gate Valves.

Globe Valves.—Test results from 15 globe valves ½ to 2 inches in diameter are given in Tables 31 to 34, pages 136 to 140.

Table 8, page 62, gives the values of k in $H = \frac{kv^2}{2g}$ obtained for each valve and also the average values for each size tested.

The curves of figure 48, page 63, were plotted using the average values of k of Table 8. They show the loss of head produced by the different sizes of valves for velocities of flow up to ten feet per second.

TABLE 8
EXPERIMENTAL VALUES OF COEFFICIENTS FOR LOSS OF HEAD DUE
TO GLOBE VALVES

$$\text{Values of } k \text{ in } H = \frac{k \cdot v}{2g}$$

Diameter of Valve	Valve No.	k Valves Fully Open		
		Wisconsin Experiments	Illinois Experiments	Purdue Experiments
½-inch	1	12.56		
½-inch	2	14.50		
½-inch	3	14.60		
	Ave.	13.55		
¾-inch	1	6.76		
¾-inch	2	6.19		
¾-inch	3	6.66		
	Ave.	6.54		
1-inch	1	5.90		
1-inch	2	6.01		
1-inch	3	6.17		
	Ave.	6.03	7.12	
1½-inch	..			7.70
2-inch	1	6.26		
2-inch	2	7.46		
2-inch	3	6.69		
2-inch	4	7.21		
2-inch	5	6.74		
2-inch	6	8.45		
	Ave.	7.13	6.00	

The water passage through globe valves is tortuous and there is enough variation between sizes to make it improbable a progressive series of values for k would be obtained from a limited series. In the tests here reported it will be noted in figure 48, page 63, the curve for the 2-inch valves shows more loss of head, at any given velocity of flow than do either the ¾ or 1-inch curves. The valve seats and discs for the different sizes differed as can be seen by figures 20, 21, and 22, page 31. The dimensions as given in Table 2, page 26, vary also. An important dimension in determining the area of the restricted passage through a valve is the rise of the disc from its seat

when the valve is fully open. Not only does this vary for different valves but the shape of this passage varies depending on style of valve disc and seat used. There were two of each of three makes of 2-inch globe valves tested; the variation of the value of k , Table 8, between two valves of the same make was as much or greater than the variation of the averages of the different designs from one another although these were quite different.

The globe valve tests were made principally to compare the loss of head due to flow through fully open valves of the two

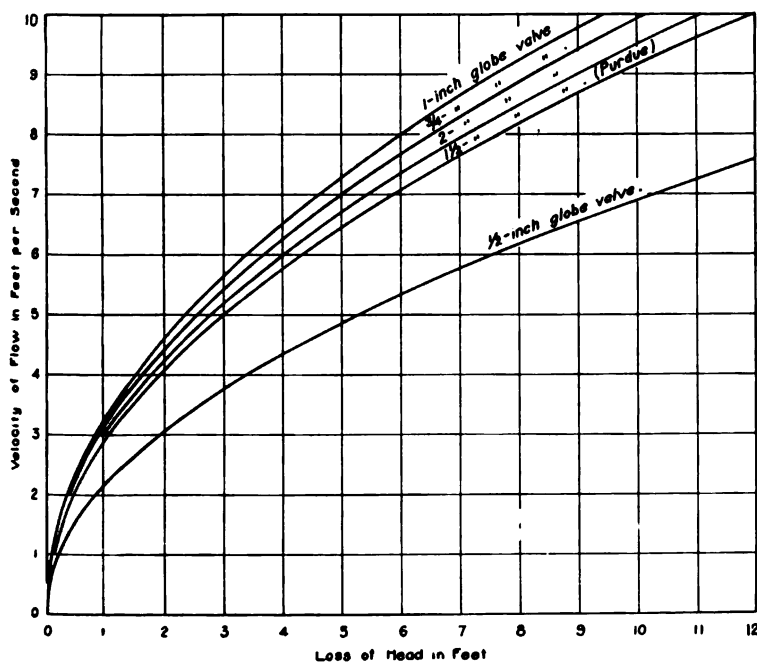


Fig. 48—Curves Showing Loss of Head for Velocities of Flow from 1 to 10 Feet per Second Through Globe Valves from $\frac{1}{2}$ to 2 inches in Diameter.

types. From the data obtained globe valves produce 16.75 times the loss due to gate valves in $\frac{1}{2}$ -inch size, 23.8 times in $\frac{3}{4}$ -inch size, 30 times in 1-inch size, and 40.7 times in 2-inch size.

The loss of head due to the flow of water through $\frac{1}{2}$ -inch globe valves fully open is approximately equivalent to the loss in 35 feet of $\frac{1}{2}$ -inch pipe at the same pipe velocity. That due to the larger sizes tested is probably equivalent to approximately 20 feet of straight pipe of the valve size.

OTHER VALVE EXPERIMENTS

The first recorded valve experiments were those of Weisbach in 1842.⁷ Of the various types tested the one illustrated by figure 49, page 65, approached nearest the modern gate valve. The pipe was 4 centimeters (1.57 inches) in diameter and the valve of simple construction as indicated by the sketch.

Fifty years later (1892) Emil Kuichling published the results of experiments on a 24-inch Ludlow gate valve.⁸ Figure 50, page 65, is a sketch of his apparatus. It will be noted that a Y connection and tee were included in the gage length.

J. Waldo Smith in 1895 published the results of tests on a 30-inch gate valve.⁹ A sketch of his apparatus is shown in figure 52, page 66. In this test entrance and exit losses as well as the loss in two conical sections of pipe were included in the gage length.

G. S. Williams, C. W. Hubbell, and G. H. Fenkell in their experiments on the water supply mains of Detroit, Michigan, found the loss of head in a fully open 30-inch gate valve to be equivalent to the loss in 15.62 feet of 30-inch straight pipe. See figure 51. Based on their computed loss per foot of pipe ($H = .000117v^2$)¹⁰ the value of k in $H = \frac{kv^2}{2g}$ will equal 0.1177.

It is evident from the special conditions that surrounded the valve experiments noted above that valve loss was complicated with other losses in such a way as to render it difficult, if not impossible, to segregate the true valve loss.

In 1902 A. P. Folwell reported the results of experiments on two 4-inch gate valves of different types¹¹ which are in close agreement with the results obtained in the Wisconsin series. He does not give any description or sketch of the apparatus used.

⁷ E. B. Coxe's translation of *Mechanics of Engineering*, by Julius Weisbach, page 903.

⁸ *Transactions A. S. C. E.*, 26 (1892) 439-468.

⁹ *Ibid.*, 34 (1895) 235-246.

¹⁰ *Ibid.*, 47 (1902) 360.

¹¹ *Engineering News*, 47 (1902) 302.



Fig. 49—J. Weisbach's 1.57-in. Gate Valve Apparatus.

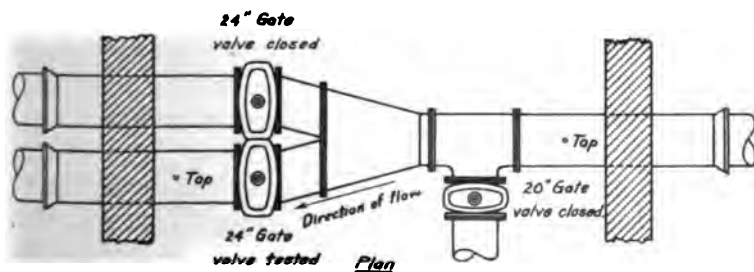
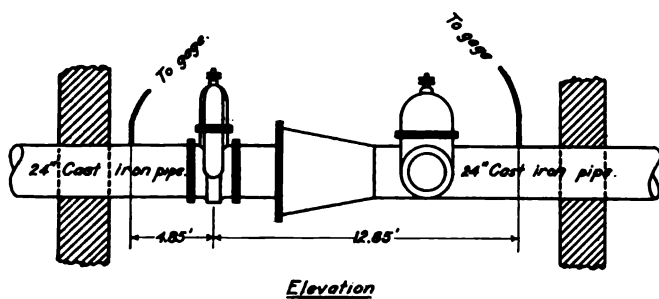


Fig. 50—E. Kuichling's 24-in. Gate Valve Apparatus.

W. E. Stanley in 1916 gave the results of tests on one 1-inch globe and one 1-inch gate valve.¹² The gage length included twenty feet of 1-inch pipe, evidently with ten feet upstream and ten feet downstream from the valve. The results given are in good agreement with those described herein.

In 1918 the University of Illinois issued a bulletin by Arthur N. Talbot and Fred B. Seeley in which appeared results of experiments on six valves of the gate, globe, and angle types.

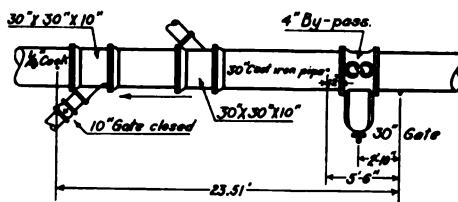


Fig. 51—Williams, Hubbell, and Fenkell's 30-in. Gate Valve Apparatus.

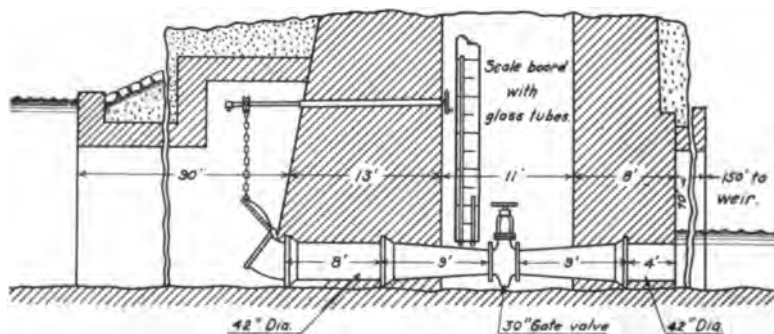


Fig. 52—J. Waldo Smith's 30-in. Gate Valve Apparatus.

There was a 1-inch and a 2-inch valve of each type tested. The gage length included one foot of straight pipe upstream and one foot downstream from the valve. The loss of head in this two feet of pipe was determined from Weston's tables of *Friction of Water in Pipes*.

In comparison with the Wisconsin experiments results on the gate valves for the fully open condition show more loss.

¹² *The Purdue Engineering Review*, No. 12, May 1916, pp. 98-106.

This is in agreement with the experiments discussed in Appendix A.

The most recent data published are those of F. W. Greve, Jr., on a gate, a globe, and a check valve, each $1\frac{1}{2}$ inches in diameter.¹³ The gage length included twenty feet of pipe upstream and twenty feet downstream from the valve. For valve openings near the fully open conditions the loss due to the valve is but a small per cent of the total gage length loss which may occasion considerable error in the valve loss since it is obtained indirectly by subtracting pipe loss from the gage length loss.

TABLE 9

VALUES OF k IN THE EXPRESSION FOR LOSS OF HEAD ($H = \frac{kv^2}{2g}$)
THROUGH GATE VALVES, AS OBTAINED BY DIFFERENT EXPERIMENTERS

Diameter of Valve Inches	Experimenter	Ratio of Height of Opening to Diameter of Valve Seat Opening						Remarks
		$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	
$\frac{1}{8}$	This Bulletin	374	53.6	18.26	7.74	2.20	0.808	4 valves
$\frac{1}{4}$	This Bulletin	308	34.9	9.91	4.23	0.92	0.280	4 valves
1	This Bulletin	211	40.3	10.15	3.54	0.88	0.233	4 valves
1	W. E. Stanley		53.0	14.60	5.40	1.29	0.240	Purdue University Values, interpolated from published curves
1	A. N. Talbot & F. B. Seeley		73.0	16.74	7.00	1.84	0.740	University of Illinois
$1\frac{1}{2}$	F. W. Greve, Jr	143	14.0	8.10	4.40	0.90	0.194	Purdue University
$1\frac{1}{2}$	This Bulletin	111	19.7	6.55	2.88	0.58	0.201	1 valve
1.57	Weisbach	98	17.0	5.52	2.06	0.26	0.000	
2	A. N. Talbot & F. B. Seeley		18.8	7.35	2.94	1.06	0.350	University of Illinois
2	This Bulletin	146	22.5	7.15	3.22	0.74	0.175	4 valves
4	This Bulletin	67	13.0	4.62	1.93	0.41	0.164	3 valves
4	A. P. Folwell {	72	17.0	6.35	2.74	0.71	0.160	Wedge gate
		104	20.7	8.16	2.88	0.82	0.160	Parallel seats
6	This Bulletin	87	17.1	6.12	2.64	0.52	0.145	10 valves
8	This Bulletin	66	13.5	4.92	2.19	0.46	0.103	2 valves
12	This Bulletin	96	17.4	5.61	2.29	0.41	0.047	1 valve
24	Emil Kuichling		22.7	8.63	3.27	0.25	0.000	
30	J. W. Smith	90	17.0	7.50	3.50	0.50	0.000	
30	Williams, Hubbell, & Fenkel						0.118	

NOTE.—Values were interpolated from curves where the original data did not include the openings listed in this table.

¹³ Bulletin No. 1, Engineering Experiment Station, Purdue University (July 1918).

This was one of the difficulties not anticipated in the Wisconsin experiments. For accurate results the length of pipe in the gage length should be reduced to the least that will include the entire valve loss in the gage length.

William T. Magruder and others have made experiments using valves as orifices to measure discharge.¹⁴ In this case there is no pipe downstream from the valve and the conditions are not comparable with those under discussion.

In Table 9, page 67, have been assembled the values of the coefficient, k , for loss of head through gate valves as determined by various experimenters. Openings of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, and fully open are given. It has been necessary to plot the published data in several cases and obtain the values for the openings given in the table by readings from the curve so drawn.

¹⁴ *Engineering Record*, 40 (1899) 78,79.

SUMMARY

1. The loss of head due to valves and other fittings occurs in part within the valve or fitting and in part as an added loss in the pipe line downstream where normal flow has been disturbed.

2. Measurement of the loss of head where the downstream piezometer is attached too near the valve will give a loss in excess of that actually produced. From 20 to 25 pipe diameters beyond the valve will probably give the best position for the downstream piezometer opening. It is undesirable to have a greater length of pipe in the gage length than is actually needed to include all valve loss.

3. The loss of head in new clean wrought iron pipe from $\frac{1}{2}$ to 12 inches in diameter is given approximately by the formula $H = \frac{0.0319}{d^{1.16}} v^{1.9}$ in which H equals loss of head in feet per 100 feet of pipe, v is the velocity of flow in the pipe in feet per second, and d is the pipe diameter in feet.

4. Globe valves offer from 15 to 40 times the resistance of gate valves for the same size. This ratio increases with the increase in the size of valves.

5. The length of straight pipe of the valve size which will produce the same loss of head varies from $\frac{3}{4}$ to 4 feet for fully open gate valves and from 20 to 35 feet for fully open globe valves. In the case of globe valves the smaller valves are equivalent to the greater length of pipe measured in pipe diameters.

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APPENDIX A

INFLUENCE OF LOCATION OF DOWNSTREAM PIEZOMETER ON VALVE LOSS OBTAINED

A study of the loss of head due to flow through valves and of the experimental data that have been published on the subject led to the feeling that sufficient study had not been given to the influence of the location of the downstream piezometer on valve loss obtained.

An investigation of this was offered as a thesis subject, and during the school year of 1919-1920 M. C. Neel and C. A. Wilson experimented with a 4-inch and an 8-inch gate valve with a series of piezometers on the downstream pipe. Figure 53, page 73, shows the location of the various piezometers with respect to the valve for both the 4-inch and the 8-inch set-ups.

Piezometer connections number 1, 5, 6, and 7 were made by tapping a single $\frac{1}{4}$ -inch pipe into the side of the main pipe. Piezometers number 2, 3, and 4 on the 4-inch valve set-up were similar to Figure 6, page 19. Piezometers number 2, 3, and 4 on the 8-inch set-up were made by tapping four $\frac{1}{4}$ -inch pipes into the main pipe on the top, bottom, and two sides. These four pipes were joined together and connected to the gage board through a single $\frac{1}{4}$ -inch pipe in each case. Wherever a piezometer pipe was tapped into the main pipe, it was examined to be sure it came just flush with the inner main pipe surface. Care was used on all details of the set-up in each case.

The differential gage consisted of seven glass tubes five and one-half feet long mounted on a board. The bottoms were connected to the different piezometer openings along the main pipe and the tops were all connected to a common pipe. This common pipe at the top was tapped for a flushing valve and for a connection to an automobile tire pump. The tire pump was

used to force air into the upper part of gage glasses so as to depress the water levels to a convenient height for reading. A graduated scale was mounted beside the glass tubes and

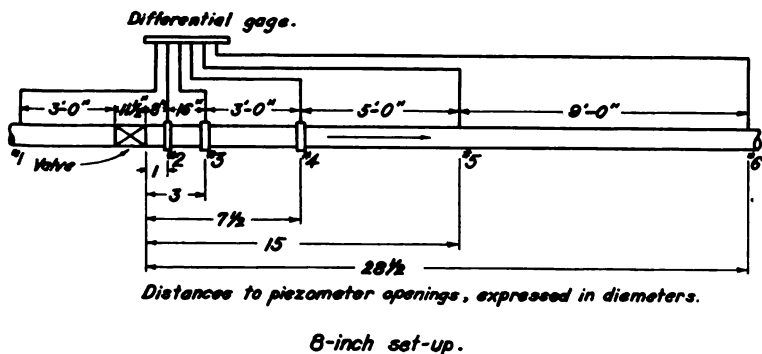
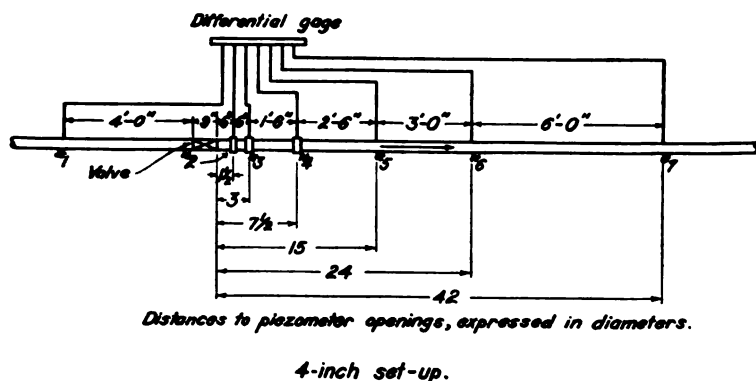


Fig. 53—Positions of Piezometer Openings for Apparatus Used to Study Effect of Location of Piezometers on Determination of Valve Loss.

arranged with a special slide by means of which the water elevation in any tube could be read from the single scale.

The supply of water, control, and means of measurement were the same as described in the main bulletin except that the

larger flows from the 4-inch set-up were measured with a 12-inch rectangular weir instead of the Venturi meter used in the original valve experiments.

The apparatus was first tested in each case without the valve in place to determine the pipe friction losses.

The valve was then inserted and the combined pipe and valve loss determined for a number of valve openings. The results for $\frac{1}{8}$, $\frac{1}{2}$, and fully open conditions only are given here. The net loss of head due to the valve, i. e., loss with valve in minus loss without valve in line, is shown in Table 10 for the different gage lengths 1-2, 1-3, 1-4, etc. An inspection of the table will show that the valve loss, as indicated by piezometers 1-2, is greater than by any other as 1-4, or 1-6. It may also be noted that there is more variation of results using 1-2, 1-3, or 1-4 than when piezometer 1 and one of the piezometers further down stream are used.

To study this better we can assume with reasonable accuracy that the loss of head will vary as the square of the velocity of flow in the pipe line. It then follows that the loss of head for any valve opening may be expressed as $H = \frac{kv^2}{2g}$ where H is the loss of head in feet due to the valve, v the velocity of flow in the pipe line in feet per second, and k an empirical coefficient.

Table 11, page 80, has been computed from the data of Table 10, giving the values of k for the different valve openings. The ratio of k , for each valve opening, as determined from piezometers 1-2, 1-3, etc., to that determined from the first and last piezometers has also been included. Figure 54, page 75, is a plotting of these data. Regarding the loss as shown by the first and last piezometers as correct, the curves show very little error is caused by using the first piezometer with any piezometer 15 or more diameters downstream from the valve to obtain valve loss. It also shows considerable error will be caused where piezometers near the valve are used.

The difference obtained for the loss in the pipe between any two piezometers (for a given velocity of flow), when the valve was in and when it was not, gives the effect produced in that section by the presence of the valve in the line. Table 12,

page 81, has been compiled showing the net loss of head, due to the valve, in the sections 1-2, 2-3, 3-4, etc.

Where the loss with the valve in place exceeded the pipe friction loss, values in the table have been marked with the plus sign. Where they were less than the pipe friction, the quantity is marked with a minus sign. It is evident that two factors are at work to cause the readings of the piezometers immediately downstream from the valve to give a false record of the true

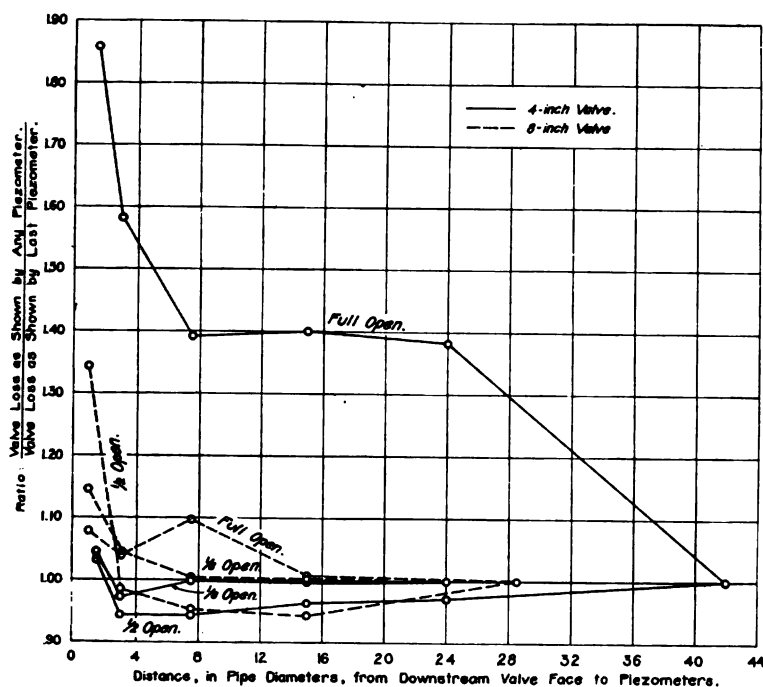


Fig. 54—Curves Showing Error Produced by Placing Downstream Piezometer too Close to Valve.

loss occasioned by the valve. First, the disturbed flow resulting from the sudden changes of section in the valve, causes velocity head effects on these piezometers. Second, where the change of section is appreciable, as when the valve is partly closed, there is a partial reconversion of velocity head back into pressure head in the pipe downstream from the valve giving an apparent negative loss. Figure 55, page 76, shows graphically the appar-

ent valve loss in each of the pipe sections between the different piezometers. For the fully open 4-inch valve there seems to have been some disturbing factor in section 6-7 which we were unable to account for. A series of check runs confirmed the first runs. It will be noted, however, that the quantity measured in the case of the fully open valve is small, being only

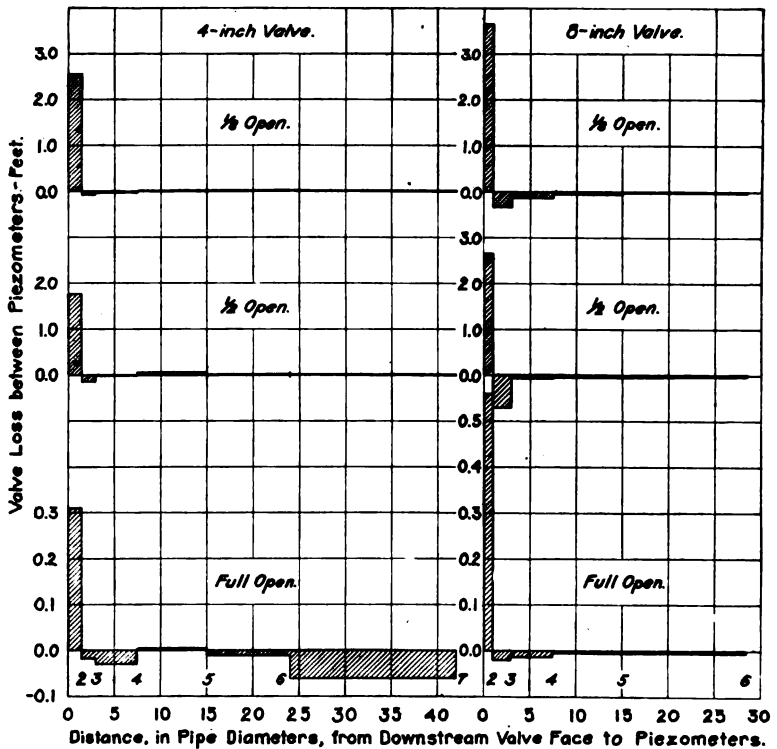


Fig. 55—Apparent Loss, Due to Presence of Valve in the Different Pipe Sections Between Piezometer Openings.

.06 foot for the section 6-7, and any small disturbance would be very noticeable.

It is concluded that a valve causes disturbed flow in the pipe line downstream for some distance from it and that a piezometer opening which is connected so as to be in this region will be affected, giving a false record of the pressure within the pipe.

Where there is considerable change of section in passing through the valve the pressure head absorbed at that point to produce the higher velocity of flow is partially recovered in the pipe downstream.

Further, if the loss of head due to flow through a valve be obtained from a piezometer within two or three diameters below the valve the result will be too large. Not until at least fifteen or twenty diameters of straight pipe intervene between the valve and piezometer will the reading show the true loss. It follows from this last and from what was pointed out in the discussion of the Purdue University experiments that it is desirable to place the downstream piezometer probably twenty to twenty-five pipe diameters downstream from the valve.

TABLE 10

VALVE LOSS OBTAINED BY USING DIFFERENT GAGE LENGTHS

Run	Diameter of Valve in Inches	Net Loss in Feet Due to Presence of Valve in Line as Obtained from Readings of Piezometers						Square of Velocity	Average Velocity Ft./Sec.
		1-2	1-3	1-4	1-5	1-6	1-7		
		Valve $\frac{1}{8}$ Open							
1	4	0.544	0.518	0.517	0.516	0.517	0.517	0.629	
2	4	1.026	0.996	0.989	0.988	0.990	0.993	1.232	
3	4	1.524	1.482	1.470	1.470	1.471	1.473	1.837	
4	4	2.067	2.011	1.989	1.990	1.989	1.996	2.440	
5	4	2.562	2.481	2.452	2.453	2.456	2.458	3.038	
6	4	3.050	2.948	2.911	2.912	2.917	2.920	3.660	
7	4	3.565	3.450	3.403	3.406	3.406	3.403	4.152	
8	4	4.092	3.960	3.907	3.914	3.915	3.916	4.800	
9	4	4.659	4.506	4.445	4.447	4.462	4.460	5.655	
Ave.		2.565	2.483	2.455	2.456	2.458	2.459	3.049	1.746
55	8	2.023	1.835	1.768	1.762	1.750		1.53	
56	8	2.831	2.571	2.494	2.489	2.484		2.24	
57	8	3.336	3.050	2.936	2.929	2.924		2.86	
58	8	3.986	3.614	3.484	3.471	3.462		3.10	
59	8	4.553	4.138	3.979	3.977	3.975		3.66	
60	8	5.169	4.718	4.541	4.538	4.535		4.14	
Ave.		3.650	3.321	3.200	3.194	3.189		2.922	1.709
		Valve $\frac{1}{2}$ Open							
28	4	0.367	0.326	0.336	0.344	0.351	0.353	10.82	
29	4	0.735	0.670	0.672	0.691	0.698	0.701	21.45	
30	4	1.089	0.999	0.993	1.021	1.034	1.162	31.60	
31	4	1.381	1.249	1.262	1.293	1.305	1.463	41.33	
32	4	1.855	1.702	1.700	1.746	1.751	1.838	52.60	
33	4	2.127	1.945	1.949	1.987	2.006	2.140	64.00	
34	4	2.297	2.095	2.094	2.131	2.151	2.162	74.50	
35	4	2.839	2.587	2.585	2.629	2.655	2.632	86.90	
36	4	3.226	2.947	2.940	2.986	2.997	2.955	100.00	
Ave.		1.768	1.613	1.615	1.648	1.661	1.712	53.69	7.327
1c	Check runs 4-inch valve	0.357	0.328	0.328	0.331	0.326	0.322	12.70	
2c		0.703	0.655	0.649	0.667	0.666	0.648	22.72	
3c		1.069	1.010	1.011	1.043	1.045	1.021	32.60	
4c		1.429	1.350	1.344	1.378	1.381	1.355	43.70	
5c		1.822	1.713	1.711	1.739	1.744	1.716	55.20	
6c		2.093	1.966	1.959	1.994	2.008	1.975	64.48	
7c		2.468	2.318	2.314	2.348	2.364	2.332	75.35	
8c		2.792	2.628	2.623	2.657	2.671	2.616	85.95	
9c		3.183	2.989	2.977	3.011	3.021	2.944	99.80	
Ave.		1.768	1.662	1.657	1.685	1.692	1.659	54.72	7.39

TABLE 10—Continued

77	8	0.522	0.376	0.362	0.352	0.350		10.57	
78	8	0.986	0.728	0.704	0.694	0.684		20.53	
79	8	1.540	1.132	1.092	1.084	1.080		31.93	
80	8	2.036	1.498	1.457	1.449	1.434		42.60	
81	8	2.666	1.960	1.908	1.902	1.886		57.13	
82	8	3.130	2.302	2.248	2.250	2.238		65.50	
83	8	3.744	2.748	2.654	2.642	2.624		78.60	
84	8	4.374	3.188	3.046	3.000	2.984		92.70	
85	8	4.950	3.616	3.460	3.414	3.418		106.30	
Ave.		2.661	1.950	1.881	1.865	1.855		56.21	7.497
Valve Fully Open									
46	4	0.066	0.059	0.060	0.085	0.080	0.056	27.04	
47	4	0.120	0.115	0.106	0.114	0.118	0.136	57.00	
48	4	0.193	0.176	0.148	0.150	0.147	0.119	85.60	
49	4	0.268	0.243	0.196	0.180	0.179	0.118	132.50	
50	4	0.543	0.507	0.447	0.383	0.343	0.233	146.80	
51	4	0.273	0.266	0.235	0.234	0.224	0.139	172.90	
52	4	0.385	0.360	0.301	0.261	0.269	0.257	210.30	
53	4	0.409	0.374	0.317	0.285	0.267	0.136	229.30	
54	4	0.440	0.420	0.359	0.390	0.280	0.153	262.50	
Ave.		0.300	0.280	0.241	0.221	0.213	0.150	147.10	12.124
10c	Check runs 4-inch valve	0.068	0.064	0.063	0.076	0.081	0.076	28.09	
11c		0.122	0.111	0.094	0.119	0.130	0.122	52.94	
12c		0.169	0.146	0.144	0.161	0.179	0.177	81.59	
13c		0.220	0.211	0.192	0.209	0.220	0.188	108.90	
14c		0.349	0.340	0.317	0.337	0.342	0.265	136.20	
15c		0.343	0.324	0.297	0.312	0.315	0.223	168.00	
16c		0.398	0.379	0.346	0.362	0.373	0.237	200.00	
17c		0.444	0.420	0.388	0.409	0.419	0.277	222.50	
18c		0.491	0.463	0.425	0.452	0.456	0.427	257.90	
Ave.		0.289	0.272	0.250	0.271	0.279	0.221	139.57	11.81
93	8	0.167	0.158	0.152	0.152	0.152		38.08	
94	8	0.356	0.342	0.322	0.322	0.302		97.20	
95	8	0.498	0.480	0.464	0.478	0.474		144.00	
96	8	0.604	0.589	0.578	0.587	0.577		186.00	
97	8	0.786	0.757	0.747	0.719	0.724		266.50	
98	8	0.945	0.905	0.887	0.869	0.863		329.30	
Ave.		0.559	0.538	0.525	0.521	0.515		176.96	13.30

TABLE 11

COEFFICIENT OF VALVE LOSS k , IN $H = \frac{kv^2}{2g}$, AS DETERMINED FROM
DIFFERENT GAGE LENGTHS

Ratio $\frac{k}{k'} = \frac{\text{Valve Loss as Shown by Any Piezometer}}{\text{Valve Loss as Shown by Last Piezometer}}$

Runs	Valve Diameter in Inches	Opening	Values of k and $\frac{k}{k'}$ for Pipe Sections						
				1-2	1-3	1-4	1-5	1-6	1-7
1 to 9	4	$\frac{1}{8}$	$\frac{k}{k'}$	54.2	52.5	51.8	51.8	51.8	51.8
			$\frac{k}{k'}$	1.046	1.013	0.999	1.000	1.000	1.000
55 to 60	8	$\frac{1}{8}$	$\frac{k}{k'}$	80.5	73.2	70.5	70.4	70.4	
			$\frac{k}{k'}$	1.143	1.040	1.002	1.001	1.000	
28 to 36	4	$\frac{1}{2}$	$\frac{k}{k'}$	2.12	1.93	1.94	1.97	1.99	2.05
			$\frac{k}{k'}$	1.033	0.942	0.944	0.962	0.970	1.000
1c to 9c (check)	4	$\frac{1}{2}$	$\frac{k}{k'}$	2.08	1.95	1.95	1.98	1.99	1.95
			$\frac{k}{k'}$	1.067	1.000	1.000	1.017	1.020	1.000
77 to 85	8	$\frac{1}{2}$	$\frac{k}{k'}$	3.05	2.24	2.16	2.14	2.13	
			$\frac{k}{k'}$	1.435	1.030	1.013	1.005	1.000	
46 to 54	4	Full	$\frac{k}{k'}$	0.131	0.123	0.105	0.097	0.093	0.066
			$\frac{k}{k'}$	2.003	1.872	1.609	1.477	1.423	1.000
10c to 18c (check)	4	Full	$\frac{k}{k'}$	0.134	0.125	0.116	0.125	0.129	0.102
			$\frac{k}{k'}$	1.310	1.230	1.133	1.226	1.265	1.000
93 to 98	8	Full	$\frac{k}{k'}$	0.203	0.196	0.191	0.190	0.188	
			$\frac{k}{k'}$	1.063	1.044	1.018	1.011	1.000	

TABLE 12

APPARENT LOSS, DUE TO PRESENCE OF VALVE, IN THE PIPE LENGTHS
BETWEEN THE SUCCESSIVE PIEZOMETER OPENINGS

Run	Diameter of Valve in Inches	Net Loss in Feet Due to Presence of Valve in Line between Piezometers						Velocity in Feet per second
		1-2	2-3	3-4	4-5	5-6	6-7	
				Valve $\frac{1}{8}$ Open				
1	4	+0.544	-0.026	-0.001	-0.001	+0.001	0.000	0.793
2	4	+1.026	-0.030	-0.007	-0.001	+0.002	+0.003	1.110
3	4	+1.524	-0.042	-0.012	0.000	+0.001	+0.002	1.355
4	4	+2.067	-0.056	-0.022	+0.001	-0.001	+0.007	1.562
5	4	+2.562	-0.081	-0.029	+0.001	+0.003	+0.002	1.743
6	4	+3.050	-0.102	-0.037	+0.001	+0.005	+0.003	1.913
7	4	+3.565	-0.115	-0.047	+0.003	0.000	-0.003	2.038
8	4	+4.092	-0.132	-0.053	+0.007	+0.001	+0.001	2.190
9	4	+4.659	-0.155	-0.059	+0.002	+0.004	-0.002	2.358
55	8	+2.023	-0.188	-0.067	-0.006	-0.012		1.235
56	8	+2.831	-0.260	-0.077	-0.005	-0.005		1.497
57	8	+3.336	-0.286	-0.114	-0.007	-0.005		1.691
58	8	+3.986	-0.372	-0.130	-0.013	-0.009		1.750
59	8	+4.553	-0.415	-0.159	-0.002	+0.002		1.913
60	8	+5.169	-0.451	-0.177	-0.003	-0.003		2.034
				Valve $\frac{1}{4}$ Open				
28	4	+0.367	-0.041	+0.010	+0.008	+0.007	+0.002	3.290
29	4	+0.735	-0.065	+0.002	+0.019	+0.007	+0.003	4.630
30	4	+1.089	-0.090	-0.006	+0.028	+0.013	+0.128	5.620
31	4	+1.381	-0.132	+0.013	+0.031	+0.012	+0.158	6.430
32	4	+1.855	-0.153	-0.002	+0.046	+0.005	+0.087	7.250
33	4	+2.127	-0.182	+0.004	+0.038	+0.019	+0.134	8.000
34	4	+2.297	-0.202	-0.001	+0.038	+0.020	+0.011	8.630
35	4	+2.839	-0.252	-0.002	+0.044	+0.026	-0.023	9.32c
36	4	+3.226	-0.279	-0.007	+0.046	+0.011	-0.042	10.000
1c	Check	+0.357	-0.029	0.000	+0.003	-0.005	-0.004	3.370
2c	runs	+0.703	-0.048	-0.006	+0.018	-0.001	-0.018	4.770
3c	4-inch	+1.069	-0.059	+0.001	+0.032	+0.002	-0.024	5.710
4c	Valve	+1.429	-0.079	-0.006	+0.034	+0.003	-0.026	6.610
5c		+1.822	-0.109	-0.002	+0.028	+0.005	-0.028	7.430
6c		+2.093	-0.127	-0.007	+0.035	+0.014	-0.033	8.030
7c		+2.468	-0.150	-0.004	+0.034	+0.016	-0.032	8.680
8c		+2.792	-0.164	-0.005	+0.034	+0.014	-0.055	9.270
9c		+3.183	-0.194	-0.012	+0.034	+0.010	-0.077	9.940

TABLE 12—Continued

77	8	+0.522	-0.146	-0.014	-0.010	-0.002		3.260
78	8	+0.986	-0.258	-0.024	-0.010	-0.010		4.530
79	8	+1.540	-0.408	-0.040	-0.008	-0.004		5.650
80	8	+2.036	-0.538	-0.041	-0.008	-0.015		6.530
81	8	+2.666	-0.706	-0.052	-0.006	-0.016		7.560
82	8	+3.130	-0.828	-0.054	+0.002	-0.012		8.090
83	8	+3.744	-0.996	-0.094	-0.012	-0.018		8.870
84	8	+4.374	-1.186	-0.142	-0.046	-0.016		9.630
85	8	+4.950	-1.334	-0.156	-0.046	0.000		10.320
Valve Fully Open								
46	4	+0.066	-0.007	+0.001	+0.025	+0.005	-0.024	5.200
47	4	+0.120	-0.005	-0.009	+0.008	+0.004	+0.018	7.550
48	4	+0.193	-0.017	-0.028	+0.002	-0.003	-0.028	9.250
49	4	+0.268	-0.025	-0.047	-0.016	-0.001	-0.061	11.510
50	4	+0.543	-0.036	-0.060	-0.064	+0.040	-0.110	12.110
51	4	+0.273	-0.007	-0.031	-0.001	-0.010	-0.085	13.140
52	4	+0.385	-0.025	-0.059	-0.040	+0.008	-0.012	14.500
53	4	+0.409	-0.035	-0.057	-0.032	-0.018	-0.131	15.140
54	4	+0.440	-0.020	-0.061	+0.031	-0.110	-0.127	16.200
10c	Check	+0.068	-0.004	-0.001	+0.013	+0.005	-0.005	5.300
11c	runs	+0.122	-0.011	-0.017	+0.025	+0.011	-0.008	7.280
12c	4-inch	+0.169	-0.023	-0.002	+0.017	+0.018	-0.002	9.030
13c	Valve	+0.220	-0.009	-0.019	+0.017	+0.011	-0.032	10.430
14c		+0.349	-0.009	-0.023	+0.020	+0.005	-0.077	11.670
15c		+0.343	-0.019	-0.027	+0.015	+0.003	-0.092	12.960
16c		+0.398	-0.019	-0.033	+0.016	+0.011	-0.136	14.140
17c		+0.444	-0.024	-0.032	+0.021	+0.010	-0.142	14.910
18c		+0.491	-0.028	-0.030	+0.027	+0.004	-0.029	16.060
93	8	+0.167	-0.009	-0.006	0.000	0.000		6.170
94	8	+0.356	-0.014	-0.020	0.000	-0.020		9.860
95	8	+0.498	-0.018	-0.016	+0.014	-0.004		12.000
96	8	+0.604	-0.015	-0.011	+0.009	-0.010		13.660
97	8	+0.786	-0.029	-0.010	-0.028	+0.005		16.330
98	8	+0.945	-0.040	-0.018	-0.018	-0.006		18.150

APPENDIX B

TABLES OF DATA

TABLE 13

LOSS OF HEAD IN 1/2-INCH PIPE

Runs 1 to 10 gage length = 20.43 ft.

Runs 11 to 33 gage length = 10.0 ft.

Cross sectional area of pipe = .00206 sq. ft. for Runs 11-33.

Cross sectional area of pipe = .002097 sq. ft. for Runs 1-10

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	Mar. 3-15	19.85	198.5	0.01989	9.490	90.00
2	"	15.18	151.8	0.01718	8.200	67.10
3	"	8.90	89.0	0.01287	6.140	37.70
4	"	5.92	59.2	0.01030	4.910	24.10
5	"	5.155	51.55	0.00956	4.560	20.80
6	"	3.596	35.96	0.00781	3.730	13.90
7	"	1.542	15.42	0.00499	2.380	5.68
8	"	3.176	31.76	0.00698	3.340	11.18
9	"	2.044	20.44	0.00571	2.725	7.43
10	"	0.578	5.78	0.00279	1.321	1.76
11	Apr. 26-19	0.243	2.43	0.00297	1.441	2.08
12	"	0.471	4.71	0.00427	2.072	4.30
13	"	0.709	7.09	0.00538	2.618	6.84
14	"	1.069	10.69	0.00672	3.264	10.65
15	"	1.558	15.58	0.00850	4.125	17.00
16	"	1.925	19.25	0.00934	4.533	20.54
17	"	3.721	37.21	0.01344	6.525	42.51
18	May 6-19	4.198	41.98	0.01438	6.975	48.60
19	"	4.734	47.34	0.01531	7.430	55.20
20	"	5.310	53.10	0.01625	7.880	62.20
21	"	2.489	24.89	0.01089	5.280	27.90
22	"	3.052	30.52	0.01220	5.920	35.00
23	June 20-19	0.270	2.70	0.00327	1.595	2.51
24	"	0.963	9.63	0.00664	3.225	10.41
25	"	1.242	12.42	0.00764	3.710	13.80
26	"	1.540	15.40	0.00852	4.132	17.10
27	"	2.079	20.79	0.01014	4.924	24.24
28	"	2.728	27.28	0.01179	5.721	32.70
29	"	3.075	30.75	0.01264	6.132	37.55
30	"	3.235	32.35	0.01292	6.270	39.26
31	"	4.094	40.94	0.01474	7.152	51.20
32	"	4.731	47.31	0.01592	7.725	59.60
33	"	4.994	49.94	0.01635	7.940	62.85

TABLE 14

LOSS OF HEAD IN $\frac{3}{4}$ -INCH PIPE

Runs 1 to 11 gage length = 20.43 ft.

Runs 12 to 35 gage length = 10.0 ft.

Cross sectional area of pipe = .00367 sq. ft. for runs 1 to 11.

Cross sectional area of pipe = .00358 sq. ft. for runs 12 to 35.

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	Apr. 20-15	13.090	64.00	0.0412	11.22	126.00
2	"	18.350	89.80	0.0494	13.46	181.00
3	"	8.670	42.40	0.0328	8.93	79.60
4	"	5.737	28.10	0.0261	7.10	50.30
5	"	4.726	23.10	0.0234	6.37	40.60
6	"	3.345	16.37	0.0193	5.25	27.50
7	"	2.175	10.63	0.0151	4.10	16.85
8	"	1.303	6.38	0.0112	3.06	9.38
9	"	0.559	2.73	0.0069	1.89	3.58
10	"	18.340	89.70	0.0496	13.50	182.00
11	"	12.300	60.20	0.0398	10.85	117.80
12	Aug. 5-19	0.477	4.77	0.0095	2.67	7.10
13	"	1.020	10.20	0.0144	4.02	16.25
14	"	1.638	15.38	0.0179	5.01	25.05
15	"	2.025	20.25	0.0207	5.80	33.60
16	"	2.565	25.65	0.0234	6.55	42.80
17	"	3.079	30.79	0.0258	7.20	51.80
18	"	3.580	35.80	0.0278	7.77	60.30
19	"	4.162	41.62	0.0300	8.40	70.20
20	"	4.663	46.63	0.0320	8.94	79.70
21	"	5.185	51.85	0.0338	9.44	88.90
22	"	5.034	50.34	0.0333	9.29	86.20
23	"	4.513	45.13	0.0315	8.80	77.30
24	"	0.923	9.23	0.0137	3.84	14.70
25	"	0.435	4.35	0.0091	2.54	6.48
26	Aug. 14-19	0.545	5.45	0.0101	2.82	7.94
27	"	1.054	10.54	0.0145	4.04	16.33
28	"	1.521	15.21	0.0175	4.89	23.90
29	"	2.027	20.27	0.0204	5.68	32.30
30	"	2.497	24.97	0.0226	6.32	40.00
31	"	3.007	30.07	0.0248	6.95	48.30
32	"	3.538	35.38	0.0271	7.57	57.20
33	"	4.014	40.14	0.0289	8.09	65.30
34	"	4.559	45.59	0.0309	8.62	74.30
35	"	5.011	50.11	0.0324	9.05	81.80

TABLE 15

LOSS OF HEAD IN 1-INCH PIPE

Runs 1 to 9 gage length = 20.3 ft.

Runs 10 to 26 gage length = 10.0 ft.

Cross sectional area of pipe = .00604 sq. ft. for runs 1 to 9.

Cross sectional area of pipe = .00601 sq. ft. for runs 10 to 26.

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	Mar. 29-15	23.300	114.80	0.1022	17.00	289.00
2	"	17.360	85.50	0.0872	14.49	210.00
3	"	10.660	52.50	0.0672	11.16	124.60
4	"	7.240	35.60	0.0543	9.02	81.30
5	"	5.700	28.10	0.0476	7.91	62.70
6	"	4.763	23.45	0.0430	7.15	51.00
7	"	3.322	16.38	0.0352	5.85	34.10
8	"	1.847	9.10	0.0252	4.19	17.60
9	"	0.947	4.66	0.0172	2.86	8.18
10	July 21-19	0.329	3.29	0.0158	2.62	6.85
11	"	0.638	6.38	0.0230	3.81	14.45
12	"	1.045	10.45	0.0300	4.97	24.60
13	"	1.536	15.36	0.0366	6.06	36.70
14	"	2.051	20.51	0.0427	7.02	49.20
15	"	2.530	25.30	0.0472	7.82	61.00
16	"	3.053	30.53	0.0519	8.60	73.80
17	"	4.029	40.29	0.0601	9.95	99.00
18	Aug. 6-19	4.608	46.08	0.0648	10.72	115.00
19	"	3.890	38.90	0.0594	9.84	96.50
20	"	3.404	34.04	0.0556	9.21	84.50
21	"	2.952	29.52	0.0515	8.53	72.50
22	"	2.464	24.64	0.0468	7.75	60.00
23	"	1.889	18.89	0.0409	6.77	45.80
24	"	1.434	14.34	0.0353	5.84	34.10
25	"	1.025	10.25	0.0296	4.90	24.00
26	"	0.562	5.62	0.0213	3.52	12.40

TABLE 16

LOSS OF HEAD IN 1½-INCH PIPE

Gage length = 23.85 ft.

Cross sectional area of pipe = .0141 sq. ft.

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	Apr. 26-15	15.560	65.25	0.2100	14.90	222.00
2	"	12.490	52.38	0.1865	13.23	175.00
3	"	10.180	42.70	0.1683	11.93	142.50
4	"	8.220	34.46	0.1513	10.73	115.10
5	"	5.705	23.93	0.1260	8.94	79.70
6	"	3.337	13.99	0.0952	6.75	45.50

TABLE 17

LOSS OF HEAD IN 2-INCH PIPE

Runs 1 to 12 gage length = 19.86 ft.

Runs 13 to 30 gage length = 19.85 ft.

Runs 31 to 98 gage length = 9.98 ft.

Cross sectional area of pipe = .0233 sq. ft. for runs 1 to 12.

Cross sectional area of pipe = .02334 sq. ft. for runs 13 to 30.

Cross sectional area of pipe = .02338 sq. ft. for runs 31 to 98.

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	Aug. 4-14	0.432	2.17	0.0704	3.02	9.15
2	"	0.431	2.17	0.0702	3.01	9.05
3	"	0.921	4.64	0.1067	4.58	21.00
4	"	0.939	4.73	0.1080	4.63	21.45
5	"	1.604	8.07	0.1398	6.00	36.00
6	"	1.600	8.05	0.1398	6.00	36.00
7	"	2.473	12.47	0.1755	7.53	56.60
8	"	2.478	12.49	0.1755	7.53	56.60
9	"	4.036	20.34	0.2235	9.60	92.00
10	"	4.061	20.46	0.2235	9.60	92.00
11	"	3.159	15.90	0.1968	8.45	71.40
12	"	3.149	15.87	0.1968	8.53	72.60
13	Feb. 11-15	0.382	1.92	0.0624	2.67	7.12
14	"	0.376	1.89	0.0619	2.65	7.01
15	"	0.386	1.94	0.0624	2.67	7.12
16	"	0.801	4.03	0.0936	3.94	16.10
17	"	0.780	3.92	0.0920	4.01	15.54
18	"	0.789	3.98	0.0926	3.97	15.80
19	"	1.497	7.54	0.1303	5.59	31.20
20	"	1.511	7.61	0.1311	5.42	31.50
21	"	1.412	7.11	0.1263	5.43	29.20
22	"	2.410	12.14	0.1681	7.20	51.90
23	Feb. 15-15	9.560	48.12	0.3350	14.36	206.00
24	"	11.025	55.60	0.3565	15.25	233.00
25	"	12.302	61.98	0.3320	14.22	202.60
26	"	5.451	27.47	0.2527	10.83	117.10
27	"	7.180	36.20	0.2910	12.46	155.40
28	"	7.018	35.32	0.2880	12.34	152.40
29	"	12.330	62.15	0.3840	16.43	270.40
30	"	12.330	62.15	0.3840	16.43	270.40
31	May 14-19	0.023	0.23	0.0214	0.91	0.84
32	"	0.060	0.60	0.0366	1.57	2.45
33	"	0.095	0.95	0.0462	1.98	3.90
34	"	0.214	2.14	0.0713	3.05	9.31
35	"	0.533	5.34	0.1164	4.98	24.80
36	"	0.807	8.08	0.1463	6.26	39.30
37	"	0.989	9.90	0.1636	7.00	49.10
38	"	1.231	12.33	0.1829	7.82	61.30
39	"	1.483	14.86	0.2015	8.62	74.30
40	"	1.968	19.70	0.2320	9.93	98.80
41	May 14-19	2.689	26.92	0.2752	11.79	138.80
42	"	3.081	30.87	0.2966	12.70	161.20
43	"	3.525	35.50	0.3180	13.60	185.50
44	"	3.976	29.79	0.3312	14.18	201.50

TABLE 17—Continued

45	May 14-19	2.756	27.59	0.2768	12.37	153.20
46	"	2.732	27.36	0.2749	11.77	138.50
47	"	3.343	33.49	0.3070	13.14	173.00
48	"	3.659	36.80	0.3200	13.70	187.90
49	"	4.069	40.72	0.3360	14.38	207.00
50	May 28-19	0.028	0.28	0.0224	0.96	0.92
51	"	0.058	0.58	0.0332	1.42	2.02
52	"	0.095	0.95	0.0443	1.90	3.60
53	"	0.202	2.02	0.0673	2.88	8.30
54	"	0.506	5.07	0.1093	4.68	21.90
55	"	0.837	8.38	0.1439	6.16	38.00
56	"	0.999	10.00	0.1582	6.76	45.80
57	"	1.308	13.10	0.1833	7.84	61.70
58	"	1.468	14.70	0.1963	8.40	70.70
59	"	1.980	19.82	0.2280	9.75	95.20
60	"	2.505	25.09	0.2576	11.02	121.60
61	"	2.939	29.40	0.2800	11.98	143.70
62	"	3.437	34.40	0.3032	12.97	168.40
63	"	4.144	41.48	0.3320	14.20	202.00
64	"	1.293	12.96	0.1816	7.77	60.60
65	"	1.634	16.37	0.2050	8.77	77.00
66	"	2.304	23.20	0.2395	10.25	105.20
67	"	2.812	28.14	0.2723	11.66	136.00
68	"	3.228	32.31	0.2930	12.54	157.20
69	"	3.784	37.90	0.3217	13.77	190.00
70	"	0.977	9.78	0.1550	6.63	44.00
71	"	0.663	6.64	0.1264	5.41	29.30
72	June 28-19	0.111	1.11	0.0502	2.15	4.62
73	"	0.225	2.25	0.0732	3.13	9.81
74	"	0.395	3.95	0.0998	4.27	18.38
75	"	0.675	6.76	0.1331	5.70	32.50
76	"	1.023	10.26	0.1662	7.11	50.85
77	"	1.497	14.99	0.2040	8.72	76.25
78	"	1.973	19.77	0.2296	9.83	96.80
79	"	2.550	25.54	0.2646	11.32	128.10
80	"	3.041	30.44	0.2900	12.40	154.00
81	"	3.459	34.61	0.3112	13.32	177.40
82	"	4.045	40.50	0.3380	14.48	210.00
83	"	4.719	47.23	0.3660	15.66	245.20
84	"	2.040	20.44	0.2372	10.15	103.10
85	"	1.449	14.50	0.1969	8.42	71.00
86	"	3.592	36.00	0.3194	13.68	187.20
87	"	3.867	38.80	0.3329	14.25	203.50
88	"	4.522	45.30	0.3581	15.34	235.00
89	July 18-19	0.322	3.22	0.0873	3.74	13.95
90	"	0.639	6.39	0.1263	5.42	29.25
91	"	1.028	10.30	0.1628	6.98	48.60
92	"	1.581	15.85	0.2031	8.71	75.50
93	"	2.028	20.32	0.2298	9.83	96.40
94	"	2.480	24.85	0.2570	11.00	121.00
95	"	3.046	30.53	0.2838	12.13	147.30
96	"	3.446	34.54	0.3019	12.92	166.50
97	"	3.992	40.00	0.3277	14.02	196.00
98	"	4.646	46.50	0.3520	15.06	226.00

TABLE 18

LOSS OF HEAD IN 4-INCH PIPE

Runs 1 to 15 gage length = 19.0 ft.

Runs 16 to 54 gage length = 16.927 ft.

Cross sectional area of pipe = .08755 sq. ft. for runs 16-64.

Cross sectional area of pipe (from Kent) = .0884 sq. ft. for runs 1-15

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	July 29-14	0.190	1.00	0.3067	3.465	12.00
2	"	0.221	1.16	0.3052	3.460	11.98
3	"	0.218	1.15	0.3067	3.465	12.00
4	"	0.768	4.04	0.5935	6.725	45.25
5	"	0.848	4.47	0.6085	6.880	47.40
6	"	0.846	4.46	0.6035	6.840	46.70
7	"	1.232	6.48	0.7600	8.600	74.00
8	"	1.273	6.70	0.7670	8.670	75.30
9	"	2.100	11.05	1.0070	11.400	130.00
10	"	2.137	11.24	1.0205	11.560	133.80
11	"	2.107	11.08	1.0140	11.500	132.20
12	"	2.698	14.20	1.0865	12.320	152.00
13	"	2.498	13.14	1.0900	12.350	152.50
14	"	1.708	8.98	0.8920	10.110	102.00
15	"	1.790	9.42	0.9080	10.270	105.50
16	April 18-19	0.035	0.21	0.1382	1.579	2.49
17	"	0.031	0.18	0.1349	1.540	2.37
18	"					
19	"	0.026	0.15	0.1330	1.519	2.32
20	"	0.027	0.16	0.1089	1.244	1.55
21	"	0.023	0.14	0.1065	1.217	1.48
22	"	0.045	0.27	0.1466	1.673	2.81
23	"	0.043	0.25	0.1408	1.609	2.58
24	April 19-19	0.043	0.25	0.1416	1.617	2.62
25	"	0.077	0.45	0.1920	2.191	4.81
26	"					
27	"	1.009	5.95	0.7780	8.880	79.10
28	"	1.556	9.19	0.9810	11.210	125.80
29	"	1.983	11.71	1.1180	12.780	163.50
30	"	2.522	14.90	1.2730	14.540	212.00
31	"	2.971	17.54	1.4580	16.650	277.50
32	"	3.381	19.97	1.4970	17.080	292.00
33	May 2-19	0.022	0.13	0.1305	1.490	2.22
34	"	1.442	8.50	0.9460	10.810	116.90
35	"	2.925	17.26	1.3700	15.650	245.30
36	"	0.033	0.19	0.1241	1.418	2.02
37	"	0.315	1.86	0.4160	4.750	22.60
38	"	0.506	2.99	0.5380	6.150	37.90
39	"	0.756	4.46	0.6595	7.530	56.90
40	"	4.042	23.85	1.6450	18.790	354.00
41	June 26-19	0.027	0.16	0.1133	1.29	1.68
42	"	0.054	0.32	0.1657	1.89	3.58
43	"	0.069	0.40	0.1918	2.19	4.80

TABLE 18—Continued

44	June 26-19	0.107	0.63	0.2402	2.75	7.53
45	"	0.323	1.91	0.4320	4.93	24.35
46	"	0.502	2.96	0.5470	6.25	39.10
47	"	0.713	4.21	0.6600	7.54	57.00
48	"	0.897	5.29	0.7465	8.53	73.00
49	"	0.997	5.89	0.7890	9.01	81.30
50	"	1.498	8.84	0.9930	11.36	129.00
51	"	1.993	11.77	1.1500	13.14	172.80
52	"	2.547	15.04	1.3010	14.87	221.50
53	"	3.102	18.31	1.4400	16.45	271.00
54	"	3.473	20.52	1.5300	17.47	306.00

TABLE 19

LOSS OF HEAD IN 6-INCH PIPE

Runs 1 to 7 gage length = 21 ft.

Runs 8 to 23 gage length = 20 ft.

Runs 24 to 94 gage length = 23 ft.

Cross sectional area of pipe = .1962 sq. ft. for runs 1 to 7.

Cross sectional area of pipe = .2006 sq. ft. for runs 8 to 23.

Cross sectional area of pipe = .2062 sq. ft. for runs 24 to 94.

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	May 15-14	0.003	0.02	0.107	0.55	0.30
2	"	0.020	0.10	0.293	1.49	2.23
3	"	0.080	0.38	0.519	2.64	6.99
4	"	0.168	0.80	0.750	3.82	14.59
5	"	0.265	1.26	1.048	5.34	28.50
6	"	0.500	2.38	1.446	7.36	54.20
7	"	0.860	4.09	1.915	9.75	95.00
8	Aug. 8-14	2.099	10.48	3.015	15.00	225.00
9	"	2.064	10.32	2.990	14.90	222.00
10	"	1.699	8.48	2.640	13.15	173.00
11	"	1.791	8.95	2.590	12.90	166.60
12	"	1.819	9.08	2.742	13.68	186.70
13	"	1.326	6.63	2.372	11.82	139.90
14	"	1.330	6.64	2.325	11.59	134.00
15	"	1.133	5.66	2.200	10.96	120.00
16	"	1.136	5.68	2.147	10.70	114.20
17	"	0.805	4.02	1.715	8.54	72.90
18	"	0.824	4.12	1.867	9.30	86.30
19	"	0.823	4.11	1.856	9.24	85.20
20	"	0.601	3.00	1.544	7.68	59.00
21	"	0.593	2.96	1.549	7.70	59.30
22	"	0.292	1.47	1.002	4.98	24.80
23	"	0.262	1.31	1.025	5.10	26.00
24	July 9-18	0.025	0.11	0.282	1.37	1.86
25	"	0.062	0.27	0.462	2.24	5.01
26	"	0.103	0.45	0.609	2.85	8.72
27	"	0.198	0.86	0.871	4.22	17.80
28	"	0.302	1.33	1.080	5.24	27.40
29	"	0.496	2.16	1.394	6.76	45.70
30	"	0.715	3.11	1.695	8.22	67.50
31	"	0.919	4.00	1.933	9.38	87.80
32	"	1.079	4.69	2.085	10.11	102.00
33	"	1.479	6.43	2.464	11.95	142.50
34	July 22-18	0.025	0.11	0.270	1.31	1.71
35	"	0.062	0.27	0.464	2.25	5.06
36	"	0.107	0.47	0.605	2.93	8.59
37	"	0.207	0.90	0.870	4.22	17.80
38	"	0.306	1.33	1.078	5.23	27.30
39	"	0.502	2.18	1.385	6.72	45.10
40	"	0.729	3.17	1.698	8.24	67.80
41	"	0.922	4.01	1.910	9.26	85.70
42	"	1.111	4.83	2.122	10.30	106.10
43	"	1.523	6.62	2.500	12.12	146.60

TABLE 19—Continued

44	Aug 26-18	0.025	0.10	0.282	1.37	1.87
45	"	0.065	0.28	0.471	2.28	5.21
46	"	0.103	0.45	0.611	2.96	8.78
47	"	0.201	0.88	0.873	4.23	17.98
48	"	0.298	1.30	1.080	5.24	27.50
49	"	0.475	2.07	1.381	6.71	45.10
50	"	0.684	2.98	1.679	8.15	66.50
51	"	0.888	3.87	1.920	9.32	87.00
52	"	1.124	4.89	2.178	10.55	111.80
53	"	1.502	6.53	2.519	12.20	148.80
54	Oct. 11-18	0.024	0.10	0.286	1.39	1.93
55	"	0.066	0.29	0.468	2.27	5.18
56	"	0.102	0.44	0.606	2.94	8.62
57	"	0.194	0.88	0.850	4.12	17.04
58	"	0.297	1.29	1.065	5.17	26.70
59	"	0.482	2.10	1.368	6.64	44.10
60	"	0.681	2.96	1.650	8.00	64.00
61	"	0.879	3.82	1.888	9.16	84.00
62	"	1.142	4.97	2.165	10.50	110.50
63	"	1.469	6.39	2.460	11.92	142.40
64	Mar. 14-19	0.031	0.14	0.273	1.32	1.76
65	"	0.073	0.32	0.469	2.28	5.20
66	"	0.120	0.52	0.617	2.99	8.94
67	"	0.232	1.00	0.884	4.28	18.41
68	"	0.345	1.50	1.089	5.28	28.00
69	"	0.533	2.32	1.374	6.66	44.40
70	"	0.780	3.39	1.685	8.17	66.70
71	"	0.987	4.29	1.907	9.25	85.90
72	"	1.242	5.40	2.155	10.45	109.50
73	"	1.632	7.10	2.489	12.08	146.00
74	May 16-19	0.025	0.11	0.289	1.40	1.96
75	"	0.090	0.39	0.494	2.40	5.74
76	"	0.136	0.59	0.620	3.01	9.05
77	"	0.256	1.11	0.868	4.21	17.75
78	"	0.401	1.74	1.093	5.30	28.20
79	"	0.622	2.70	1.374	6.67	44.50
80	"	0.910	3.96	1.674	8.13	66.10
81	"	1.179	5.13	1.920	9.32	87.00
82	"	1.381	6.01	2.085	10.12	102.50
83	"	1.894	8.24	2.440	11.82	140.50
84	"	1.889	8.22	2.424	11.75	138.50
85	"	0.455	1.98	1.162	5.63	31.70
86	May 23-19	1.522	6.62	2.173	10.53	111.50
87	"	1.635	7.11	2.260	10.97	120.60
88	"	0.274	1.19	0.895	4.34	18.90
89	"	0.538	2.34	1.268	6.15	37.80
90	"	0.787	3.42	1.542	7.49	56.62
91	"	1.049	4.56	1.795	8.72	76.00
92	"	1.320	5.74	2.022	9.82	96.80
93	"	1.493	6.50	2.155	10.45	109.50
94	"	1.963	8.54	2.454	11.90	142.00

TABLE 20

LOSS OF HEAD IN 8-INCH PIPE

Gage length = 22.0 ft.

Cross sectional area of pipe = .358 sq. ft.

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	July 1-19	3.171	14.41	6.809	19.01	362.00
2	"	3.072	13.96	6.624	18.50	342.50
3	"	2.932	13.32	6.355	17.75	315.00
4	"	2.499	11.34	5.844	16.32	266.00
5	"	1.908	8.67	5.164	14.43	208.00
6	"	1.598	7.26	4.750	13.27	175.80
7	"	1.200	5.45	4.190	11.70	136.80
8	"	0.836	3.80	3.629	10.14	102.80
9	"	0.680	3.09	3.079	8.59	73.80
10	"	0.401	1.82	2.345	6.55	42.80
11	"	0.228	1.04	1.757	4.90	24.00
12	"	0.098	0.45	1.087	3.04	9.20
13	July 2-19	0.109	0.50	1.084	3.03	9.17
14	"	0.186	0.85	1.555	4.34	18.84
15	"	0.323	1.47	2.090	5.83	34.00
16	"	0.420	1.91	2.423	6.77	45.80
17	"	0.521	2.37	2.711	7.57	57.20
18	"	0.704	3.20	3.660	8.84	78.00
19	"	0.831	3.88	3.446	9.62	92.50
20	"	2.793	12.68	6.166	17.22	296.00
21	"	1.032	4.68	3.884	10.84	117.80

TABLE 21

LOSS OF HEAD IN 12-INCH PIPE

Gage length = 33.0 ft.

Cross sectional area of pipe = .7854 sq. ft.

Run	Date of Test	Head Loss		Cubic feet per second	Velocity in feet per second	Square of Velocity
		in Gage Length ft.	in Length of 100 feet ft.			
1	June 14-19	0.070	0.21	2.353	2.99	8.98
2	"	0.383	1.16	5.274	6.70	44.90
3	"	0.198	0.60	3.815	4.86	23.55
4	"	0.520	1.58	6.111	7.78	60.50
5	"	0.327	0.99	4.935	6.28	39.50
6	"	0.621	1.88	6.608	8.41	70.70
7	"	0.779	2.36	7.378	9.40	88.00
8	"	0.989	3.00	8.731	11.11	123.50
9	"	1.048	3.18	9.115	11.60	134.60
10	"	0.501	1.52	6.124	7.80	60.70
11	June 16-19	1.088	3.30	9.255	11.79	139.00
12	"	0.815	2.47	7.508	9.55	91.20
13	"	0.584	1.77	6.543	8.32	69.20
14	"	0.379	1.15	5.350	6.81	46.40
15	"	0.220	0.67	4.074	5.19	26.90
16	"	0.124	0.37	3.008	3.82	14.65
17	"	0.062	0.18	2.072	2.64	6.95
18	"	0.645	1.96	6.872	8.75	76.60
19	"	0.813	2.46	7.542	9.60	92.00
20	"	0.907	2.75	7.901	10.06	101.40
21	"	0.801	2.43	7.503	9.55	91.00
22	"	1.063	3.22	9.015	11.47	131.80
23	"	0.938	2.84	8.055	10.25	105.10
24	"	0.829	2.51	8.250	10.50	110.50
25	"	0.684	2.08	7.448	9.48	89.90
26	"	0.495	1.50	6.071	7.74	59.80
27	"	0.320	0.97	4.937	6.28	39.40
28	"	0.151	0.46	3.363	4.28	18.30
29	"	0.950	2.88	8.781	11.18	125.00
30	June 17-19	0.999	3.03	8.220	10.46	109.50
31	"	0.840	2.54	7.639	9.72	94.30
32	"	0.817	2.48	7.542	9.60	92.00
33	"	0.618	1.87	6.730	8.56	73.30
34	"	0.681	2.06	6.998	8.91	79.30
35	"	0.758	2.30	7.296	9.29	86.10
36	"	1.118	3.39	8.954	11.40	130.00
37	"	0.894	2.71	7.885	10.04	101.00
38	"	0.011	0.03	0.803	1.02	1.05
39	"	0.123	0.37	3.041	3.87	15.00
40	"	0.060	0.18	2.072	2.64	6.95

TABLE 21—Continued

41	June 18-19	0.988	2.99	8.520	10.85	117.70
42	"	0.985	2.98	8.787	11.18	125.20
43	"	0.999	3.03	8.873	11.29	127.50
44	"	0.971	2.94	8.390	10.68	114.20
45	"	0.961	2.91	8.430	10.73	115.20
46	June 24-19	0.916	2.78	7.954	10.13	102.70
47	"	0.784	2.38	7.421	9.45	89.10
48	"	0.671	2.03	6.938	8.83	77.80
49	"	0.594	1.80	6.534	8.32	69.20
50	"	0.497	1.51	6.044	7.68	59.00
51	"	0.417	1.26	5.543	7.05	49.70
52	"	0.343	1.04	5.064	6.45	41.50
53	"	0.270	0.82	4.416	5.62	31.60
54	"	0.121	0.37	2.974	3.78	14.30
55	"	0.071	0.22	2.268	2.89	8.32
56	"	1.059	3.21	9.101	11.59	134.50
57	"	1.014	3.08	8.754	11.14	124.20
58	"	0.966	2.93	8.250	10.60	110.60

TABLE 22

LOSS OF HEAD IN 1/2-INCH GATE VALVES

VALVE NO. 0

Gage Length = 20.43 feet.

Pipe Cross Section = 0.002097 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	$k \text{ in } \frac{v^2}{2g}$
	1915		feet	feet	feet				
1	May 10	1/8	19.890	2.38	17.51	0.00617	2.94	8.64	131.3
2	"	1/8	16.400	1.99	14.41	0.00559	2.67	7.12	
3	"	1/8	9.880	1.16	8.72	0.00430	2.05	4.21	
4	"	1/8	12.190	1.47	10.72	0.00481	2.30	5.27	
5	"	1/8	3.995	0.48	3.52	0.00272	1.30	1.68	
6	"	1/8	6.360	0.74	5.62	0.00346	1.65	2.72	
7	May 4	1/4	18.560	7.63	10.93	0.01185	5.65	31.90	21.82
8	"	1/4	11.150	4.78	6.37	0.00913	4.36	19.00	
9	"	1/4	7.100	3.15	3.95	0.00723	3.45	11.90	
10	"	1/4	3.969	1.78	2.19	0.00531	2.54	6.42	
11	"	1/4	1.411	0.62	0.79	0.00313	1.49	2.23	
12	May 5	3/8	20.840	12.95	7.89	0.01580	7.53	56.80	8.99
13	"	3/8	13.590	8.53	5.06	0.01258	6.00	36.00	
14	"	3/8	8.730	5.56	3.17	0.00998	4.76	22.60	
15	"	3/8	4.719	3.14	1.58	0.00718	3.43	11.75	
16	"	3/8	2.504	1.65	0.85	0.00512	2.44	5.96	
17	"	3/8	17.060	10.65	6.41	0.01417	6.76	45.70	
18	"	1/2	20.140	15.10	5.04	0.01713	8.17	66.80	4.84
19	"	1/2	14.680	11.02	3.66	0.01446	6.90	47.60	
20	"	1/2	10.310	8.09	2.22	0.01205	5.75	33.10	
21	"	1/2	6.570	4.98	1.59	0.00935	4.46	19.90	
22	"	1/2	17.900	13.35	4.55	0.01606	7.66	58.70	
23	"	1/2	3.611	2.90	0.71	0.00684	3.26	10.65	
24	"	1/2	1.912	1.48	0.43	0.00484	2.31	5.34	
25	"	3/4	22.110	18.47	3.64	0.01915	9.14	83.40	2.674
26	"	3/4	17.800	15.15	2.65	0.01718	8.20	67.20	
27	"	3/4	11.070	9.38	1.69	0.01326	6.33	40.00	
28	May 12	3/4	8.000	6.82	1.18	0.01114	5.32	28.20	
29	"	3/4	4.819	4.17	0.65	0.00846	4.04	16.32	
30	"	3/4	2.532	2.23	0.30	0.00597	2.85	8.12	
31	"	1	20.800	19.46	1.34	0.01970	9.40	88.30	0.954
32	"	1	7.840	7.40	0.44	0.01163	5.55	30.75	
33	"	1	2.903	2.88	0.02	0.00679	3.24	10.50	
34	"	1	4.885	4.62	0.27	0.00898	4.28	18.32	
35	"	1	15.410	14.35	1.06	0.01670	7.96	63.40	

VALVE NO. 4.

Gage Length = 10 feet.

Pipe Cross Section = 0.00206 Sq. Ft.

1919									
1	July 17	$\frac{1}{8}$	0.871	0.014	0.857	0.00073	0.35	0.12	445
2	"	$\frac{1}{8}$	1.454	0.021	1.433	0.00094	0.46	0.21	
3	"	$\frac{1}{8}$	2.663	0.038	2.625	0.00127	0.61	0.38	
4	"	$\frac{1}{8}$	3.228	0.046	3.172	0.00140	0.68	0.46	
5	"	$\frac{1}{8}$	5.231	0.072	5.159	0.00177	0.86	0.74	
6	July 16	$\frac{1}{4}$	0.455	0.041	0.414	0.00132	0.64	0.41	56.0
7	"	$\frac{1}{4}$	1.070	0.009	1.098	0.00198	0.96	0.93	
8	"	$\frac{1}{4}$	1.505	0.144	1.361	0.00250	1.21	1.47	
9	"	$\frac{1}{4}$	2.060	0.201	1.859	0.00294	1.43	2.04	
10	"	$\frac{1}{4}$	2.424	0.238	2.186	0.00318	1.54	2.38	
11	"	$\frac{1}{4}$	2.985	0.297	2.688	0.00356	1.73	2.99	
12	"	$\frac{1}{4}$	3.558	0.355	3.203	0.00388	1.88	3.55	
13	"	$\frac{1}{4}$	4.005	0.402	3.603	0.00413	2.00	4.03	
14	"	$\frac{1}{4}$	4.665	0.468	4.197	0.00445	2.16	4.67	
15	"	$\frac{1}{4}$	5.333	0.540	4.893	0.00478	2.32	5.40	
16	July 14	$\frac{3}{8}$	0.438	0.094	0.344	0.00203	0.98	0.97	19.43
17	"	$\frac{3}{8}$	0.987	0.228	0.759	0.00313	1.52	2.31	
18	"	$\frac{3}{8}$	1.536	0.452	1.084	0.00438	2.13	4.53	
19	"	$\frac{3}{8}$	1.993	0.484	1.509	0.00453	2.20	4.84	
20	"	$\frac{3}{8}$	2.511	0.627	1.884	0.00516	2.50	6.28	
21	"	$\frac{3}{8}$	3.128	0.780	2.348	0.00575	2.79	7.80	
22	"	$\frac{3}{8}$	3.572	0.891	2.681	0.00622	3.02	9.12	
23	"	$\frac{3}{8}$	4.032	0.990	3.040	0.00661	3.21	10.30	
24	"	$\frac{3}{8}$	4.680	1.130	3.550	0.00715	3.47	12.05	
25	"	$\frac{3}{8}$	5.278	1.270	4.008	0.00763	3.71	13.73	
26	"	$\frac{3}{8}$	1.924	0.604	1.320	0.00506	2.46	6.04	
27	"	$\frac{3}{8}$	2.951	0.727	2.224	0.00555	2.70	7.27	
28	"	$\frac{3}{8}$	0.471	0.102	0.369	0.00211	1.02	1.05	
29	"	$\frac{3}{8}$	1.040	0.238	0.802	0.00318	1.54	2.38	
30	"	$\frac{3}{8}$	1.481	0.348	1.133	0.00384	1.86	3.48	
31	"	$\frac{3}{8}$	2.041	0.482	1.559	0.00452	2.20	4.82	
32	"	$\frac{3}{8}$	2.565	0.620	1.945	0.00513	2.49	6.20	
33	"	$\frac{3}{8}$	3.122	0.760	2.362	0.00568	2.76	7.60	
34	"	$\frac{3}{8}$	3.780	0.846	2.934	0.00620	3.01	9.07	
35	"	$\frac{3}{8}$	4.191	1.020	3.171	0.00669	3.25	10.56	
36	"	$\frac{3}{8}$	4.590	1.110	3.480	0.00706	3.43	11.76	
37	"	$\frac{3}{8}$	5.150	1.230	3.920	0.00750	3.64	13.25	
38	July 11	$\frac{1}{2}$	0.206	0.080	0.126	0.00187	0.91	0.82	7.19
39	"	$\frac{1}{2}$	0.684	0.310	0.374	0.00363	1.76	3.10	
40	"	$\frac{1}{2}$	1.148	0.529	0.619	0.00474	2.30	5.30	
41	"	$\frac{1}{2}$	1.657	0.790	0.867	0.00578	2.81	7.89	
42	"	$\frac{1}{2}$	2.175	1.020	1.155	0.00669	3.25	10.55	
43	"	$\frac{1}{2}$	2.733	1.260	1.473	0.00756	3.67	13.50	
44	"	$\frac{1}{2}$	3.165	1.400	1.765	0.00805	3.91	15.28	
45	"	$\frac{1}{2}$	3.600	1.580	2.020	0.00859	4.17	17.40	
46	"	$\frac{1}{2}$	4.533	2.030	2.503	0.00987	4.79	23.00	
47	"	$\frac{1}{2}$	5.308	2.370	2.938	0.01075	5.22	27.25	
48	June 21	$\frac{3}{4}$	0.347	0.247	0.104	0.00322	1.56	2.45	1.892
49	"	$\frac{3}{4}$	0.761	0.580	0.181	0.00496	2.41	5.80	
50	"	$\frac{3}{4}$	1.069	0.835	0.234	0.00598	2.90	8.44	
51	"	$\frac{3}{4}$	1.651	1.250	0.401	0.00755	3.67	13.45	
52	"	$\frac{3}{4}$	2.085	1.560	0.525	0.00855	4.15	17.23	
53	"	$\frac{3}{4}$	2.565	1.930	0.635	0.00965	4.67	21.80	
54	"	$\frac{3}{4}$	3.287	2.460	0.827	0.01096	5.32	28.30	
55	"	$\frac{3}{4}$	3.534	2.650	0.889	0.01130	5.52	30.50	
56	"	$\frac{3}{4}$	4.134	3.370	0.764	0.01298	6.30	39.70	

VALVE NO. 4—Continued

57	June 21	3/4	4.678	3.470	1.208	0.01324	6.40	41.00	0.755
58	"	3/4	5.484	4.090	1.394	0.01443	6.99	48.80	
59	June 20	1	0.352	0.288	0.064	0.00350	1.70	2.89	
60	"	1	0.837	0.764	0.073	0.00568	2.76	7.63	
61	"	1	1.066	0.951	0.115	0.00648	3.15	9.90	
62	"	1	1.615	1.420	0.195	0.00820	3.98	15.43	
63	"	1	2.056	1.840	0.216	0.00935	4.54	20.60	
64	"	1	2.587	2.260	0.327	0.01047	5.08	25.80	
65	"	1	3.124	2.760	0.364	0.01164	5.65	31.95	
66	"	1	3.552	3.080	0.472	0.01240	6.02	36.20	
67	"	1	4.004	3.510	0.494	0.01328	6.44	41.50	
68	"	1	4.542	3.980	0.562	0.01419	6.89	47.50	
69	"	1	5.014	4.460	0.554	0.01508	7.31	53.60	

VALVE NO. 5.

Gage Length = 10 feet

Pipe Cross Section = 0.00206 sq. ft.

	1919		feet	feet	feet				
1	July 18	1/8	0.823	0.013	0.810	0.00076	0.37	0.14	325.8
2	"	1/8	2.145	0.041	2.104	0.00133	0.65	0.42	
3	"	1/8	3.060	0.058	3.002	0.00159	0.77	0.60	
4	"	1/8	3.878	0.072	3.806	0.00179	0.87	0.75	
5	"	1/8	4.856	0.092	4.764	0.00200	0.97	0.94	
6	July 20	1/4	C. 672	0.060	C. 612	0.00163	0.79	0.63	57.0
7	"	1/4	1.210	0.113	1.097	0.00221	1.07	1.15	
8	"	1/4	1.933	0.183	1.750	0.00279	1.35	1.84	
9	"	1/4	2.400	0.226	2.174	0.00311	1.51	2.28	
10	"	1/4	3.015	0.285	2.730	0.00348	1.69	2.86	
11	"	1/4	3.687	0.350	3.337	0.00386	1.87	3.52	
12	"	1/4	4.264	0.409	3.855	0.00417	2.06	4.10	
13	"	1/4	5.427	0.529	4.898	0.00474	2.30	5.30	
14	"	1/4	3.849	0.398	3.451	0.00411	2.00	3.99	
15	"	1/4	4.389	0.452	3.937	0.00438	2.13	4.53	
16	"	1/4	3.445	0.350	3.095	0.00387	1.88	3.54	
17	"	1/4	2.640	0.260	2.380	0.00334	1.62	2.63	
18	"	1/4	1.845	0.180	1.665	0.00276	1.34	1.80	
19	"	1/4	1.031	0.094	0.937	0.00202	0.98	0.96	
20	"	1/4	0.381	0.032	0.349	0.00121	0.59	0.35	
21	"	1/4	0.660	0.066	0.594	0.00170	0.83	0.68	
22	"	1/4	1.247	0.131	1.116	0.00237	1.15	1.33	
23	"	1/4	1.962	0.210	1.752	0.00300	1.46	2.12	
24	"	1/4	2.409	0.260	2.149	0.00333	1.62	2.62	
25	"	1/4	3.009	0.324	2.685	0.00371	1.80	3.25	
26	"	1/4	3.634	0.392	3.242	0.00408	1.98	3.93	
27	"	1/4	4.538	0.489	4.049	0.00454	2.21	4.87	
28	"	1/4	5.021	0.540	4.481	0.00478	2.32	5.40	
29	July 22	3/8	0.548	0.115	0.433	0.00232	1.13	1.27	19.58
30	"	3/8	1.320	0.300	1.020	0.00371	1.80	3.25	
31	"	3/8	1.929	0.430	1.499	0.00452	2.20	4.82	
32	"	3/8	2.444	0.550	1.894	0.00510	2.48	6.15	
33	"	3/8	3.027	0.700	2.327	0.00570	2.77	7.68	
34	"	3/8	3.594	0.825	2.769	0.00622	3.02	9.11	
35	"	3/8	4.286	0.995	3.291	0.00682	3.33	10.98	
36	"	3/8	5.000	1.160	3.840	0.00737	3.58	12.80	

VALVE NO. 5—Continued

37	July 23	$\frac{1}{8}$	0.771	0.334	0.437	0.00377	1.83	3.35	7.78
38	"	$\frac{1}{8}$	1.267	0.560	0.707	0.00487	2.37	5.60	
39	"	$\frac{1}{8}$	1.817	0.812	1.005	0.00598	2.91	8.44	
40	"	$\frac{1}{8}$	2.523	1.120	1.403	0.00709	3.44	11.88	
41	"	$\frac{1}{8}$	3.091	1.340	1.751	0.00788	3.82	14.68	
42	"	$\frac{1}{8}$	3.678	1.600	2.078	0.00865	4.20	17.63	1.868
43	"	$\frac{1}{8}$	4.233	1.820	2.413	0.00929	4.52	20.40	
44	"	$\frac{1}{8}$	5.092	2.170	2.922	0.01024	4.97	24.70	
45	July 24	$\frac{3}{4}$	0.720	0.551	0.169	0.00484	2.35	5.52	
46	"	$\frac{3}{4}$	1.197	0.928	0.269	0.00639	3.10	9.60	
47	"	$\frac{3}{4}$	1.797	1.350	0.447	0.00794	3.85	14.85	0.688
48	"	$\frac{3}{4}$	2.471	1.860	0.611	0.00941	4.57	20.90	
49	"	$\frac{3}{4}$	2.954	2.230	0.724	0.01040	5.05	25.50	
50	"	$\frac{3}{4}$	3.602	2.700	0.902	0.01152	5.59	31.20	
51	"	$\frac{3}{4}$	4.165	3.120	1.045	0.01244	6.04	36.50	
52	"	$\frac{3}{4}$	4.314	3.210	1.104	0.01266	6.14	37.70	0.688
53	"	1	0.630	0.575	0.055	0.00493	2.39	5.75	
54	"	1	1.231	1.110	0.121	0.00708	3.44	11.84	
55	"	1	1.869	1.670	0.199	0.00887	4.31	18.60	
56	"	1	2.450	2.180	0.270	0.01027	4.98	24.80	
57	"	1	3.230	2.860	0.370	0.01190	5.77	33.30	0.688
58	"	1	3.624	3.170	0.454	0.01255	6.09	37.20	
59	"	1	4.144	3.520	0.624	0.01330	6.45	41.60	
60	"	1	4.987	4.350	0.639	0.01488	7.22	52.20	
61	"	1	4.826	4.310	0.516	0.01480	7.18	51.70	

VALVE NO. 6

Gage Length = 10 feet

Pipe Cross Section = 0.00206 sq. ft.

	1919		feet	feet	feet				
1	July 25	$\frac{1}{8}$	0.844	0.007	0.837	0.00588	0.28	0.08	593
2	"	$\frac{1}{8}$	1.891	0.019	1.872	0.00895	0.44	0.19	
3	"	$\frac{1}{8}$	2.212	0.022	2.190	0.00997	0.49	0.24	
4	"	$\frac{1}{8}$	2.787	0.029	2.758	0.01130	0.55	0.30	
5	"	$\frac{1}{8}$	3.516	0.037	3.479	0.01273	0.61	0.38	
6	"	$\frac{1}{8}$	4.260	0.046	4.214	0.01411	0.69	0.47	79.6
7	"	$\frac{1}{8}$	4.850	0.051	4.799	0.01497	0.72	0.53	
8	"	$\frac{1}{8}$	1.173	0.011	1.162	0.00712	0.34	0.12	
9	July 27	$\frac{1}{4}$	0.676	0.048	0.628	0.00146	0.71	0.50	
10	"	$\frac{1}{4}$	1.215	0.089	1.126	0.00198	0.96	0.93	
11	"	$\frac{1}{4}$	1.803	0.135	1.668	0.00240	1.16	1.37	79.6
12	"	$\frac{1}{4}$	2.517	0.175	2.342	0.00272	1.32	1.75	
13	"	$\frac{1}{4}$	3.342	0.235	3.107	0.00317	1.54	2.37	
14	"	$\frac{1}{4}$	3.693	0.244	3.449	0.00323	1.57	2.46	
15	"	$\frac{1}{4}$	4.317	0.286	4.031	0.00348	1.69	2.87	
16	"	$\frac{1}{4}$	5.018	0.335	4.683	0.00377	1.83	3.35	79.6
17	"	$\frac{1}{4}$	0.574	0.420	0.532	0.00133	0.65	0.42	
18	"	$\frac{1}{4}$	1.027	0.077	0.950	0.00180	0.87	0.77	
19	"	$\frac{1}{4}$	1.668	0.112	1.556	0.00225	1.09	1.12	
20	"	$\frac{1}{4}$	2.358	0.182	2.176	0.00280	1.35	1.84	
21	"	$\frac{1}{4}$	3.032	0.226	2.806	0.00311	1.51	2.28	79.6
22	"	$\frac{1}{4}$	3.597	0.260	3.337	0.00332	1.62	2.60	
23	"	$\frac{1}{4}$	4.189	0.328	3.861	0.00373	1.81	3.28	
24	"	$\frac{1}{4}$	4.949	0.378	4.571	0.00400	1.94	3.78	

VALVE NO. 6—Continued

25	July 29	1/4	0.620	0.045	0.575	0.00140	0.68	0.46	
26	"	1/4	1.194	0.089	1.105	0.00196	0.95	0.91	
27	"	1/4	1.794	0.136	1.658	0.00240	1.17	1.37	
28	"	1/4	2.361	0.179	2.182	0.00275	1.33	1.78	
29	"	1/4	3.002	0.224	2.778	0.00309	1.50	2.25	
30	"	1/4	3.730	0.282	3.448	0.00346	1.68	2.82	
31	"	1/4	4.324	0.309	4.015	0.00362	1.76	3.09	
32	"	1/4	5.293	0.370	4.923	0.00397	1.93	3.73	
33	"	3/8	0.614	0.120	0.494	0.00226	1.10	1.21	25.43
34	"	3/8	1.324	0.264	1.060	0.00335	1.63	2.65	
35	"	3/8	1.899	0.382	1.517	0.00404	1.96	3.84	
36	"	3/8	2.562	0.522	2.040	0.00472	2.29	5.24	
37	"	3/8	3.183	0.658	2.525	0.00527	2.56	6.57	
38	"	3/8	3.852	0.770	3.082	0.00571	2.77	7.68	
39	"	3/8	4.370	0.858	3.512	0.00608	2.95	8.72	
40	"	3/8	5.146	0.970	4.176	0.00654	3.18	10.10	
41	"	3/8	1.520	0.305	1.215	0.00360	1.75	3.06	
42	"	3/8	2.564	0.521	2.043	0.00470	2.28	5.21	
43	"	3/8	3.544	0.740	2.804	0.00557	2.71	7.38	
44	"	3/8	4.453	0.904	3.549	0.00627	3.05	9.30	
45	July 30	1/2	0.620	0.213	0.407	0.00301	1.46	2.14	11.16
46	"	1/2	1.188	0.424	0.764	0.00424	2.06	4.24	
47	"	1/2	1.806	0.661	1.145	0.00530	2.57	6.62	
48	"	1/2	2.390	0.867	1.523	0.00612	2.97	8.85	
49	"	1/2	3.036	1.080	1.956	0.00692	3.36	11.32	
50	"	1/2	3.605	1.250	2.355	0.00755	3.67	13.49	
51	"	1/2	4.453	1.450	3.003	0.00817	3.97	15.75	
52	"	1/2	5.013	1.640	3.373	0.00867	4.26	18.20	
53	"	1/2	1.361	0.492	0.869	0.00458	2.23	4.95	
54	"	1/2	2.535	0.929	1.606	0.00637	3.10	9.60	
55	"	1/2	3.705	1.348	2.357	0.00794	3.86	14.88	
56	July 31	3/4	0.658	0.465	0.193	0.00444	2.15	4.65	2.383
57	"	3/4	1.267	0.881	0.326	0.00618	3.01	9.03	
58	"	3/4	1.808	1.290	0.518	0.00766	3.73	13.92	
59	"	3/4	2.393	1.700	0.693	0.00893	4.35	18.90	
60	"	3/4	2.969	2.100	0.869	0.01003	4.87	23.86	
61	"	3/4	3.599	2.530	1.069	0.01108	5.38	29.10	
62	"	3/4	4.201	2.940	1.261	0.01203	5.85	34.30	
63	"	3/4	4.790	3.320	1.476	0.01285	6.25	39.20	
64	Aug. 1	1	0.615	0.540	0.075	0.00480	2.33	5.44	0.836
65	"	1	1.316	1.150	0.166	0.00729	3.55	12.60	
66	"	1	1.834	1.600	0.234	0.00870	4.22	17.80	
67	"	1	2.394	2.090	0.304	0.01002	4.87	23.70	
68	"	1	3.005	2.620	0.385	0.01130	5.48	30.10	
69	"	1	3.558	3.070	0.488	0.01235	6.00	36.00	
70	"	1	4.394	3.810	0.584	0.01387	6.73	45.40	
71	"	1	4.984	4.320	0.664	0.01480	7.18	51.80	
72	"	1	0.531	0.446	0.085	0.00440	2.14	4.58	
73	"	1	1.103	0.990	0.113	0.00658	3.20	10.21	
74	"	1	1.576	1.390	0.186	0.00798	3.88	15.03	
75	"	1	2.767	2.410	0.357	0.01083	5.26	27.70	

TABLE 23

LOSS OF HEAD IN $\frac{1}{4}$ -INCH GATE VALVES

VALVE NO. 0

Gage Length = 20.43 feet

Pipe Cross Section = 0.00367 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	k in $\frac{v^2}{2g}$
	1915		feet.	feet	feet				
1	April 7	$\frac{1}{8}$	21.100	0.32	20.78	0.00525	1.43	2.05	635.0
2	"	$\frac{1}{8}$	13.550	0.21	13.34	0.00424	1.16	1.33	
3	"	$\frac{1}{8}$	7.050	0.11	6.94	0.00304	0.83	0.68	
4	"	$\frac{1}{8}$	9.085	0.14	8.95	0.00347	0.95	0.89	
5	"	$\frac{1}{8}$	12.530	0.15	12.38	0.00362	0.99	0.97	
6	"	$\frac{1}{8}$	15.600	2.60	13.00	0.00468	1.28	1.62	
7	"	$\frac{1}{8}$	21.420	0.36	21.06	0.00556	1.52	2.30	
8	"	$\frac{1}{8}$	17.130	0.29	16.84	0.00500	1.36	1.86	
9	"	$\frac{1}{8}$	3.487	0.05	3.44	0.00203	0.55	0.30	
10	"	$\frac{1}{8}$	2.510	0.04	2.47	0.00174	0.47	0.23	
11	"	$\frac{1}{8}$	1.121	0.015	1.11	0.00112	0.30	0.10	
12	April 13	$\frac{1}{4}$	3.853	0.65	3.20	0.00760	2.07	4.29	47.5
13	"	$\frac{1}{4}$	13.150	1.96	11.19	0.01428	3.89	15.10	
14	"	$\frac{1}{4}$	22.510	3.21	19.30	0.01882	5.13	26.30	
15	"	$\frac{1}{4}$	18.530	2.69	15.84	0.01702	4.64	21.50	
16	"	$\frac{1}{4}$	6.920	1.13	5.79	0.01035	2.82	7.93	
17	"	$\frac{1}{4}$	1.905	0.32	1.59	0.00524	1.43	2.04	
18	"	$\frac{3}{8}$	21.720	8.30	13.42	0.03218	8.76	76.70	11.28
19	"	$\frac{3}{8}$	16.960	6.60	10.36	0.02820	7.68	59.00	
20	"	$\frac{3}{8}$	12.260	4.90	7.39	0.02380	6.48	42.00	
21	"	$\frac{3}{8}$	3.530	1.56	1.97	0.01240	3.38	11.40	
22	"	$\frac{3}{8}$	1.915	0.87	1.05	0.00895	2.44	5.94	
23	April 14	$\frac{3}{8}$	8.370	3.65	4.72	0.01950	5.31	28.10	
24	"	$\frac{1}{2}$	22.100	14.00	8.10	0.04260	11.60	135.00	3.93
25	"	$\frac{1}{2}$	17.470	11.10	6.37	0.03760	10.24	105.00	
26	"	$\frac{1}{2}$	11.660	7.60	4.06	0.03040	8.29	68.60	
27	"	$\frac{1}{2}$	5.760	3.55	2.21	0.02080	5.65	31.90	
28	"	$\frac{1}{2}$	8.410	5.50	2.91	0.02545	6.93	48.00	
29	"	$\frac{1}{2}$	2.579	1.75	0.83	0.01336	3.64	13.20	
30	"	$\frac{3}{4}$	23.230	21.70	1.53	0.05360	14.60	214.00	0.583
31	"	$\frac{3}{4}$	17.350	15.80	1.55	0.04570	12.46	155.00	
32	"	$\frac{3}{4}$	11.150	10.10	1.05	0.03580	9.75	94.80	
33	"	$\frac{3}{4}$	6.510	5.90	0.61	0.02660	7.25	52.40	
34	"	$\frac{3}{4}$	3.518	3.17	0.35	0.01865	5.08	25.80	
35	"	$\frac{3}{4}$	1.772	1.60	0.17	0.01262	3.44	11.80	
36	"	$\frac{3}{4}$	20.840	19.35	1.49	0.05090	13.85	192.00	
37	April 19	1	18.540	18.20	0.34	0.04915	13.40	180.00	0.1523
38	"	1	15.120	14.80	0.32	0.04400	12.00	144.00	
39	"	1	21.600	21.10	0.50	0.05330	14.50	210.00	
40	"	1	8.900	8.60	0.30	0.03276	8.91	79.40	
41	"	1	5.015	4.95	0.07	0.02390	6.51	42.30	
42	"	1	3.552	3.46	0.09	0.01960	5.34	28.50	
43	"	1	2.533	2.47	0.06	0.01627	4.43	19.60	
44	"	1	1.279	1.27	0.01	0.01100	2.99	8.97	

VALVE NO. 4

Gage Length = 10 feet

Pipe Cross Section = 0.00358 sq. ft.

	1919		feet	feet	feet				
1	Aug. 7	1/8	0.455	0.008	0.447	0.00122	0.34	0.12	261.5
2	"	1/8	1.006	0.020	0.986	0.00180	0.50	0.25	
3	"	1/8	1.850	0.037	1.813	0.00244	0.68	0.47	
4	"	1/8	2.567	0.050	2.517	0.00287	0.80	0.64	
5	"	1/8	2.993	0.059	2.934	0.00312	0.87	0.76	
6	"	1/8	3.478	0.065	3.413	0.00335	0.94	0.88	
7	"	1/8	4.237	0.080	4.157	0.00371	1.03	1.07	
8	"	1/8	4.812	0.086	4.726	0.00392	1.09	1.20	
9	"	1/4	0.572	0.063	0.509	0.00333	0.93	0.87	35.8
10	"	1/4	1.218	0.135	1.083	0.00494	1.38	1.90	
11	"	1/4	1.847	0.203	1.644	0.00614	1.72	2.94	
12	"	1/4	2.444	0.269	2.175	0.00708	1.98	3.92	
13	"	1/4	3.095	0.342	2.753	0.00801	2.24	5.00	
14	"	1/4	3.704	0.402	3.302	0.00867	2.42	5.86	
15	"	1/4	4.267	0.471	3.796	0.00938	2.62	6.86	
16	"	1/4	5.052	0.558	4.494	0.01022	2.86	8.16	
17	Aug. 8	3/8	0.684	0.200	0.484	0.00612	1.71	2.92	10.75
18	"	3/8	1.217	0.370	0.847	0.00829	2.32	5.37	
19	"	3/8	1.821	0.538	1.283	0.01008	2.82	7.90	
20	"	3/8	2.495	0.722	1.773	0.01180	3.30	10.88	
21	"	3/8	3.101	0.863	2.238	0.01320	3.68	13.59	
22	"	3/8	3.750	1.030	2.720	0.01436	4.01	16.08	
23	"	3/8	4.311	1.180	3.131	0.01540	4.31	18.55	
24	"	3/8	4.994	1.340	3.654	0.01654	4.62	21.30	
25	"	1/2	0.606	0.289	0.317	0.00733	2.05	4.19	4.92
26	"	1/2	1.282	0.622	0.660	0.01088	3.04	9.22	
27	"	1/2	1.892	0.865	1.027	0.01322	3.69	13.60	
28	"	1/2	2.475	1.140	1.335	0.01514	4.23	17.88	
29	"	1/2	3.064	1.400	1.664	0.01687	4.71	22.20	
30	"	1/2	3.651	1.640	2.011	0.01837	5.12	26.25	
31	"	1/2	4.290	1.900	2.390	0.01983	5.54	30.70	
32	"	1/2	4.931	2.170	2.761	0.02123	5.93	35.20	
33	Aug. 9	1/4	0.595	0.465	0.130	0.00933	2.61	6.80	1.19
34	"	3/4	1.229	0.923	0.306	0.01378	3.85	14.80	
35	"	3/4	1.865	1.450	0.415	0.01718	4.79	23.00	
36	"	3/4	2.370	1.830	0.540	0.01942	5.43	29.40	
37	"	3/4	2.993	2.330	0.663	0.02199	6.14	37.70	
38	"	3/4	3.572	2.820	0.752	0.02433	6.79	46.10	
39	"	3/4	4.246	3.220	1.026	0.02610	7.29	53.20	
40	"	3/4	4.870	3.680	1.190	0.02799	7.81	61.10	
41	Aug. 11	1	0.638	0.605	0.033	0.01070	2.99	8.93	0.281
42	"	1	1.254	1.170	0.084	0.01538	4.29	18.40	
43	"	1	1.839	1.730	0.109	0.01887	5.26	27.70	
44	"	1	2.429	2.300	0.129	0.02185	6.11	37.30	
45	"	1	3.053	2.860	0.193	0.02455	6.85	46.90	
46	"	1	3.685	3.450	0.235	0.02705	7.55	57.10	
47	"	1	4.254	3.950	0.304	0.02910	8.12	66.00	
48	"	1	4.945	4.550	0.395	0.03134	8.75	76.20	

VALVE NO. 5

Gage Length = 10 feet

Pipe Cross Section = 0.00358 sq. ft.

	1919		feet	feet	feet				
1	Aug. 11	$\frac{1}{8}$	0.470	0.012	0.458	0.00139	0.39	0.15	211.0
2	"	$\frac{1}{8}$	1.213	0.029	1.184	0.00223	0.62	0.39	
3	"	$\frac{1}{8}$	1.765	0.042	1.723	0.00267	0.79	0.55	
4	"	$\frac{1}{8}$	2.381	0.054	2.327	0.00303	0.85	0.71	
5	"	$\frac{1}{8}$	3.065	0.068	2.997	0.00341	0.95	0.91	
6	"	$\frac{1}{8}$	3.728	0.092	3.636	0.00403	1.14	1.26	
7	"	$\frac{1}{8}$	4.239	0.090	4.149	0.00400	1.11	1.24	
8	"	$\frac{1}{8}$	4.948	0.103	4.845	0.00428	1.19	1.43	
9	"	$\frac{1}{8}$	3.602	0.080	3.522	0.00372	1.04	1.08	
10	Aug. 12	$\frac{1}{4}$	0.613	0.084	0.529	0.00389	1.08	1.18	31.6
11	"	$\frac{1}{4}$	1.227	0.164	1.063	0.00552	1.55	2.38	
12	"	$\frac{1}{4}$	1.901	0.242	1.659	0.00673	1.88	3.52	
13	"	$\frac{1}{4}$	2.568	0.334	2.234	0.00788	2.20	4.84	
14	"	$\frac{1}{4}$	3.039	0.390	2.649	0.00854	2.38	5.68	
15	"	$\frac{1}{4}$	3.783	0.482	3.301	0.00950	2.66	7.05	
16	"	$\frac{1}{4}$	4.300	0.547	3.753	0.01016	2.84	8.07	
17	"	$\frac{1}{4}$	5.240	0.640	4.600	0.01103	3.08	9.50	
18	"	$\frac{3}{8}$	0.536	0.180	0.356	0.00575	1.61	2.58	8.64
19	"	$\frac{3}{8}$	1.246	0.407	0.839	0.00874	2.44	5.94	
20	"	$\frac{3}{8}$	1.787	0.606	1.181	0.01070	2.99	8.95	
21	"	$\frac{3}{8}$	2.373	0.782	1.591	0.01273	3.45	11.95	
22	"	$\frac{3}{8}$	3.075	1.010	2.065	0.01420	3.97	15.72	
23	"	$\frac{3}{8}$	3.681	1.200	2.481	0.01553	4.34	18.80	
24	"	$\frac{3}{8}$	4.344	1.410	2.934	0.01695	4.73	22.40	
25	"	$\frac{3}{8}$	5.114	1.590	3.524	0.01806	5.05	25.40	
26	Aug. 13	$\frac{1}{2}$	0.630	0.343	0.287	0.00801	2.24	5.00	3.98
27	"	$\frac{1}{2}$	1.274	0.680	0.594	0.01140	3.18	10.15	
28	"	$\frac{1}{2}$	1.795	0.912	0.883	0.01366	3.81	14.54	
29	"	$\frac{1}{2}$	2.414	1.260	1.154	0.01596	4.46	19.86	
30	"	$\frac{1}{2}$	3.022	1.570	1.452	0.01790	5.00	25.00	
31	"	$\frac{1}{2}$	3.670	1.860	1.810	0.01952	5.45	29.80	
32	"	$\frac{1}{2}$	4.369	2.140	2.229	0.02103	5.88	34.60	
33	"	$\frac{1}{2}$	4.876	2.400	2.476	0.02238	6.25	39.00	
34	"	$\frac{1}{2}$	3.947	1.930	2.017	0.01999	5.58	31.20	
35	"	$\frac{3}{4}$	0.618	0.540	0.078	0.00975	2.72	7.41	0.801
36	"	$\frac{3}{4}$	1.231	1.020	0.211	0.01429	3.99	15.94	
37	"	$\frac{3}{4}$	1.774	1.470	0.304	0.01733	4.84	23.40	
38	"	$\frac{3}{4}$	2.376	1.990	0.386	0.02028	5.66	32.10	
39	"	$\frac{3}{4}$	3.039	2.540	0.499	0.02300	6.43	41.30	
40	"	$\frac{3}{4}$	3.634	3.020	0.614	0.02520	7.04	49.60	
41	"	$\frac{3}{4}$	4.241	3.510	0.731	0.02728	7.62	58.10	
42	"	$\frac{3}{4}$	4.850	4.010	0.840	0.02920	8.16	66.80	
43	Aug. 13	1	0.618	0.580	0.038	0.01046	2.93	8.52	0.340
44	"	1	1.245	1.140	0.105	0.01518	4.24	17.95	
45	"	1	1.860	1.730	0.130	0.01885	5.27	27.70	
46	"	1	2.481	2.310	0.171	0.02190	6.12	37.40	
47	"	1	3.093	2.870	0.223	0.02455	6.86	47.00	
48	"	1	3.751	3.450	0.301	0.02705	7.56	57.00	
49	"	1	4.299	3.930	0.369	0.02900	8.09	65.50	
50	"	1	4.921	4.480	0.441	0.03100	8.67	75.00	

VALVE NO. 6

Gage Length = 10 feet

Pipe Cross Section = 0.00358 sq. ft.

			feet	feet	feet					
1	1919									
2	Aug. 15	1/8	0.634	0.024	0.610	0.00204	0.57	0.32	122.7	
3	"	1/8	1.206	0.048	1.158	0.00282	0.78	0.62		
4	"	1/8	1.784	0.069	1.715	0.00343	0.99	0.92		
5	"	1/8	2.355	0.086	2.269	0.00392	1.09	1.19		
6	"	1/8	2.814	0.103	2.711	0.00426	1.19	1.42		
7	"	1/8	3.871	0.140	3.731	0.00500	1.40	1.95		
8	"	1/8	4.424	0.156	4.268	0.00533	1.49	2.22		
9	"	1/8	4.917	0.173	4.744	0.00565	1.58	2.49		
10	Aug. 16	1/4	0.685	0.109	0.576	0.00440	1.23	1.51	24.88	
11	"	1/4	1.260	0.198	1.062	0.00605	1.69	2.86		
12	"	1/4	1.848	0.292	1.556	0.00738	2.06	4.25		
13	"	1/4	2.434	0.385	2.049	0.00847	2.37	5.60		
14	"	1/4	3.113	0.470	2.643	0.00938	2.62	6.87		
15	"	1/4	3.668	0.542	3.126	0.01011	2.82	7.97		
16	"	1/4	4.155	0.608	3.547	0.01072	3.00	8.96		
17	"	1/4	4.977	0.711	4.266	0.01170	3.27	10.66		
18	Aug. 20	3/8	5.010	1.500	3.510	0.01753	4.89	23.90	8.95	
19	"	3/8	4.228	1.290	2.938	0.01612	4.50	20.30		
20	"	3/8	3.643	1.140	2.503	0.01517	4.23	17.90		
21	"	3/8	2.952	0.882	2.070	0.01337	3.73	13.92		
22	"	3/8	2.408	0.759	1.649	0.01213	3.39	11.48		
23	"	3/8	1.892	0.618	1.274	0.01082	3.02	9.14		
24	"	3/8	1.189	0.390	0.799	0.00852	2.38	5.68		
25	"	1/2	0.589	0.194	0.395	0.00598	1.67	2.80		
26	"	1/2	5.114	2.520	2.594	0.02290	6.40	40.80	4.10	
27	"	1/2	4.511	2.320	2.191	0.02198	6.14	37.60		
28	"	1/2	3.510	1.700	1.810	0.01868	5.22	27.20		
29	"	1/2	2.901	1.420	1.481	0.01708	4.77	22.70		
30	"	1/2	2.209	1.020	1.189	0.01492	3.99	15.90		
31	"	1/2	1.678	0.880	0.798	0.01330	3.71	13.80		
32	"	1/2	1.013	0.542	0.471	0.01011	2.82	7.98		
33	"	1/2	0.414	0.198	0.216	0.00607	1.70	2.87		
34	Aug. 21	3/4	2.026	1.590	0.436	0.01806	5.04	25.40	1.105	
35	"	3/4	5.043	3.890	1.153	0.02885	8.05	64.80		
36	"	3/4	4.138	3.290	0.918	0.02610	7.31	53.20		
37	"	3/4	3.543	2.770	0.773	0.02409	6.72	45.20		
38	"	3/4	2.982	2.360	0.622	0.02215	6.19	38.20		
39	"	3/4	2.407	1.880	0.527	0.01966	5.48	30.20		
40	"	3/4	1.230	0.990	0.240	0.01389	3.88	15.40		
41	"	3/4	0.555	0.440	0.115	0.00907	2.53	6.42		
42	Aug. 21	1	5.221	4.690	0.531	0.03175	8.86	78.50	0.346	
43	"	1	4.683	4.190	0.493	0.03005	8.40	70.20		
44	"	1	3.886	3.500	0.386	0.02730	7.62	58.00		
45	"	1	3.006	2.720	0.286	0.02390	6.67	44.50		
46	"	1	2.359	2.100	0.259	0.02090	5.84	34.00		
47	"	1	1.845	1.680	0.165	0.01853	5.17	26.80		
48	"	1	1.130	1.020	0.110	0.01432	4.00	16.00		
			0.441	0.396	0.045	0.00859	2.40	5.75		

TABLE 24

LOSS OF HEAD IN 1-INCH GATE VALVES

VALVE NO. 0

Gage Length = 20.3 feet

Pipe Cross Section = 0.06015 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	k in $\frac{v^2}{2g}$
	1915		feet	feet	feet				
1	Mar. 17	$\frac{1}{8}$	21.790	1.08	20.71	0.01841	3.06	9.38	140.2
2	"	$\frac{1}{8}$	16.400	0.83	15.57	0.01617	2.69	7.20	
3	"	$\frac{1}{8}$	9.300	0.47	8.83	0.01223	2.04	4.14	
4	"	$\frac{1}{8}$	13.080	0.66	12.42	0.01435	2.39	5.70	
5	"	$\frac{1}{8}$	5.250	0.26	4.99	0.00905	1.50	2.26	
6	"	$\frac{1}{4}$	21.140	3.85	17.29	0.03880	6.45	41.55	26.7
7	"	$\frac{1}{4}$	14.700	2.75	11.95	0.03220	5.36	28.65	
8	"	$\frac{1}{4}$	10.070	1.99	8.08	0.02660	4.43	19.57	
9	"	$\frac{1}{4}$	4.648	1.03	3.62	0.01800	2.99	8.96	
10	"	$\frac{1}{4}$	19.410	3.70	15.71	0.03701	6.16	37.90	
11	"	$\frac{3}{8}$	18.910	7.42	11.49	0.05500	9.15	83.60	8.90
12	"	$\frac{3}{8}$	14.810	5.83	8.98	0.04840	8.05	64.70	
13	"	$\frac{3}{8}$	9.980	4.01	5.97	0.03955	6.57	43.20	
14	"	$\frac{3}{8}$	4.397	1.88	2.52	0.02578	4.28	18.38	
15	Mar. 22	$\frac{1}{2}$	20.140	15.29	4.85	0.08160	13.56	183.50	1.718
16	"	$\frac{1}{2}$	16.640	12.70	3.94	0.07380	12.27	150.50	
17	"	$\frac{1}{2}$	11.640	8.86	2.78	0.06090	10.11	102.00	
18	"	$\frac{1}{2}$	8.200	6.25	1.95	0.05023	8.35	69.70	
19	"	$\frac{1}{2}$	8.250	6.30	1.95	0.05030	8.37	70.00	
20	"	$\frac{1}{2}$	25.820	19.70	6.12	0.09300	15.45	239.00	
21	"	$\frac{1}{2}$	20.980	15.95	5.03	0.08340	13.85	192.00	
22	"	$\frac{1}{2}$	11.750	8.92	2.83	0.06110	10.15	103.00	
23	"	$\frac{1}{2}$	5.260	4.04	1.22	0.03960	6.58	43.30	
24	"	$\frac{1}{2}$	3.899	3.00	0.90	0.03372	5.61	31.40	
25	"	$\frac{1}{2}$	3.086	2.40	0.69	0.02960	4.92	24.20	
26	"	$\frac{1}{2}$	1.702	1.35	0.35	0.02130	3.54	12.50	
27	Mar. 23	$\frac{3}{4}$	21.100	19.70	1.40	0.09310	15.47	239.50	0.407
28	"	$\frac{3}{4}$	16.000	14.98	1.02	0.08060	13.40	179.70	
29	"	$\frac{3}{4}$	11.580	10.78	0.80	0.07660	11.23	126.30	
30	"	$\frac{3}{4}$	7.440	6.95	0.49	0.05320	8.85	78.00	
31	"	$\frac{3}{4}$	4.679	4.36	0.32	0.04140	6.88	47.25	
32	"	$\frac{3}{4}$	3.418	3.20	0.22	0.03480	5.79	33.50	
33	"	$\frac{3}{4}$	1.676	1.60	0.08	0.02322	3.87	14.90	
34	"	1	24.000	22.90	1.10	0.10140	16.86	284.00	0.2532
35	"	1	18.780	17.92	0.86	0.08900	14.78	218.00	
36	"	1	13.610	13.12	0.49	0.07500	12.47	155.80	
37	"	1	9.290	8.90	0.39	0.06100	10.13	102.70	
38	"	1	5.085	4.78	0.22	0.04390	7.29	53.10	
39	"	1	2.647	2.55	0.10	0.03050	5.08	25.75	

VALVE NO. 4

Gage Length = 10 feet

Pipe Cross Section = 0.00604 sq. ft.

	1919		feet	feet	feet			
1	Aug. 2	1/8	4.955	0.075	4.880	0.00755	1.25	1.56
2	"	1/8	4.118	0.062	4.056	0.00684	1.31	1.28
3	"	1/8	3.474	0.053	3.421	0.00632	1.05	1.09
4	"	1/8	3.029	0.047	2.982	0.00593	0.98	0.96
5	"	1/8	2.281	0.035	2.246	0.00513	0.85	0.72
6	"	1/8	1.783	0.027	1.756	0.00453	0.75	0.56
7	"	1/8	1.158	0.019	1.139	0.00373	0.62	0.38
8	"	1/8	0.640	0.011	0.629	0.00278	0.46	0.21
9	"	1/8	4.826	0.357	4.469	0.01663	2.75	7.58
10	"	1/4	4.174	0.313	3.861	0.01546	2.56	6.54
11	"	1/4	3.631	0.272	3.359	0.01440	2.38	5.68
12	"	1/4	2.959	0.222	2.737	0.01301	2.15	4.63
13	"	1/4	2.208	0.167	2.041	0.01131	1.87	3.50
14	"	1/4	1.743	0.132	1.611	0.01001	1.66	2.74
15	"	1/4	1.175	0.091	1.084	0.00828	1.37	1.88
16	"	1/4	0.668	0.052	0.616	0.00627	1.04	1.08
17	Aug. 4	3/8	4.356	0.990	3.546	0.02910	4.82	23.20
18	"	3/8	4.162	0.900	3.262	0.02770	4.58	21.00
19	"	3/8	3.661	0.810	2.851	0.02600	4.30	18.55
20	"	3/8	2.945	0.640	2.305	0.02325	3.85	14.80
21	"	3/8	2.400	0.530	1.870	0.02090	3.46	11.98
22	"	3/8	1.868	0.425	1.443	0.01843	3.06	9.30
23	"	3/8	1.230	0.291	0.939	0.01488	2.46	6.06
24	"	3/8	0.598	0.138	0.460	0.01024	1.70	2.88
25	"	1/2	4.703	1.960	2.743	0.04160	6.89	47.40
26	"	1/2	4.064	1.670	2.394	0.03830	6.34	40.20
27	"	1/2	3.659	1.520	2.139	0.03640	6.03	36.30
28	"	1/2	3.019	1.250	1.769	0.03300	5.46	29.80
29	"	1/2	2.472	1.030	1.442	0.02960	4.90	24.00
30	"	1/2	1.658	0.700	0.958	0.02400	3.97	15.80
31	"	1/2	1.167	0.500	0.667	0.02000	3.31	10.98
32	"	1/2	0.835	0.365	0.470	0.01685	2.79	7.78
33	Aug. 5	3/4	4.680	3.900	0.780	0.05920	9.81	96.00
34	"	3/4	4.115	3.420	0.695	0.05540	9.17	84.00
35	"	3/4	3.500	3.980	0.580	0.05150	8.52	72.70
36	"	3/4	2.911	2.450	0.461	0.04650	7.70	59.30
37	"	3/4	2.327	1.950	0.377	0.04150	6.87	47.20
38	"	3/4	1.763	1.480	0.283	0.03600	5.96	35.50
39	"	3/4	1.300	1.100	0.200	0.03075	5.09	25.80
40	"	3/4	0.616	0.516	0.106	0.02050	3.39	11.52
41	Aug. 5	1	5.049	4.600	0.449	0.03470	10.71	114.80
42	"	1	4.279	3.920	0.359	0.05940	9.84	96.70
43	"	1	3.716	3.410	0.306	0.05530	9.16	83.90
44	"	1	3.006	2.780	0.226	0.04960	8.21	67.50
45	"	1	2.424	2.260	0.164	0.04460	7.38	54.50
46	"	1	1.810	1.680	0.130	0.03830	6.34	40.20
47	"	1	0.993	0.916	0.083	0.02780	4.61	21.20
48	"	1	0.532	0.490	0.042	0.01987	3.29	10.82
49	"	1	2.916	2.700	0.216	0.04900	8.12	65.70
50	"	1	1.741	1.610	0.131	0.03755	6.21	38.60
51	"	1	4.568	4.190	0.378	0.06150	10.18	10.38
52	"	1	3.912	3.610	0.302	0.05700	9.44	89.00
53	"	1	3.473	3.210	0.263	0.05360	8.87	78.60
54	"	1	2.716	2.530	0.186	0.04725	7.82	61.20

VALVE NO. 5

Gage Length = 10 feet

Pipe Cross Section = 0.00604 sq. ft.

	1919		feet	feet	feet				
1	Aug. 7	$\frac{1}{8}$	5.120	0.041	5.079	0.00598	0.99	0.98	
2	"	$\frac{1}{8}$	4.405	0.039	4.366	0.00555	0.92	0.84	
3	"	$\frac{1}{8}$	3.572	0.032	3.540	0.00498	0.83	0.68	
4	"	$\frac{1}{8}$	2.975	0.027	2.948	0.00453	0.75	0.56	334.0
5	"	$\frac{1}{8}$	2.356	0.021	2.335	0.00405	0.67	0.45	
6	"	$\frac{1}{8}$	1.782	0.019	1.763	0.00351	0.58	0.34	
7	"	$\frac{1}{8}$	1.085	0.011	1.074	0.00275	0.45	0.21	
8	"	$\frac{1}{8}$	0.470	0.006	0.464	0.00181	0.30	0.09	
9	"	$\frac{1}{4}$	0.703	0.045	0.658	0.00622	1.03	1.06	
10	"	$\frac{1}{4}$	1.196	0.078	1.118	0.00810	1.34	1.79	
11	"	$\frac{1}{4}$	1.786	0.115	1.671	0.00994	1.64	2.70	
12	"	$\frac{1}{4}$	2.377	0.153	2.224	0.01150	1.90	3.62	39.6
13	"	$\frac{1}{4}$	3.118	0.200	2.918	0.01320	2.18	4.76	
14	"	$\frac{1}{4}$	3.661	0.236	3.425	0.01430	2.36	5.58	
15	"	$\frac{1}{4}$	4.193	0.270	3.923	0.01530	2.53	6.40	
16	"	$\frac{1}{4}$	4.969	0.328	4.631	0.01660	2.74	7.52	
17	"	$\frac{3}{8}$	0.668	0.120	0.548	0.01018	1.68	2.84	
18	"	$\frac{3}{8}$	1.130	0.209	0.921	0.01348	2.23	4.96	
19	"	$\frac{3}{8}$	1.869	0.332	1.537	0.01700	2.81	7.90	
20	"	$\frac{3}{8}$	2.349	0.415	1.934	0.01895	3.14	9.85	12.57
21	"	$\frac{3}{8}$	2.991	0.530	2.461	0.02130	3.52	12.41	
22	"	$\frac{3}{8}$	3.612	0.670	2.942	0.02340	3.87	15.00	
23	"	$\frac{3}{8}$	4.093	0.760	3.333	0.02500	4.14	17.15	
24	"	$\frac{3}{8}$	4.858	0.885	3.973	0.02720	4.50	20.30	
25	Aug. 8	$\frac{1}{2}$	4.957	1.850	3.107	0.04020	6.66	44.30	
26	"	$\frac{1}{2}$	4.752	1.810	2.942	0.03970	6.59	43.20	
27	"	$\frac{1}{2}$	3.941	1.480	2.461	0.03575	5.92	35.00	
28	"	$\frac{1}{2}$	2.997	1.160	1.837	0.03142	5.21	27.10	
29	"	$\frac{1}{2}$	2.388	0.970	1.418	0.02850	4.73	22.30	
30	"	$\frac{1}{2}$	1.792	0.740	1.052	0.02480	4.11	16.85	
31	"	$\frac{1}{2}$	1.234	0.520	0.714	0.02042	3.39	11.45	
32	"	$\frac{1}{2}$	0.628	0.239	0.389	0.01430	2.38	5.65	
33	"	$\frac{1}{2}$	4.494	1.690	2.804	0.03840	6.36	40.40	
34	"	$\frac{1}{2}$	2.069	0.870	1.199	0.02700	4.47	19.95	
35	"	$\frac{1}{2}$	1.557	0.650	0.907	0.02310	3.83	14.60	
36	"	$\frac{1}{2}$	4.803	1.710	3.093	0.03860	6.40	40.80	
37	Aug. 11	$\frac{1}{2}$	4.111	1.440	2.671	0.03540	5.87	34.40	
38	"	$\frac{1}{2}$	3.563	1.270	2.293	0.03300	5.27	29.80	
39	"	$\frac{1}{2}$	3.015	1.110	1.905	0.03090	5.12	26.10	
40	"	$\frac{1}{2}$	2.286	0.880	1.406	0.02720	4.51	20.30	
41	"	$\frac{1}{2}$	1.688	0.680	1.008	0.02340	3.87	15.00	
42	"	$\frac{1}{2}$	1.363	0.540	0.823	0.02090	3.47	12.00	
43	"	$\frac{1}{2}$	0.590	0.209	0.391	0.01340	2.23	4.94	4.61
44	"	$\frac{1}{2}$	5.054	1.780	3.274	0.03940	6.54	42.60	
45	"	$\frac{1}{2}$	4.595	1.600	2.995	0.03720	6.18	38.00	
46	"	$\frac{1}{2}$	3.775	1.310	2.465	0.03370	5.59	31.10	
47	"	$\frac{1}{2}$	1.867	0.730	1.137	0.02460	4.08	16.70	
48	"	$\frac{1}{2}$	0.923	0.340	0.583	0.01710	2.84	8.03	
49	"	$\frac{1}{2}$	3.299	1.180	2.119	0.02880	5.29	27.90	
50	Aug. 8	$\frac{3}{4}$	4.794	2.920	1.874	0.05090	8.44	71.00	
51	"	$\frac{3}{4}$	4.276	2.640	1.636	0.04810	7.98	63.50	
52	"	$\frac{3}{4}$	3.732	2.290	1.442	0.04480	7.42	54.90	
53	"	$\frac{3}{4}$	3.082	1.900	1.182	0.04070	6.75	45.50	1.69
54	"	$\frac{3}{4}$	2.447	1.500	0.947	0.03610	5.98	35.70	
55	"	$\frac{3}{4}$	1.883	1.170	0.713	0.03160	5.24	27.30	
56	"	$\frac{3}{4}$	1.193	0.760	0.433	0.02510	4.16	17.30	
57	"	$\frac{3}{4}$	0.638	0.370	0.268	0.01790	2.96	8.75	

VALVE NO. 5—Continued

58	Aug. 9	Full	4.899	4.500	0.399	0.06400	10.61	112.20	0.217
59	"	"	4.301	3.970	0.331	0.05970	9.90	97.80	
60	"	"	3.706	3.430	0.276	0.05540	9.18	84.00	
61	"	"	3.051	2.830	0.221	0.05000	8.29	68.60	
62	"	"	2.397	2.220	0.177	0.04420	7.32	53.50	
63	"	"	1.817	1.670	0.147	0.03810	6.31	39.70	
64	"	"	1.182	1.100	0.082	0.03060	5.06	25.60	
65	"	"	0.578	0.550	0.028	0.02100	3.47	12.02	

VALVE NO. 6

Gage Length = 10 feet

Pipe Cross Section = 0.00604 sq. ft.

	1919		feet	feet	feet				
1	Aug. 12	$\frac{1}{8}$	4.966	0.081	4.885	0.00831	1.38	1.89	164.0
2	"	$\frac{1}{8}$	4.181	0.070	4.111	0.00762	1.26	1.60	
3	"	$\frac{1}{8}$	3.341	0.055	3.286	0.00680	1.13	1.27	
4	"	$\frac{1}{8}$	3.091	0.052	3.039	0.00658	1.09	1.19	
5	"	$\frac{1}{8}$	2.395	0.040	2.355	0.00575	0.95	0.90	
6	"	$\frac{1}{8}$	1.786	0.032	1.754	0.00496	0.82	0.67	
7	"	$\frac{1}{8}$	1.204	0.021	1.183	0.00406	0.67	0.45	
8	"	$\frac{1}{8}$	0.689	0.014	0.665	0.00308	0.51	0.26	
9	Aug. 12	$\frac{1}{4}$	4.998	0.376	4.622	0.01805	2.99	8.94	
10	"	$\frac{1}{4}$	4.221	0.313	3.908	0.01652	2.74	7.48	
11	"	$\frac{1}{4}$	3.587	0.266	3.321	0.01520	2.52	6.32	
12	"	$\frac{1}{4}$	2.945	0.220	2.725	0.01380	2.29	5.22	33.75
13	"	$\frac{1}{4}$	2.415	0.179	2.236	0.01242	2.06	4.23	
14	"	$\frac{1}{4}$	1.852	0.137	1.715	0.01082	1.80	3.22	
15	"	$\frac{1}{4}$	1.206	0.088	1.118	0.00872	1.45	2.08	
16	"	$\frac{1}{4}$	0.609	0.045	0.564	0.00618	1.02	1.05	
17	"	$\frac{3}{8}$	4.978	1.170	3.808	0.03140	5.21	27.10	
18	"	$\frac{3}{8}$	4.344	1.010	3.334	0.02930	4.86	23.50	
19	"	$\frac{3}{8}$	3.360	0.800	2.560	0.02570	4.27	18.15	9.15
20	"	$\frac{3}{8}$	2.940	0.700	2.240	0.02400	3.98	15.78	
21	"	$\frac{3}{8}$	2.407	0.590	1.817	0.02170	3.60	12.90	
22	"	$\frac{3}{8}$	1.827	0.408	1.419	0.01880	3.12	9.70	
23	"	$\frac{3}{8}$	1.219	0.270	0.949	0.01530	2.54	6.44	
24	"	$\frac{3}{8}$	0.654	0.143	0.511	0.01110	1.84	3.37	
25	"	$\frac{1}{2}$	4.505	1.800	2.705	0.03970	6.58	43.30	
26	"	$\frac{1}{2}$	5.037	2.020	3.017	0.04210	6.98	48.60	
27	"	$\frac{1}{2}$	3.639	1.470	2.169	0.03560	5.90	34.70	
28	"	$\frac{1}{2}$	2.848	1.160	1.688	0.03140	5.21	27.00	4.00
29	"	$\frac{1}{2}$	2.434	1.000	1.434	0.02900	4.80	23.00	
30	"	$\frac{1}{2}$	1.768	0.740	1.028	0.02460	4.08	16.60	
31	"	$\frac{1}{2}$	1.245	0.530	0.715	0.02060	3.41	11.60	
32	"	$\frac{1}{2}$	0.588	0.230	0.358	0.01390	2.30	5.26	
33	Aug. 14	$\frac{3}{4}$	5.008	3.710	1.288	0.05770	9.57	91.20	
34	"	$\frac{3}{4}$	4.346	3.230	1.116	0.05360	8.89	78.80	
35	"	$\frac{3}{4}$	3.654	2.750	0.904	0.04920	8.15	66.40	
36	"	$\frac{3}{4}$	3.084	2.310	0.774	0.04500	7.46	55.50	0.91
37	"	$\frac{3}{4}$	2.478	1.850	0.628	0.04020	6.67	44.40	
38	"	$\frac{3}{4}$	1.914	1.440	0.474	0.03520	5.84	34.00	
39	"	$\frac{3}{4}$	1.233	0.930	0.303	0.02800	4.64	21.50	
40	"	$\frac{3}{4}$	0.574	0.398	0.176	0.01860	3.08	9.46	
41	"	1	4.796	4.350	0.446	0.06280	10.40	108.00	

VALVE NO. 6—Continued

42	Aug. 14	1	4.238	3.880	0.358	0.05900	9.79	95.50	0.238
43	"	1	3.730	3.430	0.300	0.05540	9.18	84.00	
44	"	1	2.983	2.760	0.223	0.04940	8.18	66.60	
45	"	1	2.392	2.210	0.182	0.04410	7.30	53.20	
46	"	1	1.636	1.500	0.136	0.03610	5.98	35.70	
47	"	1	1.078	1.000	0.078	0.02900	4.80	23.00	
48	"	1	0.513	0.480	0.033	0.01930	3.20	10.20	

TABLE 25

LOSS OF HEAD IN 1½-INCH GATE VALVE

VALVE NO. 0

Gage Length = 23.85 feet

Pipe Cross Section = 0.01415 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	$k \text{ in } \frac{v^2}{2g}$
	1915		feet	feet	feet				
1	April 26	$\frac{1}{8}$	17.990	0.710	17.28	0.0450	3.18	10.10	110.7
2	"	$\frac{1}{8}$	14.720	0.605	14.11	0.0405	2.86	8.20	
3	"	$\frac{1}{8}$	9.980	0.413	9.57	0.0334	2.36	5.58	
4	"	$\frac{1}{8}$	5.252	0.214	5.04	0.0240	1.69	2.86	
5	"	$\frac{1}{4}$	21.100	4.030	17.07	0.1057	7.46	55.70	19.72
6	"	$\frac{1}{4}$	16.940	3.240	13.70	0.0946	6.69	44.70	
7	April 27	$\frac{1}{4}$	10.830	2.060	8.77	0.0755	5.34	28.50	
8	"	$\frac{1}{4}$	7.380	1.400	5.98	0.0624	4.41	19.50	
9	"	$\frac{3}{8}$	19.330	8.020	11.31	0.1494	10.56	111.80	6.55
10	"	$\frac{3}{8}$	15.790	6.530	9.26	0.1351	9.55	91.00	
11	"	$\frac{3}{8}$	10.600	4.390	6.21	0.1101	7.78	60.50	
12	"	$\frac{3}{8}$	7.070	2.920	4.15	0.0901	6.37	40.50	
13	"	$\frac{1}{2}$	20.130	12.300	7.83	0.1858	13.14	172.50	2.88
14	"	$\frac{1}{2}$	16.300	10.020	6.28	0.1674	11.83	140.00	
15	"	$\frac{1}{2}$	11.750	7.250	4.50	0.1423	10.06	101.20	
16	"	$\frac{1}{2}$	7.520	4.650	2.87	0.1137	8.02	64.30	
17	"	$\frac{3}{4}$	19.500	17.470	2.03	0.2218	15.70	246.00	0.576
18	"	$\frac{3}{4}$	15.950	14.320	1.63	0.2004	14.20	201.00	
19	"	$\frac{3}{4}$	11.960	10.600	1.36	0.1721	12.18	1.48	
20	"	$\frac{3}{4}$	8.110	7.110	1.00	0.1407	9.95	99.00	
21	"	$\frac{3}{4}$	5.515	4.830	0.69	0.1160	8.20	67.20	
22	"	$\frac{3}{4}$	14.190	12.550	1.64	0.1878	13.27	176.00	
23	"	$\frac{3}{4}$	23.450	20.600	2.85	0.2420	17.10	292.00	
24	"	$\frac{3}{4}$	17.070	15.380	1.69	0.2080	14.70	216.00	
25	"	$\frac{3}{4}$	4.190	3.690	0.50	0.1011	7.15	51.00	
26	"	1	15.350	14.940	0.41	0.2046	14.48	210.00	0.201
27	"	1	18.240	17.460	0.78	0.2219	15.68	246.00	
28	"	1	11.540	11.170	0.37	0.1767	12.48	156.00	
29	"	1	8.720	8.440	0.28	0.1534	10.84	117.70	
30	"	1	6.200	5.890	0.31	0.1281	9.05	81.90	
31	"	1	24.050	22.660	1.39	0.2535	17.90	321.00	

TABLE 26

LOSS OF HEAD IN 2-INCH GATE VALVES

VALVE NO. 0

Gage Length = 19.85 feet

Pipe Cross Section = 0.02334 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	$k \text{ in } \frac{v^2}{2g}$
	1915		feet	feet	feet				
1	Feb. 15	$\frac{1}{8}$	3.682	0.080	3.602	0.0299	1.28	1.65	
2	"	$\frac{1}{8}$	8.550	0.190	8.360	0.0467	2.00	4.00	
3	"	$\frac{1}{8}$	24.510	0.520	23.990	0.0749	3.21	10.29	147.2
4	"	$\frac{1}{8}$	24.100	0.510	23.590	0.0740	3.17	10.06	
5	"	$\frac{1}{8}$	15.295	0.320	14.975	0.0597	2.56	6.56	
6	"	$\frac{1}{8}$	16.670	0.350	16.320	0.0623	2.67	7.12	
7	"	$\frac{1}{4}$	3.420	0.370	3.050	0.0626	2.68	7.19	
8	"	$\frac{1}{4}$	5.646	0.610	5.036	0.0803	3.44	11.85	
9	"	$\frac{1}{4}$	20.580	2.050	18.530	0.1543	6.62	43.80	27.35
10	"	$\frac{1}{4}$	18.980	1.900	17.080	0.1477	6.33	40.10	
11	"	$\frac{1}{4}$	14.120	1.420	12.700	0.1278	5.48	30.00	
12	"	$\frac{1}{4}$	10.900	1.220	9.680	0.1118	4.79	22.90	
13	Feb. 16	$\frac{3}{8}$	24.680	5.830	18.850	0.2610	11.19	125.20	
14	"	$\frac{3}{8}$	17.180	4.100	13.080	0.2111	9.32	86.50	9.70
15	"	$\frac{3}{8}$	8.850	2.100	6.750	0.1560	6.69	44.70	
16	"	$\frac{3}{8}$	3.994	1.010	2.984	0.1043	4.47	20.01	
17	Feb. 17	$\frac{1}{2}$	19.640	8.010	11.630	0.3045	13.04	170.10	
18	"	$\frac{1}{2}$	20.260	8.400	11.860	0.3120	13.37	178.60	
19	"	$\frac{1}{2}$	14.310	5.930	8.380	0.2620	11.23	126.00	4.33
20	"	$\frac{1}{2}$	7.420	3.030	4.390	0.1881	8.08	65.00	
21	"	$\frac{1}{2}$	5.366	2.180	3.186	0.1598	6.85	46.90	
22	"	$\frac{3}{4}$	16.860	12.510	4.350	0.3801	16.29	265.50	
23	"	$\frac{3}{4}$	16.525	11.900	4.625	0.3715	15.91	253.60	
24	"	$\frac{3}{4}$	12.840	9.430	3.410	0.3305	14.16	201.00	1.081
25	"	$\frac{3}{4}$	7.840	5.800	2.040	0.2600	11.13	124.00	
26	"	$\frac{3}{4}$	4.231	3.120	1.111	0.1900	8.15	66.30	
27	"	$\frac{3}{4}$	16.630	12.430	4.200	0.3790	16.23	262.00	
28	"	1	12.820	12.220	0.600	0.3760	16.10	259.00	
29	"	1	13.700	13.000	0.700	0.3870	16.60	275.00	
30	"	1	10.150	9.320	0.830	0.3325	14.25	203.50	.194
31	"	1	5.910	5.480	0.430	0.2520	10.80	116.60	
32	"	1	3.106	2.900	0.206	0.1840	7.90	62.30	

VALVE NO. 00

Test by H. N. Lendall Summer of 1914

Gage Length = 19.86 feet

Pipe Cross Section = 0.0233 sq. ft.

	1914		feet	feet	feet				
1	Aug. 2	$\frac{1}{8}$	1.392	0.018	1.374	0.0151	0.65	0.42	
2	"	$\frac{1}{8}$	1.410	0.018	1.392	0.0150	0.64	0.41	
3	"	$\frac{1}{8}$	3.750	0.048	3.702	0.0245	1.05	1.10	
4	"	$\frac{1}{8}$	3.603	0.045	3.558	0.0238	1.01	1.03	
5	"	$\frac{1}{8}$	7.060	0.092	6.968	0.0336	1.44	2.10	223
6	"	$\frac{1}{8}$	7.070	0.087	6.983	0.0335	1.44	1.99	

VALVE NO. 00—Continued

7	Aug. 2	$\frac{1}{8}$	11.370	0.146	11.224	0.0426	1.83	3.33	
8	"	$\frac{1}{8}$	11.190	0.138	11.052	0.0415	1.78	3.16	
9	"	$\frac{1}{8}$	14.920	0.183	14.737	0.0477	2.04	4.19	
10	"	$\frac{1}{8}$	14.530	0.175	14.355	0.0466	2.00	3.99	
11	Aug. 1	$\frac{1}{4}$	0.402	0.053	0.349	0.0256	1.00	1.21	
12	"	$\frac{1}{4}$	0.421	0.056	0.365	0.0264	1.12	1.27	
13	"	$\frac{1}{4}$	1.403	C.190	1.213	0.0485	2.08	4.34	
14	"	$\frac{1}{4}$	1.403	0.190	1.213	0.0486	2.08	4.34	
15	"	$\frac{1}{4}$	4.120	0.055	4.065	0.0825	3.53	12.50	19.9
16	"	$\frac{1}{4}$	3.113	0.395	2.718	0.0701	3.05	9.02	
17	"	$\frac{1}{4}$	3.031	0.395	2.636	0.0701	3.05	9.02	
18	"	$\frac{1}{4}$	4.167	0.052	4.115	0.0802	3.44	11.80	
19	"	$\frac{1}{4}$	2.139	0.274	1.865	0.0588	2.50	6.25	
20	"	$\frac{1}{4}$	2.084	0.260	1.824	0.0571	2.44	5.95	
21	Aug. 1	$\frac{3}{8}$	0.410	0.128	0.282	0.0398	1.71	2.92	
22	"	$\frac{3}{8}$	0.410	0.126	0.284	0.0398	1.70	2.88	
23	"	$\frac{3}{8}$	0.961	0.330	0.631	0.0640	2.75	7.54	
24	"	$\frac{3}{8}$	0.961	0.330	0.631	0.0640	2.75	7.54	
25	"	$\frac{3}{8}$	1.679	0.590	1.089	C.0857	3.68	13.48	5.25
26	"	$\frac{3}{8}$	1.696	0.590	1.106	0.0857	3.68	13.48	
27	"	$\frac{3}{8}$	3.163	0.113	3.050	0.1185	5.07	25.75	
28	"	$\frac{3}{8}$	3.165	0.113	3.052	0.1185	5.07	25.75	
29	"	$\frac{3}{8}$	3.927	0.136	3.791	0.1299	5.57	31.00	
30	"	$\frac{3}{8}$	3.921	0.136	3.785	0.1299	5.57	31.00	
31	"	$\frac{3}{8}$	6.510	0.207	6.303	0.1607	6.88	47.40	
32	"	$\frac{3}{8}$	6.500	0.208	6.292	0.1590	6.82	46.50	
33	"	$\frac{1}{2}$	2.384	1.307	1.077	0.1271	5.45	29.80	
34	"	$\frac{1}{2}$	3.216	1.760	1.456	0.1479	6.33	40.10	
35	"	$\frac{1}{2}$	3.195	1.762	1.433	0.148C	6.34	40.20	
36	"	$\frac{1}{2}$	3.947	2.140	1.807	0.1629	6.99	48.80	2.365
37	"	$\frac{1}{2}$	3.952	2.140	1.812	0.1629	6.99	48.80	
38	"	$\frac{1}{2}$	0.972	0.534	0.438	0.0814	3.49	12.18	
39	"	$\frac{1}{2}$	0.972	0.537	0.435	0.0818	3.50	12.25	
40	"	$\frac{1}{2}$	0.422	0.211	0.211	0.0511	2.19	4.81	
41	"	$\frac{1}{2}$	0.422	0.200	0.222	0.0511	2.14	4.58	
42	"	$\frac{1}{2}$	1.541	0.833	0.708	0.1017	4.36	19.10	
43	"	$\frac{1}{2}$	1.544	C.849	0.695	C.1028	4.40	19.35	
44	"	$\frac{1}{2}$	2.395	1.310	1.085	0.1271	5.46	29.85	
45	July 31	$\frac{3}{4}$	0.501	0.371	0.130	0.0679	2.91	8.46	
46	"	$\frac{3}{4}$	0.492	0.358	0.134	0.0669	2.86	8.17	
47	"	$\frac{3}{4}$	1.162	0.634	0.528	0.0888	3.80	14.45	
48	"	$\frac{3}{4}$	1.144	0.634	0.510	0.0888	3.80	14.45	
49	"	$\frac{3}{4}$	1.161	0.625	0.536	0.0881	3.77	14.25	
50	"	$\frac{3}{4}$	2.234	1.142	1.092	0.1209	5.18	26.15	
51	"	$\frac{3}{4}$	2.298	1.218	1.070	0.1230	5.27	27.80	
52	"	$\frac{3}{4}$	3.065	1.962	1.103	0.1558	6.69	44.80	.533
53	"	$\frac{3}{4}$	3.043	1.962	1.081	0.1558	6.69	44.80	
54	"	$\frac{3}{4}$	1.479	1.266	0.293	0.1254	5.37	28.90	
55	"	$\frac{3}{4}$	1.485	1.253	0.232	0.1247	5.35	28.60	
56	"	$\frac{3}{4}$	3.142	2.650	0.492	0.1820	7.78	60.50	
57	"	$\frac{3}{4}$	3.138	2.618	0.520	0.1805	7.73	59.70	
58	"	$\frac{3}{4}$	4.134	3.482	0.652	0.2076	8.91	79.50	
59	"	$\frac{3}{4}$	4.155	3.430	0.725	0.2068	8.85	78.40	
60	"	$\frac{3}{4}$	2.147	1.805	0.342	0.1498	6.42	41.20	
61	"	$\frac{3}{4}$	2.167	1.840	0.327	0.1511	6.48	42.00	
62	"	$\frac{3}{4}$	1.142	0.947	0.195	0.1083	4.65	21.60	
63	"	$\frac{3}{4}$	1.162	0.991	C.171	0.1107	4.75	22.60	
64	"	1	0.560	0.508	0.058	0.0798	3.41	11.60	
65	"	1	0.562	0.502	0.060	0.0791	3.38	11.48	
66	"	1	1.131	1.010	0.121	0.1119	4.80	23.05	

VALVE NO. 00—Continued

67	July 31	1	1.073	0.983	0.080	0.1111	4.76	22.65	.256
68	"	1	2.779	2.520	0.259	0.1770	7.58	57.50	
69	"	1	2.604	2.418	0.186	0.1731	7.42	55.10	
70	"	1	3.586	3.318	0.270	0.2029	8.70	75.60	
71	"	1	3.650	3.403	0.247	0.2056	8.81	77.70	
72	"	1	4.410	4.010	0.400	0.2235	9.56	91.60	
73	"	1	4.417	4.030	0.387	0.2243	9.58	92.00	
74	"	1	1.753	1.608	0.145	0.1414	6.05	36.75	
75	"	1	1.750	1.608	0.142	0.1414	6.05	36.75	

VALVE NO. 7

Gage Length = 9.98 feet

Pipe Cross Section = 0.02338 sq. ft.

	1919		feet	feet	feet				
1	June 5	$\frac{1}{8}$	0.693	0.010	0.683	0.0145	0.62	0.38	115.3
2	"	$\frac{1}{8}$	1.020	0.014	1.006	0.0176	0.76	0.57	
3	"	$\frac{1}{8}$	1.563	0.022	1.541	0.0218	0.94	0.87	
4	"	$\frac{1}{8}$	2.070	0.029	2.041	0.0250	1.07	1.15	
5	"	$\frac{1}{8}$	2.561	0.036	2.525	0.0279	1.20	1.43	
6	"	$\frac{1}{8}$	3.064	0.042	3.022	0.0304	1.31	1.71	
7	"	$\frac{1}{8}$	3.564	0.049	3.515	0.0328	1.41	1.98	
8	"	$\frac{1}{8}$	4.088	0.055	4.033	0.0350	1.50	2.25	
9	"	$\frac{1}{8}$	4.645	0.061	4.584	0.0373	1.60	2.55	
10	"	$\frac{1}{8}$	0.298	0.004	0.294	0.0095	0.41	0.17	25.13
11	June 23	$\frac{1}{4}$	0.191	0.012	0.179	0.0154	0.66	0.44	
12	"	$\frac{1}{4}$	0.318	0.019	0.299	0.0199	0.85	0.73	
13	"	$\frac{1}{4}$	0.687	0.038	0.649	0.0292	1.26	1.57	
14	"	$\frac{1}{4}$	1.015	0.057	0.958	0.0354	1.52	2.31	
15	"	$\frac{1}{4}$	1.492	0.087	1.405	0.0443	1.90	3.60	
16	"	$\frac{1}{4}$	1.926	0.111	1.815	0.0502	2.15	4.63	
17	"	$\frac{1}{4}$	2.547	0.143	2.404	0.0579	2.48	6.15	
18	"	$\frac{1}{4}$	3.015	0.170	2.845	0.0631	2.71	7.33	
19	"	$\frac{1}{4}$	3.533	0.196	3.337	0.0681	2.92	8.52	8.03
20	"	$\frac{1}{4}$	3.967	0.218	3.749	0.0723	3.10	9.60	
21	"	$\frac{1}{4}$	4.521	0.246	4.275	0.0770	3.30	10.90	
22	"	$\frac{1}{4}$	4.975	0.268	4.707	0.0810	3.47	12.05	
23	June 27	$\frac{3}{8}$	0.190	0.031	0.159	0.0264	1.13	1.28	
24	"	$\frac{3}{8}$	0.388	0.063	0.325	0.0378	1.62	2.63	
25	"	$\frac{3}{8}$	0.800	0.130	0.670	0.0544	2.34	5.46	
26	"	$\frac{3}{8}$	0.984	0.156	0.828	0.0604	2.60	6.71	
27	"	$\frac{3}{8}$	1.500	0.230	1.270	0.0746	3.20	10.20	
28	"	$\frac{3}{8}$	2.042	0.308	1.734	0.0875	3.75	14.05	3.465
29	"	$\frac{3}{8}$	2.460	0.361	2.099	0.0954	4.10	16.80	
30	"	$\frac{3}{8}$	3.089	0.425	2.664	0.1074	4.61	21.20	
31	"	$\frac{3}{8}$	3.590	0.490	3.100	0.1153	4.96	24.50	
32	"	$\frac{3}{8}$	4.007	0.550	3.457	0.1228	5.27	27.70	
33	"	$\frac{3}{8}$	4.870	0.680	4.190	0.1360	5.84	34.00	
34	June 25	$\frac{1}{2}$	0.165	0.050	0.115	0.0336	1.44	2.07	
35	"	$\frac{1}{2}$	0.394	0.121	0.273	0.0523	2.24	5.06	
36	"	$\frac{1}{2}$	0.711	0.209	0.502	0.0707	3.03	9.20	
37	"	$\frac{1}{2}$	1.026	0.295	0.731	0.0855	3.66	13.40	3.465
38	"	$\frac{1}{2}$	1.498	0.430	1.068	0.1035	4.44	19.70	
39	"	$\frac{1}{2}$	2.122	0.590	1.532	0.1233	5.30	28.00	
40	"	$\frac{1}{2}$	2.449	0.675	1.774	0.1333	5.71	32.50	
41	"	$\frac{1}{2}$	3.121	0.860	2.261	0.1510	6.47	41.80	

VALVE NO. 7—Continued

42	June 25	$\frac{1}{8}$	3.577	0.997	2.580	0.1630	6.98	48.59	
43	"	$\frac{1}{8}$	4.101	1.130	2.971	0.1738	7.44	25.10	
44	"	$\frac{1}{8}$	4.884	1.330	3.554	0.1896	8.13	66.00	
45	June 26	$\frac{5}{8}$	0.209	0.100	0.109	0.0491	2.11	4.42	
46	"	$\frac{5}{8}$	0.418	0.210	0.208	0.0711	3.04	9.26	
47	"	$\frac{5}{8}$	0.672	0.340	0.332	0.0906	3.89	15.08	
48	"	$\frac{5}{8}$	1.028	0.500	0.528	0.1127	4.84	23.30	
49	"	$\frac{5}{8}$	1.493	0.712	0.781	0.1373	5.89	34.60	
50	"	$\frac{5}{8}$	2.002	0.955	1.047	0.1592	6.83	46.60	
51	"	$\frac{5}{8}$	2.437	1.150	1.287	0.1758	7.55	56.80	1.418
52	"	$\frac{5}{8}$	3.071	1.460	1.611	0.1988	8.53	72.70	
53	"	$\frac{5}{8}$	3.488	1.650	1.838	0.2108	9.09	82.50	
54	"	$\frac{5}{8}$	4.040	1.935	2.105	0.2306	9.90	97.00	
55	"	$\frac{5}{8}$	4.830	2.290	2.540	0.2513	10.78	116.10	
56	"	$\frac{5}{8}$	3.723	1.796	1.927	0.2218	9.51	90.10	
57	"	$\frac{5}{8}$	4.012	1.890	2.122	0.2278	9.77	95.10	
58	"	$\frac{5}{8}$	4.386	2.080	2.306	0.2380	10.22	104.20	
59	"	$\frac{5}{8}$	4.890	2.350	2.540	0.2545	10.93	119.60	
60	"	$\frac{3}{4}$	0.200	0.135	0.065	0.0562	2.41	5.82	
61	"	$\frac{3}{4}$	0.361	0.240	0.121	0.0768	3.29	10.83	
62	"	$\frac{3}{4}$	0.677	0.460	0.217	0.1077	4.62	21.30	
63	"	$\frac{3}{4}$	1.002	0.665	0.337	0.1320	5.68	32.20	
64	"	$\frac{3}{4}$	2.004	1.347	0.657	0.1910	8.20	67.20	.680
65	"	$\frac{3}{4}$	2.468	1.635	0.833	0.2109	9.05	81.80	
66	"	$\frac{3}{4}$	3.001	1.893	1.108	0.2308	9.90	98.00	
67	"	$\frac{3}{4}$	3.388	2.170	1.218	0.2453	10.52	110.80	
68	"	$\frac{3}{4}$	4.030	2.635	1.395	0.2699	11.58	134.00	
69	"	$\frac{3}{4}$	4.591	3.000	1.591	0.2880	12.37	152.80	
70	"	1	0.226	0.199	0.027	0.0686	2.94	8.66	
71	"	1	0.353	0.319	0.034	0.0891	3.82	14.60	
72	"	1	0.681	0.619	0.062	0.1270	5.45	29.70	
73	"	1	1.005	0.858	0.147	0.1552	6.67	44.40	
74	"	1	1.505	1.300	0.205	0.1900	8.16	66.50	.140
75	"	1	1.983	1.780	0.203	0.2220	9.53	90.80	
76	"	1	2.476	2.170	0.306	0.2460	10.52	110.80	
77	"	1	2.964	2.680	0.284	0.2730	11.71	137.00	
78	"	1	3.453	3.100	0.353	0.2940	12.62	159.10	
79	"	1	3.909	3.520	0.389	0.3132	13.42	180.00	
80	"	1	4.300	3.880	0.420	0.3280	14.07	198.00	

VALVE NO. 8

Gage Length = 9.48 feet

Pipe Cross Section = 0.02338 sq. ft.

	1919		feet	feet	feet				
1	June 30	$\frac{1}{8}$	0.228	0.003	0.225	0.0086	0.36	0.13	
2	"	$\frac{1}{8}$	0.373	0.007	0.366	0.0114	0.49	0.24	
3	"	$\frac{1}{8}$	0.793	0.012	0.781	0.0166	0.71	0.51	
4	"	$\frac{1}{8}$	1.044	0.017	1.027	0.0191	0.82	0.67	
5	"	$\frac{1}{8}$	1.588	0.026	1.562	0.0236	1.01	1.02	98.9
6	"	$\frac{1}{8}$	2.091	0.033	2.058	0.0270	1.16	1.34	
7	"	$\frac{1}{8}$	2.716	0.042	2.674	0.0308	1.32	1.75	
8	"	$\frac{1}{8}$	3.094	0.049	3.045	0.0330	1.41	2.00	
9	"	$\frac{1}{8}$	3.553	0.056	3.497	0.0351	1.50	2.27	
10	"	$\frac{1}{8}$	4.000	0.062	3.938	0.0374	1.60	2.57	
11	"	$\frac{1}{8}$	5.131	0.079	5.052	0.0423	1.81	3.28	

VALVE NO. 8—Continued

12	July 11	1/4	0.165	0.020	0.145	0.0181	0.77	0.60	
13	"	1/4	0.427	0.040	0.387	0.0292	1.24	1.55	
14	"	1/4	0.661	0.051	0.610	0.0361	1.55	2.39	
15	"	1/4	0.822	0.070	0.752	0.0400	1.71	2.92	
16	"	1/4	1.926	0.160	1.766	0.0618	2.65	6.98	
17	"	1/4	2.363	0.190	2.173	0.0684	2.93	8.55	16.46
18	"	1/4	2.898	0.240	2.658	0.0757	3.24	10.50	
19	"	1/4	3.670	0.290	3.380	0.0842	3.61	13.00	
20	"	1/4	4.001	0.310	3.691	0.0870	3.73	13.85	
21	"	1/4	5.040	0.380	4.660	0.0980	4.18	17.45	
22	"	1/4	1.057	0.090	0.967	0.0458	1.96	3.85	
23	"	1/4	1.557	0.130	1.427	0.0556	2.38	5.67	
24	"	1/4	2.031	0.168	1.863	0.0636	2.72	7.40	
25	"	1/4	3.005	0.250	2.755	0.0775	3.32	11.00	
26	"	1/4	4.057	0.336	3.721	0.0901	3.86	14.90	
27	"	1/4	5.109	0.410	4.699	0.1012	4.34	18.80	
28	July 14	3/8	0.459	0.100	0.359	0.0476	2.04	4.15	
29	"	3/8	0.836	0.150	0.686	0.0645	2.76	7.62	
30	"	3/8	1.227	0.220	1.007	0.0782	3.35	11.20	
31	"	3/8	1.618	0.300	1.318	0.0902	3.87	14.90	
32	"	3/8	2.042	0.375	1.667	0.1014	4.35	18.80	
33	"	3/8	2.522	0.470	2.052	0.1132	4.85	23.50	5.62
34	"	3/8	2.985	0.555	2.430	0.1228	5.26	27.60	
35	"	3/8	3.465	0.645	2.820	0.1325	5.67	32.10	
36	"	3/8	4.070	0.760	3.310	0.1441	6.18	38.00	
37	"	3/8	5.027	0.940	4.087	0.1600	6.85	46.80	
38	July 15	1/2	0.441	0.140	0.301	0.0606	2.59	6.72	
39	"	1/2	0.675	0.210	0.465	0.0755	3.23	10.40	
40	"	1/2	1.013	0.310	0.703	0.0928	3.97	15.75	
41	"	1/2	1.520	0.475	1.045	0.1142	4.90	23.90	
42	"	1/2	2.065	0.660	1.405	0.1341	5.75	33.00	
43	"	1/2	2.570	0.830	1.740	0.1502	6.45	41.40	2.7
44	"	1/2	2.971	0.960	2.011	0.1617	6.93	47.80	
45	"	1/2	3.544	1.150	2.394	0.1770	7.58	57.30	
46	"	1/2	4.173	1.330	2.843	0.1920	8.23	67.50	
47	"	1/2	4.027	1.320	2.707	0.1890	8.10	65.40	
48	"	3/4	0.440	0.270	0.170	0.0861	3.69	13.60	
49	"	3/4	0.819	0.525	0.294	0.1191	5.21	26.10	
50	"	3/4	1.061	0.680	0.381	0.1360	5.83	34.00	
51	"	3/4	1.565	0.960	0.605	0.1611	6.91	47.60	
52	"	3/4	2.027	1.340	0.687	0.1905	8.17	66.50	.66
53	"	3/4	2.603	1.730	0.873	0.2175	9.32	86.70	
54	"	3/4	3.080	2.020	1.060	0.2362	10.12	102.20	
55	"	3/4	3.549	2.350	1.199	0.2544	10.91	118.60	
56	"	3/4	4.083	2.680	1.403	0.2732	11.70	136.50	
57	"	3/4	4.850	3.200	1.650	0.2978	12.75	162.20	
58	"	3/4	1.515	0.990	0.525	0.1640	7.03	49.30	
59	July 16	1	0.407	0.380	0.027	0.0965	4.13	17.05	
60	"	1	0.674	0.640	0.034	0.1262	5.41	29.20	
61	"	1	0.978	0.850	0.128	0.1475	6.32	39.80	
62	"	1	1.442	1.440	0.002	0.1921	8.23	67.70	
63	"	1	1.968	1.925	0.043	0.2236	9.56	91.20	
64	"	1	2.439	2.380	0.059	0.2500	10.70	114.20	
65	"	1	3.068	2.850	0.218	0.2742	11.74	138.00	
66	"	1	3.473	3.350	0.123	0.2985	12.79	163.50	
67	"	1	4.007	3.875	0.132	0.3220	13.77	189.50	.0424
68	"	1	4.650	4.500	0.150	0.3480	14.89	221.00	
69	"	1	1.957	1.900	0.057	0.2220	9.52	90.20	
70	"	1	2.546	2.535	0.011	0.2580	11.04	121.80	
71	"	1	3.131	3.010	0.121	0.2825	12.10	146.00	

VALVE NO. 8—Continued

72	July 16	1	3.439	3.400	0.039	0.3005	12.87	165.50
73	"	1	3.897	3.800	0.097	0.3180	13.62	185.50
74	"	1	4.937	4.800	0.137	0.3600	15.42	237.00
75	July 18	1	0.329	0.300	0.029	0.0859	3.68	13.50
76	"	1	0.619	0.580	0.039	0.1208	5.18	26.70
77	"	1	1.059	1.000	0.059	0.1610	6.90	47.50
78	"	1	1.521	1.480	0.041	0.1952	8.38	70.00
79	"	1	2.044	1.990	0.054	0.2275	9.74	94.70
80	"	1	2.445	2.370	0.075	0.2490	10.66	113.50
81	"	1	2.983	2.890	0.093	0.2762	11.83	140.00
82	"	1	3.543	3.440	0.103	0.3022	12.95	167.50
83	"	1	3.958	3.870	0.088	0.3215	13.77	189.00
84	"	1	4.909	4.745	0.164	0.3560	15.25	232.00

TABLE 27

LOSS OF HEAD IN 4-INCH GATE VALVES

VALVE NO. 00

Test by Kline & Tanghe Spring of 1914

Gage Length = 19.166 feet

Pipe Cross Section = 0.0884 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	$k \text{ in } \frac{v^2}{2g}$
	1914		feet	feet	feet				
1	May 2	$\frac{1}{8}$	0.253	0.007	0.246	0.0471	0.53	0.28	58.0
2	"	$\frac{1}{8}$	0.634	0.016	0.618	0.0721	0.81	0.66	
3	"	$\frac{1}{8}$	1.200	0.031	1.169	0.1024	1.16	1.34	
4	"	$\frac{1}{8}$	2.330	0.031	2.299	0.1390	1.57	2.47	
5	"	$\frac{1}{8}$	3.530	0.084	3.446	0.1740	1.97	3.87	
6	"	$\frac{1}{8}$	4.550	0.116	4.434	0.2060	2.33	5.42	
7	May 8	$\frac{1}{8}$	0.180	0.004	0.176	0.0390	0.44	0.19	13.2
8	"	$\frac{1}{8}$	0.570	0.015	0.555	0.0696	0.79	0.62	
9	"	$\frac{1}{8}$	1.080	0.028	1.052	0.0964	1.09	1.19	
10	"	$\frac{1}{8}$	1.910	0.044	1.866	0.1230	1.39	1.93	
11	"	$\frac{1}{8}$	2.630	0.063	2.567	0.1467	1.66	2.75	
12	"	$\frac{1}{8}$	4.140	0.092	4.048	0.1820	2.06	4.23	
13	May 2	$\frac{1}{4}$	0.150	0.017	0.133	0.0770	0.87	0.76	
14	"	$\frac{1}{4}$	0.325	0.035	0.290	0.1075	1.21	1.48	
15	"	$\frac{1}{4}$	0.630	0.070	0.560	0.1570	1.77	3.15	
16	"	$\frac{1}{4}$	1.160	0.114	1.046	0.2040	2.31	5.32	
17	"	$\frac{1}{4}$	1.820	0.172	1.648	0.2540	2.87	8.23	
18	"	$\frac{1}{4}$	2.680	0.250	2.430	0.3080	3.48	12.12	
19	"	$\frac{1}{4}$	4.520	0.420	4.100	0.4000	4.52	20.40	
20	May 8	$\frac{1}{4}$	0.085	0.008	0.077	0.0528	0.60	0.36	4.80
21	"	$\frac{1}{4}$	0.220	0.024	0.196	0.0889	1.00	1.01	
22	"	$\frac{1}{4}$	0.760	0.078	0.682	0.1663	1.88	3.54	
23	"	$\frac{1}{4}$	1.310	0.120	1.190	0.2110	2.38	5.70	
24	"	$\frac{1}{4}$	2.050	0.181	1.869	0.2600	2.94	8.63	
25	"	$\frac{1}{4}$	3.640	0.270	2.770	0.3230	3.65	13.30	
26	"	$\frac{1}{4}$	3.830	0.330	3.500	0.3550	4.01	16.10	
27	May 2	$\frac{3}{8}$	0.240	0.054	0.186	0.1380	1.56	2.43	
28	"	$\frac{3}{8}$	0.550	0.112	0.438	0.2110	2.39	5.70	
29	"	$\frac{3}{8}$	1.050	0.245	0.805	0.3050	3.45	10.19	
30	"	$\frac{3}{8}$	1.630	0.365	1.275	0.3680	4.16	17.30	
31	"	$\frac{3}{8}$	2.400	0.520	1.880	0.4490	5.08	25.80	
32	"	$\frac{3}{8}$	4.020	0.860	3.160	0.5870	6.64	44.00	

VALVE NO. 00—Continued

33	May 8	$\frac{3}{8}$	0.055	0.016	0.039	0.0717	0.81	0.66	
34	"	$\frac{3}{8}$	0.180	0.047	0.133	0.1270	1.43	2.06	
35	"	$\frac{3}{8}$	0.436	0.097	0.339	0.1884	2.13	4.54	
36	"	$\frac{3}{8}$	0.950	0.200	0.750	0.2740	3.10	9.60	
37	"	$\frac{3}{8}$	1.520	0.330	1.190	0.3530	3.99	15.90	4.80
38	"	$\frac{3}{8}$	2.240	0.480	1.760	0.4330	4.90	24.00	
39	"	$\frac{3}{8}$	3.230	0.660	2.570	0.5080	5.75	33.00	
40	May 2	$\frac{1}{2}$	0.060	0.027	0.033	0.0951	1.07	1.16	1.80
41	"	$\frac{1}{2}$	0.260	0.117	0.143	0.2080	2.35	5.11	
42	"	$\frac{1}{2}$	0.440	0.200	0.240	0.2730	3.09	9.50	
43	"	$\frac{1}{2}$	1.060	0.400	0.660	0.3890	4.40	19.30	
44	"	$\frac{1}{2}$	1.780	0.693	1.095	0.5220	5.90	34.80	
45	"	$\frac{1}{2}$	2.870	1.070	1.820	0.6580	7.45	55.30	
46	May 8	$\frac{1}{2}$	0.053	0.030	0.023	0.1000	1.13	1.28	
47	"	$\frac{1}{2}$	0.180	0.078	0.102	0.1663	1.88	3.54	
48	"	$\frac{1}{2}$	0.380	0.158	0.222	0.2430	2.74	7.53	
49	"	$\frac{1}{2}$	0.800	0.300	0.500	0.3390	3.84	14.70	
50	"	$\frac{1}{2}$	0.700	0.620	0.080	0.4930	5.58	31.10	
51	"	$\frac{1}{2}$	2.690	0.930	1.560	0.6150	6.95	48.40	
52	May 2	$\frac{3}{4}$	0.075	0.050	0.025	0.1300	1.47	2.16	0.511
53	"	$\frac{3}{4}$	0.210	0.166	0.044	0.2490	2.82	7.92	
54	"	$\frac{3}{4}$	0.560	0.420	0.140	0.4000	4.52	20.40	
55	"	$\frac{3}{4}$	1.420	1.025	0.395	0.6440	7.28	53.00	
56	"	$\frac{3}{4}$	1.830	1.290	0.540	0.7290	8.25	68.00	
57	"	$\frac{3}{4}$	2.250	1.630	0.620	0.8270	9.35	87.40	
58	May 8	$\frac{3}{4}$	0.055	0.044	0.011	0.1230	1.39	1.93	
59	"	$\frac{3}{4}$	0.290	0.220	0.070	0.2910	3.29	10.80	
60	"	$\frac{3}{4}$	0.750	0.550	0.200	0.4640	5.25	27.50	
61	"	$\frac{3}{4}$	1.310	0.865	0.445	0.5890	6.66	44.30	
62	"	$\frac{3}{4}$	1.940	1.290	0.650	0.7280	8.24	67.80	
63	"	$\frac{3}{4}$	2.370	1.730	0.640	0.8520	9.64	92.80	
64	May 1	1	0.185	0.007	0.178	0.0507	0.57	0.33	0.244
65	"	1	0.150	0.056	0.094	0.1380	1.56	2.43	
66	"	1	0.170	0.148	0.022	0.2360	2.67	7.11	
67	"	1	0.315	0.280	0.035	0.3290	3.72	13.80	
68	"	1	0.580	0.480	0.100	0.4330	4.90	24.00	
69	"	1	1.000	0.825	0.125	0.5730	6.48	41.90	
70	"	1	1.210	1.070	0.140	0.6600	7.46	55.70	
71	"	1	1.780	1.585	0.195	0.8130	9.20	84.40	
72	May 2	1	0.090	0.056	0.034	0.1390	1.57	2.47	0.244
73	"	1	0.146	0.096	0.050	0.1870	2.12	4.47	
74	"	1	0.220	0.176	0.044	0.2570	2.90	8.42	
75	"	1	0.430	0.380	0.050	0.3810	4.31	18.50	
76	"	1	0.720	0.640	0.080	0.5000	5.66	32.00	
77	"	1	1.110	0.940	0.170	0.6170	6.97	48.60	
78	"	1	1.690	1.420	0.270	0.7640	8.64	74.50	

VALVE NO. 00

Test of H. N. Lendall Summer of 1914.

Gage Length = 19.75 feet

Pipe Cross Section = 0.0884 sq. ft.

1	1914 July 18	$\frac{1}{8}$	0.222	0.002	0.220	0.0446	0.50	0.25	61.50
2	"	$\frac{1}{8}$	0.245	0.002	0.243	0.0445	0.50	0.25	
3	"	$\frac{1}{8}$	0.813	0.007	0.806	0.0503	0.91	0.82	

VALVE NO. 00—Continued

4	July 18	$\frac{1}{8}$	0.832	0.007	0.825	0.0812	0.92	0.84	13.1
5	"	$\frac{1}{8}$	1.591	0.024	1.567	0.1101	1.24	1.55	
6	"	$\frac{1}{8}$	1.599	0.024	1.575	0.1098	1.24	1.54	
7	"	$\frac{1}{8}$	2.352	0.042	2.310	0.1359	1.54	2.36	
8	"	$\frac{1}{8}$	2.345	0.042	2.303	0.1359	1.54	2.36	
9	July 20	$\frac{1}{8}$	3.850	0.070	3.780	0.1770	2.00	4.01	
10	"	$\frac{1}{8}$	3.901	0.068	3.833	0.1172	1.94	3.78	
11	"	$\frac{1}{8}$	2.980	0.050	2.930	0.1521	1.72	2.96	
12	"	$\frac{1}{8}$	3.021	0.048	2.973	0.1508	1.70	2.90	
13	"	$\frac{1}{8}$	2.549	0.049	2.500	0.1515	1.71	2.94	
14	"	$\frac{1}{8}$	4.145	0.072	4.073	0.1798	2.03	4.14	
15	"	$\frac{1}{8}$	7.610	0.160	7.450	0.2565	2.90	8.41	
16	"	$\frac{1}{8}$	12.970	1.410	11.56	0.3214	3.63	13.20	
17	"	$\frac{1}{8}$	18.810	3.330	15.48	0.3850	4.35	18.92	
18	July 24	$\frac{1}{8}$	2.570	0.470	2.523	0.1450	1.64	2.69	
19	"	$\frac{1}{8}$	2.580	0.471	2.533	0.1454	1.64	2.70	
20	"	$\frac{1}{8}$	5.250	0.100	5.150	0.2070	2.34	5.48	
21	"	$\frac{1}{8}$	7.100	0.099	7.000	0.2065	2.33	5.45	
22	"	$\frac{1}{8}$	7.220	0.141	7.079	0.2435	2.75	7.57	
23	"	$\frac{1}{8}$	10.040	0.052	9.988	0.2440	2.76	7.61	
24	"	$\frac{1}{8}$	10.570	0.203	10.367	0.2940	3.32	11.06	
25	"	$\frac{1}{8}$	10.480	0.200	10.28	0.2910	3.29	10.88	
26	"	$\frac{1}{8}$	16.850	0.311	16.539	0.3720	4.21	17.70	
27	"	$\frac{1}{8}$	16.660	0.301	16.359	0.3650	4.12	17.00	
28	"	$\frac{1}{8}$	16.800	0.309	16.491	0.3690	4.17	17.40	
29	"	$\frac{1}{8}$	16.730	0.301	16.429	0.3650	4.12	17.00	
30	"	$\frac{1}{8}$	13.010	0.240	12.770	0.3220	3.64	13.25	
31	"	$\frac{1}{8}$	12.740	0.236	12.504	0.3200	3.62	13.10	
32	"	$\frac{1}{8}$	18.940	0.350	18.590	0.3935	4.45	19.80	
33	"	$\frac{1}{8}$	18.890	0.333	18.557	0.3860	4.37	19.20	
34	JI 21-23	$\frac{1}{4}$	0.188	0.013	0.175	0.0807	0.91	0.83	
35	"	$\frac{1}{4}$	0.195	0.014	0.181	0.0814	0.92	0.85	
36	"	$\frac{1}{4}$	0.740	0.057	0.683	0.1600	1.81	3.28	
37	"	$\frac{1}{4}$	0.724	0.056	0.668	0.1595	1.80	3.26	
38	"	$\frac{1}{4}$	1.297	0.110	1.187	0.2156	2.44	5.95	
39	"	$\frac{1}{4}$	1.318	0.112	1.206	0.2172	2.46	6.02	
40	"	$\frac{1}{4}$	1.310	0.112	1.198	0.2172	2.46	6.02	
41	JI 21-23	$\frac{1}{4}$	2.201	0.191	2.010	0.2835	3.20	10.28	
42	"	$\frac{1}{4}$	2.221	0.191	2.030	0.2835	3.20	10.28	
43	"	$\frac{1}{4}$	2.226	0.187	2.039	0.2816	3.18	10.14	
44	"	$\frac{1}{4}$	2.984	0.242	2.742	0.3255	3.68	13.55	
45	"	$\frac{1}{4}$	2.965	0.242	2.723	0.3250	3.68	13.55	
46	"	$\frac{1}{4}$	2.929	0.240	2.689	0.3234	3.66	13.40	
47	"	$\frac{1}{4}$	4.010	0.320	3.690	0.3770	4.26	18.20	
48	"	$\frac{1}{4}$	4.004	0.322	3.682	0.3790	4.29	18.40	
49	"	$\frac{1}{4}$	6.670	0.520	6.150	0.4835	5.47	29.90	
50	"	$\frac{1}{4}$	6.610	0.520	6.090	0.4825	5.46	29.80	
51	"	$\frac{1}{4}$	5.630	0.441	5.189	0.4435	5.02	25.20	
52	"	$\frac{1}{4}$	5.660	0.442	5.218	0.4460	5.04	25.40	
53	"	$\frac{1}{4}$	9.190	0.741	8.449	0.5780	6.54	42.75	
54	"	$\frac{1}{4}$	9.250	0.659	8.591	0.5440	6.15	37.80	
55	"	$\frac{1}{4}$	15.660	1.210	14.45	0.7420	8.40	70.50	
56	"	$\frac{1}{4}$	17.730	1.460	16.27	0.8170	9.24	85.20	
57	"	$\frac{1}{4}$	5.240	0.412	4.828	0.4285	4.85	23.56	
58	"	$\frac{1}{4}$	5.080	0.399	4.681	0.4200	4.75	22.59	
59	"	$\frac{1}{4}$	5.360	0.422	4.938	0.4345	4.92	24.20	
60	"	$\frac{1}{4}$	5.280	0.418	4.862	0.4300	4.87	23.70	
61	"	$\frac{1}{4}$	8.760	0.682	8.078	0.5530	6.25	39.20	
62	"	$\frac{1}{4}$	8.910	0.692	8.218	0.5580	6.31	39.80	
63	"	$\frac{1}{4}$	11.420	0.850	10.57	0.6370	7.20	51.80	

VALVE NO. 00—Continued

64	July 21-23	1/4	11.550	0.902	10.65	0.6400	7.24	52.30	
65	"	1/4	15.280	1.200	15.08	0.7370	8.34	69.30	
66	"	1/4	17.420	1.330	16.09	0.7850	8.88	78.70	
67	"	1/4	18.120	1.480	16.64	0.8250	9.33	87.00	
68	July 29	3/8	4.180	0.762	3.418	0.5845	6.61	43.60	5.05
69	"	3/8	4.156	0.744	3.412	0.5800	6.56	43.00	
70	"	3/8	2.674	0.482	2.192	0.4660	5.27	27.80	
71	"	3/8	2.636	0.482	2.154	0.4650	5.26	27.70	
72	"	3/8	2.076	0.385	1.691	0.4120	4.65	21.70	
73	"	3/8	2.193	0.408	1.785	0.4270	4.83	23.30	
74	"	3/8	1.437	0.261	1.176	0.3380	3.82	14.60	
75	"	3/8	1.459	0.256	1.203	0.3350	3.79	14.40	
76	"	3/8	0.958	0.189	0.769	0.2816	3.18	10.12	
77	"	3/8	0.954	0.184	0.770	0.2790	3.16	9.95	
78	"	3/8	0.549	0.112	0.437	0.2147	2.43	5.90	
79	"	3/8	0.545	0.108	0.439	0.2125	2.40	5.78	
80	July 28	1/2	0.630	0.222	0.408	0.3115	3.52	12.40	2.14
81	"	1/2	0.608	0.235	0.373	0.3205	3.62	13.10	
82	"	1/2	1.047	0.343	0.704	0.3920	4.43	19.60	
83	"	1/2	1.000	0.341	0.659	0.3900	4.41	19.50	
84	"	1/2	2.060	0.687	1.373	0.5550	6.28	39.40	
85	"	1/2	1.966	0.680	1.286	0.5520	6.24	39.00	
86	"	1/2	3.943	1.330	2.613	0.7820	8.85	78.00	
87	"	1/2	3.832	1.320	2.512	0.7790	8.81	77.50	
88	"	1/2	3.720	1.310	2.410	0.7740	8.75	76.50	
89	"	1/2	2.778	0.938	1.840	0.6520	7.37	54.30	
90	"	1/2	2.782	0.925	1.857	0.6475	7.33	53.60	
91	July 24	3/8	0.287	0.150	0.137	0.2515	2.84	8.08	1.023
92	"	3/8	0.246	0.150	0.096	0.2500	2.83	8.00	
93	"	3/8	0.277	0.150	0.127	0.2500	2.83	8.00	
94	"	3/8	0.258	0.144	0.114	0.2470	2.79	7.80	
95	"	3/8	0.655	0.357	0.298	0.3980	4.50	20.28	
96	"	3/8	0.682	0.355	0.327	0.3960	4.48	20.00	
97	"	3/8	0.668	0.346	0.322	0.3920	4.43	19.60	
98	"	3/8	0.671	0.349	0.322	0.3930	4.45	19.80	
99	"	3/8	0.946	0.496	0.450	0.4710	5.33	28.40	
100	"	3/8	0.929	0.490	0.439	0.4690	5.30	28.10	
101	"	3/8	0.932	0.496	0.436	0.4710	5.33	28.40	
102	"	3/8	1.783	0.928	0.855	0.6500	7.35	54.10	
103	"	3/8	1.784	0.933	0.851	0.6530	7.38	54.50	
104	"	3/8	2.349	1.230	1.119	0.7520	8.50	72.20	
105	"	3/8	2.290	1.210	1.080	0.7450	8.42	71.00	
106	"	3/8	2.300	1.200	1.100	0.7410	8.38	70.30	
107	"	3/8	2.493	1.210	1.283	0.7440	8.41	70.80	
108	"	3/8	3.078	1.610	1.468	0.8570	9.70	93.80	
109	"	3/8	3.033	1.570	1.463	0.8510	9.64	92.60	
110	"	3/8	4.177	2.080	2.097	1.0000	11.31	127.50	
111	July 28	3/4	1.970	1.340	0.630	0.7880	8.91	79.30	0.527
112	"	3/4	2.030	1.380	0.650	0.7950	9.00	81.00	
113	"	3/4	2.050	1.390	0.660	0.7980	9.03	81.30	
114	"	3/4	3.434	2.340	1.094	1.0470	11.82	140.00	
115	"	3/4	3.480	2.390	1.090	1.0550	11.92	142.00	
116	"	3/4	2.470	1.660	0.810	0.8780	9.93	98.30	
117	"	3/4	1.618	1.081	0.537	0.7020	7.94	62.90	
118	"	3/4	1.617	1.081	0.536	0.7020	7.94	62.90	
119	"	3/4	1.070	0.512	0.558	0.5660	6.40	40.90	
120	"	3/4	1.052	0.708	0.344	0.5640	6.38	40.70	
121	"	3/4	0.507	0.439	0.068	0.4430	5.01	25.10	
122	"	3/4	0.694	0.455	0.239	0.4510	5.10	26.00	

VALVE NO. 00—Continued

123	July 15	1	0.032	0.020	0.012	0.0985	1.11	1.24	0.250
124	"	1	0.410	0.020	0.021	0.0985	1.11	1.24	
125	"	1	0.407	0.256	0.151	0.3340	3.78	14.34	
126	"	1	0.306	0.252	0.054	0.3310	3.74	14.00	
127	"	1	0.317	0.256	0.061	0.3340	3.78	14.25	
128	"	1	0.316	0.256	0.060	0.3340	3.78	14.25	
129	"	1	0.308	0.253	0.055	0.3320	3.75	14.05	
130	"	1	0.905	0.668	0.237	0.5460	5.18	38.20	
131	"	1	0.913	0.671	0.242	0.5490	6.21	38.60	
132	July 16	1	1.580	1.150	0.430	0.7250	8.20	67.10	
133	"	1	1.580	1.150	0.430	0.7250	8.20	67.10	
134	"	1	2.570	1.630	0.940	0.8650	9.78	95.50	
135	"	1	2.545	1.670	0.875	0.8770	9.91	98.00	
136	"	1	3.441	2.780	0.661	1.1420	12.90	116.70	
137	"	1	3.329	2.660	0.669	1.1160	12.61	159.00	
138	July 17	1	2.259	1.600	0.659	0.8560	9.68	93.50	
139	"	1	1.92	1.610	0.352	0.8550	9.66	93.30	
140	"	1	1.998	1.630	0.368	0.8600	9.72	94.50	
141	"	1	2.358	1.930	0.428	0.9440	10.67	114.00	
142	"	1	2.368	1.930	0.438	0.9440	10.67	114.00	
143	"	1	3.185	2.610	0.575	1.1050	12.49	156.00	
144	"	1	3.226	2.600	0.626	1.1000	12.42	154.50	
145	July 18	1	2.024	1.650	0.374	0.8730	9.87	97.20	0.250
146	"	1	2.198	1.880	0.318	0.9320	10.52	111.00	
47	"	1	3.634	2.950	0.684	1.1760	13.28	176.00	
148	"	1	3.009	2.500	0.509	1.0750	12.15	148.00	
149	July 20	1	2.514	1.960	0.554	0.9500	10.74	115.50	
150	"	1	2.593	2.000	0.593	0.9560	10.82	117.40	
151	"	1	2.327	2.000	0.327	0.9560	10.82	117.40	
152	"	1	2.602	1.970	0.632	0.9540	10.78	116.50	
153	"	1	4.265	3.620	0.645	1.3050	14.76	218.50	
154	"	1	0.855	0.625	0.230	0.5320	6.02	36.20	
155	"	1	1.496	1.250	0.246	0.7560	8.55	73.10	
156	"	1	1.572	1.260	0.312	0.7580	8.58	73.50	
157	"	1	1.273	1.070	0.203	0.6990	7.90	62.50	
158	"	1	0.709	0.562	0.147	0.5020	5.68	32.20	
159	"	1	1.005	0.824	0.181	0.6110	6.91	47.70	

VALVE NO. 1

Gage Length = 16.93 feet

Pipe Cross Section = 0.08855 sq. ft.

	1919		feet	feet	feet				
1	May 3	$\frac{1}{8}$	0.453	0.0075	0.4457	0.0594	0.68	0.46	69.75
2	"	$\frac{1}{8}$	0.966	0.0148	0.9512	0.0836	0.95	0.91	
3	"	$\frac{1}{8}$	1.528	0.0230	1.505	0.1045	1.19	1.42	
4	"	$\frac{1}{8}$	2.045	0.0300	2.015	0.1210	1.38	1.91	
5	"	$\frac{1}{8}$	3.014	0.0247	2.989	0.1484	1.68	2.66	
6	"	$\frac{1}{8}$	3.493	0.0490	3.444	0.1586	1.81	3.27	
7	"	$\frac{1}{8}$	3.490	0.0470	3.443	0.1550	1.77	3.13	
8	"	$\frac{1}{8}$	3.970	0.0523	3.918	0.1650	1.88	3.54	
9	"	$\frac{1}{8}$	4.458	0.0603	4.398	0.1780	2.03	4.13	
10	"	$\frac{1}{8}$	5.000	0.0648	4.935	0.1850	2.11	4.45	
11	"	$\frac{1}{8}$							
12	May 10	$\frac{1}{4}$	0.500	0.0378	0.4622	0.1374	1.57	2.46	12.49
13	"	$\frac{1}{4}$	1.008	0.0692	0.939	0.1918	2.19	4.78	
14	"	$\frac{1}{4}$	1.494	0.1010	1.395	0.2340	2.67	7.12	

VALVE NO. 1—Continued

15	May 10	$\frac{1}{4}$	1.972	0.1340	1.838	0.2700	3.08	9.48	
16	"	$\frac{1}{4}$	2.461	0.1670	2.294	0.2996	3.42	11.69	
17	"	$\frac{1}{4}$	2.956	0.2000	2.756	0.3307	3.78	14.25	
18	"	$\frac{1}{4}$	3.481	0.2370	3.244	0.3587	4.10	16.74	
19	"	$\frac{1}{4}$	3.982	0.2650	3.717	0.3837	4.38	19.16	
20	"	$\frac{1}{4}$	4.570	0.3000	4.270	0.4120	4.71	22.10	
21	"	$\frac{1}{4}$	5.037	0.3400	4.697	0.4325	4.94	24.35	
22	May 16	$\frac{3}{8}$	0.506	0.0852	0.421	0.2124	2.44	5.96	4.27
23	"	$\frac{3}{8}$	0.993	0.1700	0.823	0.3060	3.49	12.18	
24	"	$\frac{3}{8}$	1.490	0.2500	1.240	0.3760	4.29	18.40	
25	"	$\frac{3}{8}$	2.005	0.3470	1.658	0.4370	4.99	24.80	
26	"	$\frac{3}{8}$	2.479	0.4200	2.059	0.4880	5.57	31.00	
27	"	$\frac{3}{8}$	2.951	0.5040	2.447	0.5330	6.09	37.00	
28	"	$\frac{3}{8}$	3.492	0.5840	2.908	0.5810	6.64	44.00	
29	"	$\frac{3}{8}$	3.981	0.6600	3.321	0.6190	7.07	49.80	
30	"	$\frac{3}{8}$	4.494	0.7450	3.749	0.6610	7.55	56.80	
31	"	$\frac{3}{8}$	4.955	0.8150	4.140	0.6940	7.93	62.80	
32	May 23	$\frac{1}{2}$	0.482	0.0168	0.465	0.0915	1.04	1.09	1.816
33	"	$\frac{1}{2}$	0.978	0.3100	0.668	0.4180	4.78	22.80	
34	"	$\frac{1}{2}$	1.522	0.4750	1.047	0.5250	6.00	36.00	
35	"	$\frac{1}{2}$	2.133	0.6900	1.443	0.6350	7.25	52.50	
36	"	$\frac{1}{2}$	2.507	0.8000	1.707	0.6890	7.87	61.90	
37	"	$\frac{1}{2}$	2.560	0.8100	1.750	0.6940	7.93	62.80	
38	"	$\frac{1}{2}$	3.041	0.9300	2.111	0.7600	8.70	75.30	
39	"	$\frac{1}{2}$	3.535	1.0700	2.465	0.8160	9.32	86.70	
40	"	$\frac{1}{2}$	3.867	1.1550	2.712	0.8500	9.71	94.10	
41	"	$\frac{1}{2}$	4.594	1.4000	3.194	0.9380	10.72	114.80	
42	"	$\frac{1}{2}$	5.222	1.5800	3.642	1.0030	11.45	131.00	
43	May 24	$\frac{3}{4}$	0.526	0.3900	0.136	0.4720	5.38	28.90	.2923
44	"	$\frac{3}{4}$	1.019	0.7700	0.249	0.6740	7.70	59.10	
45	"	$\frac{3}{4}$	1.538	1.1100	0.428	0.8330	9.51	90.10	
46	"	$\frac{3}{4}$	2.028	1.4800	0.548	0.9680	11.05	122.00	
47	"	$\frac{3}{4}$	2.495	1.8100	0.685	1.0760	12.28	150.20	
48	"	$\frac{3}{4}$	3.006	2.1700	0.836	1.1800	13.48	181.00	
49	"	$\frac{3}{4}$	3.474	2.5200	0.944	1.2700	14.50	210.00	
50	"	$\frac{3}{4}$	3.980	2.8600	1.120	1.3640	15.58	242.00	
51	"	$\frac{3}{4}$	4.423	3.1700	1.253	1.4430	16.49	271.00	
52	"	$\frac{3}{4}$	4.694	3.3800	1.314	1.4940	17.06	290.00	
53	"	$\frac{3}{4}$	4.295	3.1300	1.165	1.4340	16.38	267.50	
54	May 31	1	0.503	0.4200	0.073	0.4970	5.69	32.30	.125
55	"	1	1.019	0.8500	0.149	0.7290	8.32	69.10	
56	"	1	1.516	1.2500	0.206	0.9050	10.34	103.70	
57	"	1	2.012	1.6700	0.282	1.0500	11.99	143.50	
58	"	1	2.529	2.1300	0.339	1.1850	13.52	182.60	
59	"	1	3.001	2.5300	0.401	1.2960	14.80	218.00	
60	"	1	3.525	2.9700	0.495	1.4100	16.10	258.00	
61	"	1	4.390	3.7600	0.640	1.5780	18.02	324.00	
62	"	1	4.211	3.4600	0.611	1.5460	17.65	310.50	

VALVE NO. 2

Gage Length = 16.93 feet

Pipe Cross Section = 0.08855 sq. ft.

1	1919		feet	feet	feet				
July 21	$\frac{1}{8}$	0.066	0.0007	0.0653	0.0212	0.24	0.05	79.35	
2	"	$\frac{1}{8}$	0.503	0.0070	0.496	0.0578	0.66	0.44	
3	"	$\frac{1}{8}$	1.008	0.0132	0.9948	0.0802	0.92	0.84	
4	"	$\frac{1}{8}$							
5	"	$\frac{1}{8}$	2.040	0.0269	2.0131	0.1130	1.29	1.66	
6	"	$\frac{1}{8}$	2.562	0.0339	2.5281	0.1263	1.44	2.17	

VALVE NO. 2—Continued

7	July 21	$\frac{1}{8}$	3.149	0.0390	3.110	0.1401	1.60	2.56	
8	"	$\frac{1}{8}$	3.539	0.0419	3.4971	0.1455	1.66	2.76	
9	"	$\frac{1}{8}$	3.994	0.0470	3.947	0.1554	1.78	3.15	
10	"	$\frac{1}{8}$	4.563	0.0525	4.5105	0.1651	1.89	3.56	
11	"	$\frac{1}{8}$	5.163	0.0600	5.103	0.1774	2.02	4.11	
12	"	$\frac{1}{8}$	0.135	0.0016	0.1334	0.0297	0.34	0.11	
13	"	$\frac{1}{8}$	0.263	0.0035	0.2595	0.0418	0.48	0.23	
14	"	$\frac{1}{8}$	1.509	0.0200	1.489	0.0977	1.12	1.25	
15	"	$\frac{1}{8}$	1.508	0.0191	1.4889	0.0953	1.09	1.18	
16	"	$\frac{1}{8}$	3.546	0.0428	3.5032	0.1470	1.68	2.81	
17	"	$\frac{1}{8}$	5.037	0.0591	4.9779	0.1756	2.00	4.02	
18	July 22	$\frac{1}{4}$	0.576	0.0400	0.536	0.1420	1.62	2.62	13.26
19	"	$\frac{1}{4}$	1.077	0.0720	1.005	0.1951	2.23	4.96	
20	"	$\frac{1}{4}$	1.523	0.0990	1.424	0.2312	2.64	6.97	
21	"	$\frac{1}{4}$	2.009	0.1300	1.879	0.2653	3.03	9.18	
22	"	$\frac{1}{4}$	2.559	0.1600	2.399	0.2995	3.42	11.70	
23	"	$\frac{1}{4}$	3.071	0.1750	2.896	0.3280	3.74	14.01	
24	"	$\frac{1}{4}$	3.476	0.2150	3.261	0.3480	3.97	15.79	
25	"	$\frac{1}{4}$	4.079	0.2600	3.819	0.3780	4.32	18.60	
26	"	$\frac{1}{4}$	4.617	0.2900	4.327	0.4020	4.59	21.10	
27	"	$\frac{1}{4}$	5.133	0.3240	4.809	0.4230	4.83	23.35	
28	July 23	$\frac{3}{8}$	0.548	0.0865	0.461	0.2240	2.56	6.52	4.35
29	"	$\frac{3}{8}$	1.015	0.1700	0.845	0.3080	3.52	12.38	
30	"	$\frac{3}{8}$	1.496	0.2500	1.246	0.3730	4.26	18.20	
31	"	$\frac{3}{8}$	2.025	0.3500	1.675	0.4380	5.01	25.10	
32	"	$\frac{3}{8}$	2.511	0.4240	2.087	0.4860	5.55	30.82	
33	"	$\frac{3}{8}$	3.051	0.5000	2.551	0.5370	6.13	37.60	
34	"	$\frac{3}{8}$	3.530	0.5800	2.950	0.5790	6.61	43.75	
35	"	$\frac{3}{8}$	4.004	0.6550	3.349	0.6170	7.04	49.60	
36	"	$\frac{3}{8}$	4.499	0.7300	3.769	0.6550	7.48	56.00	
37	"	$\frac{3}{8}$	5.054	0.8050	4.249	0.6930	7.92	62.75	
38	July 24	$\frac{1}{2}$	0.526	0.1650	0.361	0.3043	3.48	12.09	1.95
39	"	$\frac{1}{2}$	1.018	0.3300	0.688	0.4260	4.86	23.65	
40	"	$\frac{1}{2}$	1.545	0.4800	1.065	0.5260	6.00	36.00	
41	"	$\frac{1}{2}$	2.010	0.6200	1.390	0.6010	6.86	47.10	
42	"	$\frac{1}{2}$	2.582	0.7450	1.837	0.6590	7.53	56.70	
43	"	$\frac{1}{2}$	2.979	0.8850	2.099	0.7320	8.36	69.95	
44	"	$\frac{1}{2}$	3.519	1.0150	2.504	0.7940	9.07	82.20	
45	"	$\frac{1}{2}$	4.017	1.1700	2.847	0.8540	9.75	95.00	
46	"	$\frac{1}{2}$	4.571	1.3080	3.263	0.9050	10.34	106.90	
47	"	$\frac{1}{2}$	5.122	1.4620	3.660	0.9600	10.97	120.30	
48	"	$\frac{1}{2}$	2.540	0.7650	1.775	0.6740	7.70	59.30	
49	July 24	$\frac{3}{4}$	0.532	0.3900	0.142	0.4710	5.37	28.90	0.3185
50	"	$\frac{3}{4}$	1.035	0.7580	0.277	0.6680	7.64	58.20	
51	"	$\frac{3}{4}$	1.521	1.0600	0.461	0.8140	9.29	86.20	
52	"	$\frac{3}{4}$	2.025	1.4200	0.605	0.9430	10.78	116.10	
53	"	$\frac{3}{4}$	2.550	1.7800	0.770	1.0630	12.15	147.60	
54	"	$\frac{3}{4}$	2.996	2.1100	0.886	1.1600	13.25	175.70	
55	"	$\frac{3}{4}$	3.540	2.5340	1.006	1.2740	14.55	211.50	
56	"	$\frac{3}{4}$	4.036	2.8500	1.186	1.3610	15.55	242.00	
57	"	$\frac{3}{4}$	4.505	3.1700	1.335	1.4420	16.49	271.00	
58	"	$\frac{3}{4}$	4.840	3.4200	1.420	1.5000	17.12	293.50	
59	July 25	1	0.543	0.053	0.053	0.5320	6.07	36.80	1.1217
60	"	1	1.058	0.138	0.138	0.7480	8.54	73.00	
61	"	1	1.515	0.221	0.221	0.8980	10.26	105.50	
62	"	1	2.033	0.245	0.245	1.0530	12.03	144.30	
63	"	1	2.516	0.346	0.346	1.1780	13.45	181.00	
64	"	1	3.097	0.427	0.427	1.3140	15.00	225.00	
65	"	1	3.999	0.569	0.569	1.5050	17.19	295.00	
66	"	1	3.535	0.485	0.485	1.4130	16.13	260.00	
67	"	1	4.597	0.657	0.657	1.6190	18.49	341.05	

TABLE 28

LOSS OF HEAD IN 6-INCH GATE VALVES

VALVE NO. 00

Test by Kline and Tanghe Spring 1914

Gage Length = 21 feet

Pipe Cross Section = 0.1962 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	$k \text{ in } \frac{v^2}{2g}$
	1914		feet	feet	feet				
1	May 16	$\frac{1}{8}$	0.092	0.0004	0.092	0.053	0.21	0.04	76.4
2	"	$\frac{1}{8}$	0.977	0.0080	0.969	0.174	0.89	0.79	
3	"	$\frac{1}{8}$	2.970	0.0240	2.946	0.306	1.56	2.44	
4	"	$\frac{1}{8}$	3.820	0.0310	3.789	0.348	1.77	3.14	
5	"	$\frac{1}{8}$	1.940	0.0160	1.924	0.246	1.25	1.56	
6	"	$\frac{1}{8}$	1.920	0.1400	1.780	0.749	3.82	14.59	
7	"	$\frac{1}{4}$	0.113	0.0040	0.109	0.121	0.61	0.38	16.01
8	"	$\frac{1}{4}$	0.360	0.0140	0.346	0.236	1.20	1.43	
9	"	$\frac{1}{4}$	0.830	0.0320	0.798	0.352	1.79	3.20	
10	"	$\frac{1}{4}$	1.650	0.0700	1.580	0.525	2.67	7.13	
11	"	$\frac{1}{4}$	2.670	0.1000	2.570	0.631	3.21	10.30	
12	"	$\frac{1}{4}$	3.930	0.1500	3.780	0.776	3.95	15.60	
13	"	$\frac{3}{8}$	3.765	0.367	3.398	1.216	6.18	38.19	6.01
14	"	$\frac{3}{8}$	2.880	0.284	2.593	1.074	5.47	29.92	
15	"	$\frac{3}{8}$	2.070	0.207	1.863	0.712	4.64	21.53	
16	"	$\frac{3}{8}$	1.140	0.111	1.029	0.668	3.40	11.56	
17	"	$\frac{3}{8}$	0.310	0.031	0.279	0.348	1.77	3.13	
18	"	$\frac{3}{8}$	0.120	0.009	0.111	0.189	0.96	0.92	
19	"	$\frac{1}{2}$	0.040	0.006	0.034	0.153	0.78	0.61	2.67
20	"	$\frac{1}{2}$	0.230	0.044	0.186	0.419	2.13	4.54	
21	"	$\frac{1}{2}$	0.870	0.170	0.700	0.826	4.20	17.68	
22	"	$\frac{1}{2}$	1.440	0.298	1.142	1.082	5.57	31.02	
23	"	$\frac{3}{4}$	2.250	0.453	1.797	1.350	6.87	47.20	
24	"	$\frac{3}{4}$	3.380	0.684	2.696	1.660	8.45	71.40	
25	May 19	$\frac{3}{4}$	1.770	1.337	0.433	2.307	11.75	138.06	.476
26	"	$\frac{3}{4}$	1.510	0.843	0.667	1.841	9.37	87.80	
27	"	$\frac{3}{4}$	0.930	0.508	0.422	1.429	7.27	52.85	
28	"	$\frac{3}{4}$	0.590	0.333	0.257	1.154	5.88	34.57	
29	"	$\frac{3}{4}$	0.354	0.200	0.154	0.897	4.57	20.88	
30	"	$\frac{3}{4}$	0.140	0.066	0.074	0.508	2.59	6.71	
31	"	1	0.080	0.033	0.047	0.355	1.81	3.28	.408
32	"	1	0.190	0.105	0.085	0.647	3.29	10.82	
33	"	1	0.273	0.160	0.113	0.803	4.08	16.65	
34	"	1	0.310	0.225	0.085	0.950	4.85	23.52	
35	"	1	0.500	0.394	0.106	1.256	6.40	40.96	
36	"	1	1.230	0.950	0.280	1.950	9.93	98.60	

VALVE NO. 03

Test of H. N. Lendall Summer of 1914

Gage Length = 20.92 feet

Pipe Cross Section = 0.2006 sq. ft.

	1914		feet	feet	feet				
1	Aug. 6	$\frac{1}{8}$	0.400	0.003	0.397	0.121	0.61	0.37	71.2
2	"	$\frac{1}{8}$	1.090	0.009	1.081	0.197	0.98	0.96	
3	"	$\frac{1}{8}$	1.090	0.009	1.081	0.197	0.98	0.96	
4	"	$\frac{1}{8}$	2.003	0.018	1.985	0.268	1.34	1.79	
5	"	$\frac{1}{8}$	2.043	0.018	2.025	0.272	1.36	1.84	
6	"	$\frac{1}{8}$	2.813	0.025	2.788	0.316	1.57	2.48	
7	"	$\frac{1}{8}$	2.838	0.026	2.812	0.322	1.60	2.56	
8	"	$\frac{1}{8}$	3.645	0.033	3.612	0.364	1.81	3.28	
9	"	$\frac{1}{8}$	3.575	0.032	3.543	0.358	1.78	3.19	
10	"	$\frac{1}{8}$	4.488	0.041	4.447	0.407	2.03	4.12	
11	"	$\frac{1}{8}$	4.470	0.038	4.432	0.401	2.00	3.99	
12	"	$\frac{1}{4}$	4.208	0.166	4.042	0.835	4.16	17.30	15.44
13	"	$\frac{1}{4}$	4.187	0.164	4.023	0.829	4.13	17.06	
14	"	$\frac{1}{4}$	3.635	0.140	3.495	0.766	3.82	14.60	
15	"	$\frac{1}{4}$	3.650	0.142	3.508	0.770	3.83	14.70	
16	"	$\frac{1}{4}$	2.452	0.095	2.357	0.629	3.13	9.82	
17	"	$\frac{1}{4}$	2.425	0.095	2.330	0.627	3.13	9.80	
18	"	$\frac{1}{4}$	1.638	0.064	1.574	0.516	2.57	6.53	
19	"	$\frac{1}{4}$	1.667	0.065	1.602	0.516	2.57	6.60	
20	"	$\frac{1}{4}$	0.920	0.036	0.884	0.382	1.90	3.62	
21	"	$\frac{1}{4}$	0.432	0.017	0.415	0.262	1.31	1.71	
22	"	$\frac{3}{8}$	0.718	0.073	0.648	0.539	2.68	7.20	5.75
23	"	$\frac{3}{8}$	0.720	0.070	0.650	0.539	2.68	7.20	
24	"	$\frac{3}{8}$	1.535	0.145	1.387	0.790	3.95	15.45	
25	"	$\frac{3}{8}$	1.566	0.152	1.414	0.796	3.96	15.70	
26	"	$\frac{3}{8}$	2.335	0.227	2.108	0.974	4.86	23.65	
27	"	$\frac{3}{8}$	2.344	0.224	2.120	0.970	4.83	23.38	
28	"	$\frac{3}{8}$	3.247	0.320	2.927	1.160	5.78	33.40	
29	"	$\frac{3}{8}$	3.288	0.330	2.958	1.173	5.85	34.25	
30	"	$\frac{3}{8}$	4.501	0.428	4.073	1.340	6.67	44.50	
31	"	$\frac{3}{8}$	4.468	0.445	4.023	1.366	6.80	46.40	
32	Aug. 5	$\frac{1}{2}$	4.211	0.853	3.358	1.890	9.43	88.90	2.51
33	"	$\frac{1}{2}$	4.184	0.836	3.348	1.871	9.34	87.20	
34	"	$\frac{1}{2}$	3.791	0.712	3.079	1.728	8.61	74.20	
35	"	$\frac{1}{2}$	3.753	0.767	2.986	1.794	8.95	80.00	
36	Aug. 6	$\frac{1}{2}$	2.780	0.556	2.224	1.525	7.60	57.80	
37	"	$\frac{1}{2}$	2.765	0.553	2.212	1.520	7.58	57.50	
38	"	$\frac{1}{2}$	2.140	0.423	1.717	1.332	6.64	44.00	
39	"	$\frac{1}{2}$	2.145	0.432	1.713	1.350	6.72	45.10	
40	"	$\frac{1}{2}$	1.424	0.285	1.139	1.090	5.43	29.48	
41	"	$\frac{1}{2}$	1.445	0.286	1.159	1.093	5.45	29.70	
42	"	$\frac{1}{2}$	0.927	0.177	0.750	0.861	4.29	18.42	
43	"	$\frac{1}{2}$	0.917	0.177	0.740	0.860	4.28	18.40	
44	"	$\frac{1}{2}$	0.352	0.070	0.282	0.540	2.69	7.25	
45	Aug. 7	$\frac{3}{4}$	2.100	1.157	0.943	2.200	10.99	120.70	549
46	"	$\frac{3}{4}$	2.098	1.278	0.920	2.210	11.02	121.90	
47	"	$\frac{3}{4}$	1.786	1.000	0.786	2.040	10.19	103.80	
48	"	$\frac{3}{4}$	1.826	1.020	0.806	2.060	10.30	106.00	
49	"	$\frac{3}{4}$	1.105	0.693	0.512	1.580	7.86	61.80	
50	"	$\frac{3}{4}$	1.097	0.602	0.495	1.590	7.92	62.80	
51	"	$\frac{3}{4}$	0.775	0.437	0.338	1.356	6.75	45.50	
52	"	$\frac{3}{4}$	0.790	0.441	0.349	1.363	6.78	46.00	
53	"	$\frac{3}{4}$	0.535	0.287	0.248	1.100	5.46	29.90	
54	"	$\frac{3}{4}$	0.530	0.286	0.244	1.093	5.45	29.70	
55	"	$\frac{3}{4}$	0.368	0.203	0.165	0.922	4.60	21.19	
56	"	$\frac{3}{4}$	0.167	0.073	0.094	0.550	2.74	7.54	
57	"	$\frac{3}{4}$	0.179	0.076	0.103	0.558	2.78	7.74	
58	"	$\frac{3}{4}$	1.448	0.808	0.640	1.840	9.18	84.40	

VALVE NO. 1

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1918		feet	feet	feet				
1	July 10	$\frac{1}{8}$	0.621	0.0080	0.612	0.160	0.77	0.60	76.5
2	"	$\frac{1}{8}$	1.185	0.0139	1.1711	0.210	1.02	1.04	
3	"	$\frac{1}{8}$	1.831	0.0201	1.8109	0.255	1.23	1.53	
4	"	$\frac{1}{8}$	2.439	0.0262	2.4128	0.292	1.41	2.00	
5	"	$\frac{1}{8}$	2.987	0.0313	2.9557	0.321	1.55	2.42	
6	"	$\frac{1}{8}$	3.554	0.0367	3.5173	0.355	1.72	2.96	
7	"	$\frac{1}{8}$	4.244	0.0442	4.1998	0.388	1.88	3.54	
8	"	$\frac{1}{8}$	4.915	0.0502	4.8648	0.415	2.01	4.04	
9	July 11	$\frac{1}{4}$	0.592	0.0292	0.5628	0.310	1.50	2.26	16.26
10	"	$\frac{1}{4}$	1.182	0.0558	1.1262	0.437	2.14	4.49	
11	"	$\frac{1}{4}$	1.705	0.0802	1.6248	0.533	2.58	6.69	
12	"	$\frac{1}{4}$	2.393	0.1064	2.2866	0.621	3.00	9.05	
13	"	$\frac{1}{4}$	3.019	0.1303	2.8887	0.696	3.37	11.3	
14	"	$\frac{1}{4}$	3.563	0.1518	3.4112	0.758	3.67	13.45	
15	"	$\frac{1}{4}$	4.207	0.1762	4.0308	0.822	3.98	15.82	
16	"	$\frac{1}{4}$	4.746	0.1976	4.5484	0.874	4.23	17.90	
17	July 12	$\frac{3}{8}$	0.606	0.0703	0.5357	0.497	2.41	5.79	6.12
18	"	$\frac{3}{8}$	1.212	0.1338	1.0782	0.707	3.42	11.70	
19	"	$\frac{3}{8}$	1.789	0.1903	1.5987	0.856	4.14	17.20	
20	"	$\frac{3}{8}$	2.369	0.2568	2.1122	0.986	4.77	22.80	
21	"	$\frac{3}{8}$	3.010	0.3090	2.701	1.099	5.33	28.40	
22	"	$\frac{3}{8}$	3.578	0.3750	3.203	1.206	5.84	34.30	
23	"	$\frac{3}{8}$	4.105	0.4250	3.680	1.292	6.27	39.30	
24	"	$\frac{3}{8}$	4.764	0.4920	4.272	1.389	6.76	45.80	
25	July 13	$\frac{1}{2}$	0.601	0.1335	0.4675	0.708	3.43	11.79	2.63
26	"	$\frac{1}{2}$	1.190	0.2588	0.9312	0.990	4.79	23.00	
27	"	$\frac{1}{2}$	1.778	0.3750	1.403	1.210	5.86	34.40	
28	"	$\frac{1}{2}$	2.377	0.5000	1.877	1.402	6.80	46.40	
29	"	$\frac{1}{2}$	2.987	0.6100	2.377	1.567	7.59	57.80	
30	"	$\frac{1}{2}$	3.582	0.7300	2.852	1.720	8.34	69.80	
31	"	$\frac{1}{2}$	4.192	0.8520	3.340	1.865	9.05	82.00	
32	"	$\frac{1}{2}$	4.722	0.9500	3.772	1.970	9.55	91.50	
33	July 16	$\frac{3}{4}$	0.395	0.2304	0.1646	0.943	4.56	20.80	5175
34	"	$\frac{3}{4}$	0.807	0.4700	0.337	1.365	6.62	43.80	
35	"	$\frac{3}{4}$	1.192	0.6780	0.514	1.655	8.03	64.50	
36	"	$\frac{3}{4}$	1.596	0.9200	0.676	1.939	9.40	88.30	
37	"	$\frac{3}{4}$	1.952	1.0920	0.860	2.122	10.30	106.00	
38	"	$\frac{3}{4}$	2.382	1.3260	1.056	2.353	11.39	129.00	
39	"	$\frac{3}{4}$	2.756	1.5270	1.229	2.522	12.42	149.00	
40	"	$\frac{3}{4}$	0.173	0.1032	0.0698	0.610	2.95	8.75	
41	July 18	1	0.201	0.1819	0.0191	0.832	4.03	16.20	0735
42	"	1	0.404	0.3700	0.034	1.203	5.84	34.20	
43	"	1	0.625	0.5760	0.049	1.515	7.35	54.20	
44	"	1	0.815	0.7490	0.066	1.740	8.44	71.20	
45	"	1	1.006	0.9100	0.096	1.929	9.34	87.50	
46	"	1	1.217	1.0920	0.125	2.123	10.29	105.90	
47	"	1	1.422	1.2700	0.152	2.295	11.15	123.50	
48	"	1	1.690	1.5010	0.189	2.500	12.12	147.00	

VALVE NO. 2

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1918		feet	feet	feet				
1	July 23	$\frac{1}{8}$	0.650	0.0095	0.6405	0.170	0.82	0.68	70.7
2	"	$\frac{1}{8}$	1.204	0.0195	1.1845	0.220	1.07	1.14	
3	"	$\frac{1}{8}$	1.825	0.0230	1.8020	0.265	1.28	1.65	
4	"	$\frac{1}{8}$	2.426	0.0291	2.3969	0.303	1.47	2.16	
5	"	$\frac{1}{8}$	3.087	0.0358	3.0512	0.340	1.65	2.72	
6	"	$\frac{1}{8}$	3.666	0.0421	3.6239	0.373	1.81	3.27	
7	"	$\frac{1}{8}$	4.335	0.0496	4.2854	0.408	1.98	3.92	
8	"	$\frac{1}{8}$	4.944	0.0552	4.8888	0.432	2.09	4.39	
9	July 24	$\frac{1}{4}$	0.582	0.0340	0.5480	0.331	1.60	2.57	14.05
10	"	$\frac{1}{4}$	1.112	0.0608	1.0512	0.456	2.21	4.89	
11	"	$\frac{1}{4}$	1.861	0.0977	1.7633	0.585	2.84	8.04	
12	"	$\frac{1}{4}$	2.471	0.1235	2.3475	0.665	3.22	10.40	
13	"	$\frac{1}{4}$	3.077	0.1570	2.92	0.758	3.67	13.52	
14	"	$\frac{1}{4}$	3.600	0.1819	3.4181	0.820	3.97	15.81	
15	"	$\frac{1}{4}$	4.207	0.2073	3.9997	0.888	4.26	18.20	
16	"	$\frac{1}{4}$	4.933	0.3439	4.6891	0.959	4.65	21.60	
17	July 25	$\frac{3}{8}$	0.421	0.0580	0.3630	0.444	2.15	4.64	5.12
18	"	$\frac{3}{8}$	0.928	0.1216	0.8064	0.660	3.20	10.22	
19	"	$\frac{3}{8}$	1.662	0.2088	1.4532	0.883	4.28	18.32	
20	"	$\frac{3}{8}$	2.273	0.2859	1.9871	1.036	5.05	25.50	
21	"	$\frac{3}{8}$	2.884	0.3500	2.534	1.170	5.67	32.20	
22	"	$\frac{3}{8}$	3.478	0.4190	3.059	1.277	6.19	38.40	
23	"	$\frac{3}{8}$	4.096	0.4810	3.615	1.380	6.69	44.80	
24	"	$\frac{3}{8}$	4.861	0.5700	4.291	1.506	7.30	53.40	
25	July 26	$\frac{1}{2}$	0.366	0.0948	0.2712	0.576	2.79	7.80	2.36
26	"	$\frac{1}{2}$	0.783	0.1912	0.5918	0.843	4.08	16.70	
27	"	$\frac{1}{2}$	1.341	0.3200	1.021	1.110	5.38	29.00	
28	"	$\frac{1}{2}$	1.822	0.4250	1.397	1.290	6.25	39.20	
29	"	$\frac{1}{2}$	2.391	0.5500	1.841	1.479	7.17	51.50	
30	"	$\frac{1}{2}$	3.077	0.6800	2.397	1.662	8.06	65.00	
31	"	$\frac{1}{2}$	3.585	0.7940	2.791	1.796	8.70	76.00	
32	"	$\frac{1}{2}$	4.244	0.9250	3.319	1.944	9.44	89.10	
33	July 27	$\frac{3}{4}$	0.146	0.0921	0.0539	0.567	2.75	7.57	.464
34	"	$\frac{3}{4}$	0.417	0.2558	0.1612	0.982	4.76	22.70	
35	"	$\frac{3}{4}$	0.755	0.4550	0.300	1.338	6.49	42.20	
36	"	$\frac{3}{4}$	1.087	0.6690	0.418	1.664	8.07	65.20	
37	"	$\frac{3}{4}$	1.492	0.8750	0.617	1.888	9.16	84.00	
38	"	$\frac{3}{4}$	1.794	1.0550	0.739	2.081	10.10	102.00	
39	"	$\frac{3}{4}$	2.071	1.2200	0.851	2.245	10.90	118.50	
40	"	$\frac{3}{4}$	2.328	1.3650	0.963	2.375	11.50	132.50	
41	"	$\frac{3}{4}$	2.565	1.4830	1.082	2.480	12.02	144.20	
42	July 30	1	0.110	0.1072	0.0028	0.597	2.89	8.38	.0914
43	"	1	0.301	0.2730	0.028	1.016	4.93	24.30	
44	"	1	0.497	0.4400	0.057	1.315	6.38	40.80	
45	"	1	0.706	0.6250	0.081	1.585	7.69	59.20	
46	"	1	0.902	0.7960	0.106	1.800	8.73	76.30	
47	"	1	1.104	0.9770	0.127	1.999	9.70	94.20	
48	"	1	1.283	1.1230	0.160	2.151	10.42	108.80	
49	"	1	1.475	1.2900	0.185	2.310	11.20	125.50	
50	"	1	1.701	1.4800	0.221	2.475	11.99	144.00	

VALVE NO. 3

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1918		feet	feet	feet				
1	July 31	$\frac{1}{8}$	0.619	0.0094	0.6096	0.170	0.82	0.68	67.7
2	"	$\frac{1}{8}$	1.258	0.0172	1.2408	0.228	1.10	1.22	
3	"	$\frac{1}{8}$	1.842	0.0243	1.8177	0.272	1.32	1.74	
4	"	$\frac{1}{8}$	2.463	0.0312	2.4318	0.314	1.52	2.32	
5	"	$\frac{1}{8}$	3.046	0.0369	3.0091	0.347	1.68	2.83	
6	"	$\frac{1}{8}$	3.672	0.0439	3.6281	0.381	1.85	3.41	
7	"	$\frac{1}{8}$	4.244	0.5000	4.1940	0.410	1.99	3.95	
8	"	$\frac{1}{8}$	4.838	0.564	4.8816	0.437	2.12	4.50	
9	Aug. 1	$\frac{1}{4}$	0.577	0.344	0.5426	0.333	1.61	2.61	13.8
10	"	$\frac{1}{4}$	1.204	0.662	1.1378	0.477	2.81	5.36	
11	"	$\frac{1}{4}$	1.828	0.980	1.7300	0.586	2.84	8.08	
12	"	$\frac{1}{4}$	2.407	0.1268	2.2802	0.675	3.27	10.71	
13	"	$\frac{1}{4}$	3.001	0.1548	2.8462	0.753	3.65	13.32	
14	"	$\frac{1}{4}$	3.614	0.1839	3.4301	0.825	4.00	16.00	
15	"	$\frac{1}{4}$	4.208	0.2122	3.9958	0.891	4.32	18.66	
16	"	$\frac{1}{4}$	4.862	0.2443	4.6177	0.959	4.65	21.65	
17	Aug. 1	$\frac{3}{8}$	0.589	0.0751	0.5139	0.511	2.48	6.13	5.28
18	"	$\frac{3}{8}$	1.188	0.1488	1.0392	0.736	3.57	12.75	
19	"	$\frac{3}{8}$	1.807	0.2218	1.5852	0.911	4.41	19.52	
20	"	$\frac{3}{8}$	2.384	0.2890	2.095	1.048	5.08	25.80	
21	"	$\frac{3}{8}$	2.998	0.3510	2.647	1.170	5.67	32.20	
22	"	$\frac{3}{8}$	3.593	0.4200	3.173	1.280	6.21	38.60	
23	"	$\frac{3}{8}$	4.148	0.4800	3.668	1.376	6.67	44.60	
24	"	$\frac{3}{8}$	4.756	0.5440	4.212	1.470	7.13	51.00	
25	Aug. 2	$\frac{1}{2}$	0.378	0.0960	0.2820	0.580	2.81	7.90	2.39
26	"	$\frac{1}{2}$	0.941	0.2210	0.7200	0.910	4.41	19.48	
27	"	$\frac{1}{2}$	1.491	0.3440	1.147	1.151	5.58	31.20	
28	"	$\frac{1}{2}$	1.954	0.4450	1.509	1.326	6.43	41.40	
29	"	$\frac{1}{2}$	2.529	0.5650	1.964	1.496	7.25	52.80	
30	"	$\frac{1}{2}$	3.067	0.6720	2.395	1.651	8.01	64.20	
31	"	$\frac{1}{2}$	3.700	0.8100	2.890	1.818	8.81	77.80	
32	"	$\frac{1}{2}$	4.131	0.9000	3.231	1.920	9.30	86.90	
33	"	$\frac{1}{2}$	4.713	1.0250	3.688	2.050	9.94	98.80	
34	Aug. 3	$\frac{3}{4}$	0.211	0.1323	0.0787	0.691	3.35	11.24	0.493
35	"	$\frac{3}{4}$	0.531	0.3200	0.211	1.105	5.36	28.80	
36	"	$\frac{3}{4}$	0.858	0.5000	0.358	1.410	6.84	46.80	
37	"	$\frac{3}{4}$	1.161	0.6740	0.487	1.650	8.00	64.00	
38	"	$\frac{3}{4}$	1.528	0.8900	0.638	1.906	9.24	85.70	
39	"	$\frac{3}{4}$	1.831	1.0600	0.771	2.087	10.13	102.60	
40	"	$\frac{3}{4}$	2.106	1.2100	0.896	2.235	10.82	117.50	
41	"	$\frac{3}{4}$	2.401	1.3620	1.039	2.375	11.50	132.50	
42	"	$\frac{3}{4}$	2.602	1.4750	1.127	2.475	11.99	143.80	
43	Aug. 5	1	0.140	0.1259	0.0141	0.672	3.25	10.61	.0966
44	"	1	0.358	0.3220	0.036	1.111	5.39	29.1C	
45	"	1	0.572	0.5100	0.062	1.420	6.89	47.50	
46	"	1	0.748	0.6580	0.090	1.630	7.90	62.60	
47	"	1	0.969	0.8510	0.118	1.866	9.04	81.70	
48	"	1	1.188	1.0500	0.138	2.080	10.09	101.80	
49	"	1	1.374	1.1950	0.179	2.221	10.77	116.00	
50	"	1	1.545	1.3350	0.210	2.355	11.40	130.00	
51	"	1	1.755	1.5280	0.235	2.510	12.17	148.00	

VALVE NO. 4

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1918		feet	feet	feet				
1	Aug. 6	$\frac{1}{8}$	0.657	0.0086	0.6484	0.161	0.78	0.61	79.9
2	"	$\frac{1}{8}$	1.241	0.0144	1.2266	0.209	1.01	1.03	
3	"	$\frac{1}{8}$	1.804	0.0198	1.784	0.246	1.19	1.42	
4	"	$\frac{1}{8}$	2.431	0.0260	2.405	0.284	1.38	1.89	
5	"	$\frac{1}{8}$	3.061	0.0318	3.029	0.318	1.54	2.38	
6	"	$\frac{1}{8}$	3.726	0.0370	3.689	0.348	1.69	2.85	
7	"	$\frac{1}{8}$	4.277	0.0472	4.230	0.373	1.81	3.72	
8	"	$\frac{1}{8}$	4.928	0.0476	4.880	0.399	1.93	3.74	
9	Aug. 7	$\frac{1}{4}$	0.607	0.0338	0.573	0.329	1.59	2.54	14.92
10	"	$\frac{1}{4}$	1.260	0.0524	1.208	0.469	2.27	5.18	
11	"	$\frac{1}{4}$	1.806	0.0913	1.715	0.565	2.74	7.50	
12	"	$\frac{1}{4}$	2.404	0.1184	2.286	0.650	3.15	9.94	
13	"	$\frac{1}{4}$	3.028	0.1451	2.883	0.727	3.52	12.41	
14	"	$\frac{1}{4}$	3.630	0.1714	3.459	0.795	3.85	14.85	
15	"	$\frac{1}{4}$	4.177	0.1895	3.988	0.853	4.13	17.10	
16	"	$\frac{1}{4}$	4.805	0.2240	4.581	0.916	4.44	19.75	
17	Aug. 7	$\frac{3}{8}$	0.548	0.0738	0.474	0.505	2.45	6.00	5.35
18	"	$\frac{3}{8}$	1.208	0.1488	1.059	0.737	3.57	12.76	
19	"	$\frac{3}{8}$	1.780	0.2161	1.564	0.900	4.36	19.00	
20	"	$\frac{3}{8}$	2.420	0.2890	2.131	1.048	5.08	25.80	
21	"	$\frac{3}{8}$	3.010	0.3500	2.660	1.164	5.65	32.00	
22	"	$\frac{3}{8}$	3.668	0.4200	3.248	1.287	6.24	39.00	
23	"	$\frac{3}{8}$	4.195	0.4820	3.713	1.378	6.68	44.60	
24	"	$\frac{3}{8}$	4.891	0.5550	4.336	1.483	7.20	52.00	
25	Aug. 8	$\frac{1}{2}$	0.520	0.1312	0.389	0.685	3.32	11.10	2.42
26	"	$\frac{1}{2}$	1.143	0.2730	0.870	1.015	4.93	24.30	
27	"	$\frac{1}{2}$	1.801	0.4100	1.391	1.269	6.15	37.90	
28	"	$\frac{1}{2}$	2.358	0.5320	1.826	1.455	7.06	49.90	
29	"	$\frac{1}{2}$	2.935	0.6530	2.282	1.620	7.86	61.90	
30	"	$\frac{1}{2}$	3.604	0.7920	2.812	1.794	8.71	76.00	
31	"	$\frac{1}{2}$	4.109	0.8900	3.219	1.910	9.25	85.60	
32	"	$\frac{1}{2}$	4.699	1.0200	3.679	2.042	9.92	98.80	
33	Aug. 9	$\frac{3}{4}$	0.240	0.1472	0.093	0.732	3.55	12.60	0.472
34	"	$\frac{3}{4}$	0.568	0.3350	0.233	1.141	5.54	30.70	
35	"	$\frac{3}{4}$	0.911	0.5300	0.381	1.452	7.05	49.70	
36	"	$\frac{3}{4}$	1.196	0.8000	0.396	1.666	8.08	65.50	
37	"	$\frac{3}{4}$	1.495	0.9990	0.496	1.869	9.06	82.20	
38	"	$\frac{3}{4}$	1.711	1.0200	0.691	2.041	9.90	98.40	
39	"	$\frac{3}{4}$	2.107	1.1950	0.912	2.222	10.78	116.00	
40	"	$\frac{3}{4}$	2.395	1.3570	1.038	2.370	11.49	132.00	
41	"	$\frac{3}{4}$	2.696	1.5350	1.161	2.527	12.26	150.00	0.083
42	Aug. 9	1	0.153	0.1406	0.012	0.714	3.46	12.00	
43	"	1	0.336	0.3070	0.029	1.081	5.25	27.50	
44	"	1	0.539	0.4850	0.054	1.380	6.70	45.00	
45	"	1	0.727	0.6550	0.072	1.620	7.86	62.00	
46	"	1	0.903	0.8100	0.093	1.813	8.80	77.50	
47	"	1	1.094	0.9760	0.118	1.999	9.70	94.10	
48	"	1	1.258	1.1150	0.143	2.146	10.40	108.00	
49	"	1	1.437	1.2700	0.167	2.292	11.10	123.50	
50	"	1	1.645	1.4450	0.200	2.448	11.87	140.80	

VALVE NO. 5

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1918		feet	feet	feet				
1	Oct. 14	$\frac{1}{8}$	0.592	0.0050	0.587	0.123	0.60	0.36	146.2
2	"	$\frac{1}{8}$	1.249	0.0084	1.2406	0.163	0.79	0.63	
3	"	$\frac{1}{8}$	1.829	0.0119	1.8171	0.191	0.93	0.86	
4	"	$\frac{1}{8}$	2.457	0.0151	2.4419	0.218	1.05	1.11	
5	"	$\frac{1}{8}$	3.032	0.0180	3.0140	0.235	1.14	1.29	
6	"	$\frac{1}{8}$	3.611	0.0210	3.5900	0.255	1.23	1.52	
7	"	$\frac{1}{8}$	4.280	0.0241	4.2559	0.275	1.33	1.78	
8	"	$\frac{1}{8}$	5.027	0.0280	4.9990	0.298	1.44	2.08	
9	Oct. 16	$\frac{1}{4}$	0.626	0.0228	0.6032	0.265	1.28	1.65	23.78
10	"	$\frac{1}{4}$	1.227	0.0417	1.1853	0.372	1.80	3.25	
11	"	$\frac{1}{4}$	1.812	0.0590	1.7530	0.450	2.18	4.75	
12	"	$\frac{1}{4}$	2.426	0.0781	2.3479	0.521	2.52	6.39	
13	"	$\frac{1}{4}$	3.019	0.0952	2.9238	0.578	2.80	7.85	
14	"	$\frac{1}{4}$	3.626	0.1122	3.5138	0.631	3.05	9.35	
15	"	$\frac{1}{4}$	4.146	0.1282	4.0178	0.680	3.29	10.85	
16	"	$\frac{1}{4}$	4.970	0.1540	4.8160	0.750	3.63	13.23	
17	Oct. 17	$\frac{3}{8}$	0.562	0.0555	0.5065	0.435	2.12	4.45	7.55
18	"	$\frac{3}{8}$	1.112	0.1062	1.0052	0.613	2.96	8.80	
19	"	$\frac{3}{8}$	1.777	0.1640	1.6130	0.775	3.76	14.18	
20	"	$\frac{3}{8}$	2.416	0.2181	2.1979	0.903	4.37	19.20	
21	"	$\frac{3}{8}$	3.051	0.2666	2.7844	1.006	4.87	23.70	
22	"	$\frac{3}{8}$	3.652	0.3170	3.3350	1.099	5.33	28.40	
23	"	$\frac{3}{8}$	4.280	0.3600	3.9200	1.187	5.75	33.20	
24	"	$\frac{3}{8}$	4.884	0.4050	4.4790	1.263	6.13	37.50	
25	Oct. 18	$\frac{1}{2}$	0.549	0.1040	0.4450	0.602	2.92	8.60	3.45
26	"	$\frac{1}{2}$	1.168	0.2073	0.9607	0.876	4.24	18.20	
27	"	$\frac{1}{2}$	1.834	0.3149	1.5191	1.095	5.31	28.20	
28	"	$\frac{1}{2}$	2.456	0.4100	2.046	1.272	6.17	38.10	
29	"	$\frac{1}{2}$	3.004	0.4980	2.506	1.401	6.80	46.40	
30	"	$\frac{1}{2}$	3.620	0.5900	3.030	1.540	7.47	55.90	
31	"	$\frac{1}{2}$	4.232	0.6900	3.542	1.672	8.10	65.80	
32	"	$\frac{1}{2}$	5.092	0.8330	4.259	1.845	8.94	80.01	
33	Oct. 21	$\frac{3}{4}$	0.270	0.1461	0.1239	0.730	3.54	12.50	.644
34	"	$\frac{3}{4}$	0.660	0.3450	0.315	1.160	5.63	31.70	
35	"	$\frac{3}{4}$	1.006	0.5200	0.486	1.435	6.95	48.50	
36	"	$\frac{3}{4}$	1.341	0.6820	0.659	1.665	8.07	65.20	
37	"	$\frac{3}{4}$	1.655	0.8490	0.806	1.860	9.02	81.20	
38	"	$\frac{3}{4}$	1.937	0.9820	0.955	2.030	9.84	96.90	
39	"	$\frac{3}{4}$	2.296	1.1750	1.121	2.201	10.68	114.00	
40	"	$\frac{3}{4}$	2.632	1.3310	1.301	2.350	11.38	129.70	
41	"	$\frac{3}{4}$	2.916	1.4660	1.450	2.463	11.95	143.00	
42	Nov. 6	1	0.153	0.1271	0.0259	0.676	3.27	10.75	0.184
43	"	1	0.409	0.3250	0.084	1.125	5.45	29.80	
44	"	1	0.601	0.4790	0.122	1.370	6.64	44.20	
45	"	1	0.792	0.6300	0.162	1.590	7.72	59.60	
46	"	1	1.008	0.8000	0.208	1.802	8.74	76.50	
47	"	1	1.226	0.9600	0.266	1.982	9.60	92.50	
48	"	1	1.418	1.1180	0.300	2.142	10.38	107.90	
49	"	1	1.625	1.2680	0.357	2.289	11.11	123.00	
50	"	1	1.920	1.4800	0.440	2.480	12.00	144.00	

VALVE NO. 6

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1919		feet	feet	feet				
1	Jan. 2	$\frac{1}{8}$	0.576	0.0089	0.567	0.163	0.79	0.62	70.3
2	"	$\frac{1}{8}$	1.190	0.0142	1.176	0.218	1.06	1.12	
3	"	$\frac{1}{8}$	1.892	0.0235	1.869	0.269	1.30	1.69	
4	"	$\frac{1}{8}$	2.422	0.0299	2.392	0.303	1.47	2.15	
5	"	$\frac{1}{8}$	3.079	0.0378	3.041	0.343	1.66	2.76	
6	"	$\frac{1}{8}$	3.716	0.0446	3.671	0.375	1.81	3.30	
7	"	$\frac{1}{8}$	4.130	0.0497	4.080	0.396	1.92	3.68	
8	"	$\frac{1}{8}$	5.114	0.0603	5.054	0.439	2.12	4.52	
9	"	$\frac{1}{8}$	0.613	0.0090	0.604	0.168	0.81	0.66	
10	"	$\frac{1}{8}$	1.262	0.0142	1.248	0.225	1.09	1.19	15.58
11	"	$\frac{1}{8}$	3.075	0.0380	3.037	0.342	1.67	2.77	
12	Jan. 3	$\frac{1}{4}$	0.599	0.0323	0.567	0.318	1.54	2.37	
13	"	$\frac{1}{4}$	1.188	0.0623	1.126	0.446	2.16	4.66	
14	"	$\frac{1}{4}$	1.848	0.0937	1.754	0.556	2.69	7.25	
15	"	$\frac{1}{4}$	2.446	0.1272	2.319	0.640	3.10	9.62	
16	"	$\frac{1}{4}$	3.120	0.1593	2.961	0.725	3.50	12.30	
17	"	$\frac{1}{4}$	3.863	0.1941	3.669	0.807	3.91	15.25	
18	"	$\frac{1}{4}$	4.396	0.2100	4.186	0.862	4.17	17.40	
19	"	$\frac{1}{4}$	5.016	0.2364	4.779	0.916	4.44	19.70	5.83
20	Jan. 31	$\frac{3}{8}$	0.554	0.0732	0.481	0.486	2.35	5.52	
21	"	$\frac{3}{8}$	1.245	0.1580	1.087	0.720	3.48	12.19	
22	"	$\frac{3}{8}$	1.949	0.2284	1.721	0.900	4.36	19.00	
23	"	$\frac{3}{8}$	2.535	0.2900	2.245	1.020	4.94	24.50	
24	"	$\frac{3}{8}$	3.025	0.3500	2.675	1.122	5.44	29.70	
25	"	$\frac{3}{8}$	3.778	0.4200	3.358	1.245	6.04	36.50	
26	"	$\frac{3}{8}$	4.225	0.4650	3.760	1.317	6.39	40.90	
27	"	$\frac{3}{8}$	4.939	0.5350	4.404	1.428	6.92	47.90	
28	Feb. 3	$\frac{1}{2}$	0.531	0.1380	0.393	0.670	3.24	10.50	2.543
29	"	$\frac{1}{2}$	1.167	0.2720	0.895	0.986	4.77	22.80	
30	"	$\frac{1}{2}$	1.771	0.4080	1.363	1.223	5.94	35.30	
31	"	$\frac{1}{2}$	2.539	0.5540	1.985	1.450	7.03	49.50	
32	"	$\frac{1}{2}$	3.103	0.6750	2.428	1.615	7.84	61.30	
33	"	$\frac{1}{2}$	3.661	0.7850	2.876	1.751	8.50	72.20	
34	"	$\frac{1}{2}$	4.272	0.9080	3.364	1.890	9.16	84.10	
35	"	$\frac{1}{2}$	4.946	1.0660	3.880	2.050	9.94	99.20	
36	Feb. 4	$\frac{3}{4}$	0.289	0.1892	0.100	0.795	3.85	14.82	0.506
37	"	$\frac{3}{4}$	0.596	0.3580	0.238	1.143	5.54	30.80	
38	"	$\frac{3}{4}$	0.905	0.5310	0.374	1.421	6.89	47.60	
39	"	$\frac{3}{4}$	1.226	0.7230	0.503	1.675	8.13	66.00	
40	"	$\frac{3}{4}$	1.531	0.9000	0.631	1.880	9.12	83.10	
41	"	$\frac{3}{4}$	1.841	1.0660	0.775	2.055	9.95	99.20	
42	"	$\frac{3}{4}$	2.137	1.2200	0.917	2.205	10.69	114.00	
43	"	$\frac{3}{4}$	2.384	1.3510	1.033	2.325	11.48	127.00	
44	"	$\frac{3}{4}$	2.722	1.5500	1.172	2.495	12.09	146.00	.1214
45	Feb. 10	1	0.178	0.1692	0.0088	0.749	3.62	13.15	
46	"	1	0.380	0.3450	0.035	1.117	5.42	29.40	
47	"	1	0.621	0.5380	0.083	1.435	6.9	48.50	
48	"	1	0.847	0.7350	0.112	1.690	8.19	67.10	
49	"	1	1.015	0.8700	0.145	1.848	8.96	80.40	
50	"	1	1.210	1.0200	0.190	2.015	9.76	95.50	
51	"	1	1.383	1.1650	0.218	2.160	10.48	109.00	
52	"	1	1.573	1.3300	0.243	2.305	11.90	125.00	
53	"	1	1.743	1.4490	0.294	2.410	11.69	136.20	

VALVE NO. 7

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1919		feet	feet	feet				
1	Feb. 18	1/8	0.611	0.0079	0.603	0.155	0.75	0.57	87.1
2	"	1/8	1.213	0.0132	1.200	0.201	0.97	0.95	
3	"	1/8	1.789	0.0184	1.771	0.238	1.15	1.33	
4	"	1/8	2.421	0.0183	2.403	0.273	1.32	1.75	
5	"	1/8	3.082	0.0312	3.051	0.310	1.51	2.26	
6	"	1/8	3.658	0.0380	3.620	0.334	1.61	2.62	
7	"	1/8	4.478	0.0439	4.434	0.370	1.79	3.22	
8	"	1/8	5.178	0.0502	5.128	0.398	1.93	3.72	
9	Feb. 21	1/4	0.596	0.0321	0.564	0.313	1.51	2.31	16.34
10	"	1/4	1.191	0.0585	1.133	0.431	2.09	4.36	
11	"	1/4	1.804	0.0871	1.717	0.535	2.59	6.72	
12	"	1/4	2.444	0.1172	2.327	0.628	3.04	9.28	
13	"	1/4	2.989	0.1391	2.850	0.690	3.34	11.20	
14	"	1/4	3.622	0.1666	3.455	0.760	3.68	13.60	
15	"	1/4	4.190	0.1929	3.997	0.823	3.99	15.90	
16	"	1/4	4.975	0.2224	4.753	0.888	4.31	18.49	
17	Mar. 6	3/8	0.581	0.0729	0.508	0.483	2.34	5.49	5.93
18	"	3/8	1.207	0.1435	1.063	0.701	3.40	11.58	
19	"	3/8	1.870	0.2249	1.645	0.883	4.28	18.40	
20	"	3/8	2.454	0.2810	2.173	1.004	4.86	23.70	
21	"	3/8	2.990	0.3055	2.685	1.100	5.33	28.40	
22	"	3/8	3.608	0.4000	3.208	1.218	5.90	34.90	
23	"	3/8	4.270	0.4650	3.805	1.320	6.40	41.00	
24	"	3/8	4.962	0.5400	4.422	1.430	6.94	48.20	
25	Mar. 11	1/2	0.399	0.1001	0.299	0.577	2.79	7.82	2.48
26	"	1/2	1.098	0.2606	0.837	0.965	4.67	21.80	
27	"	1/2	1.777	0.4008	1.376	1.230	5.96	35.60	
28	"	1/2	2.442	0.5500	1.892	1.445	7.03	49.40	
29	"	1/2	3.000	0.6760	2.324	1.602	7.77	61.50	
30	"	1/2	3.559	0.7840	2.775	1.749	8.48	72.00	
31	"	1/2	4.183	0.9080	3.275	1.890	9.15	84.00	
32	"	1/2	4.835	1.0550	3.780	2.039	9.87	98.00	
33	"	3/4	0.260	0.1525	0.108	0.725	3.51	12.38	0.571
34	"	3/4	0.574	0.3294	0.245	1.090	5.28	28.00	
35	"	3/4	0.903	0.5000	0.403	1.375	6.67	44.4	
36	"	3/4	1.220	0.6720	0.548	1.608	7.80	61.00	
37	"	3/4	1.518	0.8350	0.683	1.811	8.78	77.20	
38	"	3/4	1.808	0.9300	0.878	1.979	9.59	92.10	
39	"	3/4	2.094	1.1650	0.929	2.148	10.45	109.00	
40	"	3/4	2.393	1.3100	1.083	2.289	11.20	123.00	
41	"	3/4	2.767	1.5240	1.243	3.470	11.95	143.50	
42	Mar. 12	1	0.179	0.1492	0.0298	0.716	3.47	12.08	0.1516
43	"	1	0.377	0.3239	0.0531	1.077	5.25	27.50	
44	"	1	0.605	0.5000	0.105	1.374	6.67	44.60	
45	"	1	0.810	0.6750	0.135	1.610	7.82	61.20	
46	"	1	1.004	0.8250	0.179	1.796	8.72	76.00	
47	"	1	1.202	0.9950	0.207	1.980	9.60	92.50	
48	"	1	1.419	1.1640	0.255	2.150	10.42	108.80	
49	"	1	1.591	1.2830	0.308	2.277	11.05	120.70	
50	"	1	1.870	1.5250	0.345	2.475	11.95	143.90	

VALVE NO. 8

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1919		feet	feet	feet				
1	Mar. 19	$\frac{1}{8}$	0.609	0.0099	0.599	0.151	0.73	0.54	87.0
2	"	$\frac{1}{8}$	1.218	0.0170	1.201	0.202	0.98	0.96	
3	"	$\frac{1}{8}$	1.751	0.0229	1.728	0.235	1.14	1.30	
4	"	$\frac{1}{8}$	2.434	0.0310	2.403	0.274	1.33	1.76	
5	"	$\frac{1}{8}$	3.020	0.0361	2.983	0.305	1.48	2.19	
6	"	$\frac{1}{8}$	3.659	0.0415	3.618	0.335	1.62	2.64	
7	"	$\frac{1}{8}$	4.295	0.0470	4.248	0.362	1.75	3.08	
8	"	$\frac{1}{8}$	5.144	0.0549	5.089	0.397	1.92	3.70	
9	Mar. 20	$\frac{1}{4}$	0.645	0.0342	0.611	0.296	1.44	2.06	19.47
10	"	$\frac{1}{4}$	1.230	0.0570	1.173	0.407	1.97	3.89	
11	"	$\frac{1}{4}$	1.847	0.0817	1.765	0.500	2.42	5.88	
12	"	$\frac{1}{4}$	2.510	0.1066	2.403	0.580	2.81	7.90	
13	"	$\frac{1}{4}$	3.102	0.1300	2.972	0.647	3.14	9.85	
14	"	$\frac{1}{4}$	3.677	0.1510	3.526	0.703	3.41	11.60	
15	"	$\frac{1}{4}$	4.332	0.1767	4.155	0.765	3.71	13.75	
16	"	$\frac{1}{4}$	5.029	0.2026	4.826	0.824	3.99	15.95	
17	Mar. 20	$\frac{3}{8}$	0.563	0.0660	0.497	0.443	2.15	4.61	6.93
18	"	$\frac{3}{8}$	1.159	0.1270	1.032	0.638	3.09	9.58	
19	"	$\frac{3}{8}$	1.792	0.1912	1.601	0.799	3.87	15.00	
20	"	$\frac{3}{8}$	2.416	0.2531	2.163	0.928	4.50	20.20	
21	"	$\frac{3}{8}$	3.081	0.3180	2.763	1.045	5.06	25.70	
22	"	$\frac{3}{8}$	3.711	0.3820	3.329	1.149	5.57	31.10	
23	"	$\frac{3}{8}$	4.301	0.4350	3.866	1.235	5.99	35.90	
24	"	$\frac{3}{8}$	4.973	0.5000	4.473	1.330	6.45	41.60	
25	Mar. 21	$\frac{1}{2}$	0.487	0.1178	0.369	0.612	2.97	8.80	2.683
26	"	$\frac{1}{2}$	1.088	0.2472	0.841	0.929	4.50	20.30	
27	"	$\frac{1}{2}$	1.786	0.4080	1.378	1.190	5.77	33.30	
28	"	$\frac{1}{2}$	2.462	0.5500	1.912	1.397	6.77	46.00	
29	"	$\frac{1}{2}$	3.059	0.6700	2.389	1.561	7.57	57.40	
30	"	$\frac{1}{2}$	3.643	0.7850	2.858	1.702	8.25	68.20	
31	"	$\frac{1}{2}$	4.192	0.9050	3.287	1.828	8.86	78.70	
32	"	$\frac{1}{2}$	4.869	1.0490	3.820	1.975	9.57	92.00	
33	Mar. 22	$\frac{3}{4}$	0.214	0.1381	0.076	0.660	3.20	10.24	5264
34	"	$\frac{3}{4}$	0.492	0.2980	0.194	1.010	4.89	24.00	
35	"	$\frac{3}{4}$	0.881	0.5320	0.349	1.372	6.65	44.30	
36	"	$\frac{3}{4}$	1.240	0.7300	0.510	1.632	7.91	62.80	
37	"	$\frac{3}{4}$	1.517	0.8820	0.635	1.809	8.76	77.00	
38	"	$\frac{3}{4}$	1.791	1.0420	0.749	1.973	9.56	91.70	
39	"	$\frac{3}{4}$	2.095	1.2150	0.880	2.140	10.38	107.50	
40	"	$\frac{3}{4}$	2.390	1.3800	1.010	2.285	11.08	122.80	
41	"	$\frac{3}{4}$	2.755	1.5810	1.174	2.452	11.88	141.00	
42	Mar. 24	1	0.166	0.1340	0.032	0.657	3.19	10.16	1644
43	"	1	0.350	0.2980	0.052	1.011	4.90	24.00	
44	"	1	0.583	0.4900	0.093	1.312	6.36	40.50	
45	"	1	0.800	0.6600	0.140	1.544	7.49	56.00	
46	"	1	0.985	0.8230	0.162	1.740	8.44	71.30	
47	"	1	1.188	0.9700	0.218	1.900	9.20	85.00	
48	"	1	1.396	1.1350	0.261	2.067	10.01	100.05	
49	"	1	1.582	1.2900	0.292	2.208	10.70	114.60	
50	"	1	1.958	1.5700	0.388	2.440	11.84	140.00	

VALVE NO. 9

Gage Length = 23 feet

Pipe Cross Section = 0.2062 sq. ft.

	1919		feet	feet	feet				
1	April 11	$\frac{1}{8}$	0.667	0.0081	0.659	0.141	0.68	0.47	114.0
2	"	$\frac{1}{8}$	1.243	0.0136	1.230	0.181	0.88	0.77	
3	"	$\frac{1}{8}$	1.726	0.0180	1.708	0.207	1.00	1.01	
4	"	$\frac{1}{8}$	2.474	0.0240	2.450	0.240	1.16	1.36	
5	"	$\frac{1}{8}$	2.920	0.0281	2.892	0.261	1.26	1.60	
6	"	$\frac{1}{8}$	3.600	0.0342	3.566	0.292	1.41	2.01	
7	"	$\frac{1}{8}$	4.098	0.0375	4.061	0.308	1.49	2.23	
8	"	$\frac{1}{8}$	4.968	0.0440	4.922	0.339	1.64	2.70	
9	April 18	$\frac{1}{4}$	0.634	0.0322	0.601	0.282	1.37	1.87	20.85
10	"	$\frac{1}{4}$	1.232	0.0565	1.175	0.393	1.91	3.63	
11	"	$\frac{1}{4}$	1.856	0.0809	1.775	0.480	2.33	5.41	
12	"	$\frac{1}{4}$	2.386	0.1014	2.284	0.546	2.67	7.00	
13	"	$\frac{1}{4}$	3.062	0.1282	2.933	0.620	3.00	9.04	
14	"	$\frac{1}{4}$	3.631	0.1499	3.481	0.676	3.28	10.72	
15	"	$\frac{1}{4}$	4.402	0.1795	4.222	0.745	3.61	13.05	
16	"	$\frac{1}{4}$	5.093	0.2058	4.887	0.803	3.89	15.20	
17	April 19	$\frac{3}{8}$	0.586	0.0695	0.517	0.441	2.14	4.57	7.214
18	"	$\frac{3}{8}$	1.203	0.1342	1.069	0.636	3.08	9.51	
19	"	$\frac{3}{8}$	1.892	0.2058	1.686	0.803	3.89	15.20	
20	"	$\frac{3}{8}$	2.397	0.2563	2.140	0.900	4.36	19.05	
21	"	$\frac{3}{8}$	3.083	0.3290	2.754	1.023	4.97	24.70	
22	"	$\frac{3}{8}$	3.515	0.3680	3.147	1.088	5.27	27.80	
23	"	$\frac{3}{8}$	4.228	0.4440	3.784	1.200	5.82	33.90	
24	"	$\frac{3}{8}$	5.075	0.5300	4.545	1.310	6.35	40.50	
25	April 30	$\frac{1}{2}$	0.717	0.1668	0.550	0.716	3.47	12.05	2.872
26	"	$\frac{1}{2}$	1.173	0.2672	0.905	0.920	4.46	19.90	
27	"	$\frac{1}{2}$	1.816	0.4150	1.401	1.150	5.58	31.20	
28	"	$\frac{1}{2}$	2.512	0.5650	1.947	1.357	6.58	43.40	
29	"	$\frac{1}{2}$	2.912	0.6500	2.262	1.463	7.10	50.40	
30	"	$\frac{1}{2}$	3.002	0.6650	2.337	1.486	7.21	52.00	
31	"	$\frac{1}{2}$	3.737	0.8300	2.907	1.668	8.09	65.50	
32	"	$\frac{1}{2}$	4.310	0.9600	3.350	1.800	8.73	76.30	
33	"	$\frac{1}{2}$	5.144	1.1500	3.994	1.957	9.49	90.20	
34	May 3	$\frac{3}{4}$	0.312	0.1904	0.122	0.770	3.73	13.90	0.509
35	"	$\frac{3}{4}$	0.644	0.4050	0.239	1.136	5.51	30.40	
36	"	$\frac{3}{4}$	0.890	0.5620	0.328	1.350	6.55	43.00	
37	"	$\frac{3}{4}$	1.181	0.7350	0.446	1.560	7.56	57.40	
38	"	$\frac{3}{4}$	1.487	0.9150	0.572	1.752	8.50	72.50	
39	"	$\frac{3}{4}$	1.845	1.1300	0.715	1.968	9.49	90.30	
40	"	$\frac{3}{4}$	2.178	1.3350	0.843	2.135	10.36	107.40	
41	"	$\frac{3}{4}$	2.441	1.4820	0.959	2.255	10.93	119.50	
42	May 8	1	0.382	0.2991	0.0829	1.010	4.90	24.10	1.782
43	"	1	0.792	0.6550	0.137	1.467	7.12	50.80	
44	"	1	1.124	0.9350	0.189	1.774	8.61	74.20	
45	"	1	1.518	1.2400	0.278	2.060	10.00	100.00	
46	"	1	1.558	1.2750	0.283	2.090	10.13	102.50	
47	"	1	1.994	1.6350	0.359	2.374	11.50	132.50	
48	"	1	2.195	1.7850	0.410	2.480	12.02	144.60	

TABLE 29

LOSS OF HEAD IN 8-INCH GATE VALVES

VALVE NO. 1

Gage Length = 22 feet

Pipe Cross Section = 0.358 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	$k \text{ in } \frac{v^2}{H=2g}$
	1919		feet	feet	feet				
1	July 3	$\frac{1}{8}$	5.087	0.046	5.041	0.745	2.08	4.33	75.2
2	"	$\frac{1}{8}$	4.259	0.039	4.220	0.680	1.90	3.60	
3	"	$\frac{1}{8}$	3.929	0.036	3.893	0.653	1.82	3.33	
4	"	$\frac{1}{8}$	3.179	0.029	3.150	0.587	1.64	2.69	
5	"	$\frac{1}{8}$	2.580	0.023	2.557	0.533	1.49	2.22	
6	"	$\frac{1}{8}$	1.980	0.018	1.962	0.468	1.31	1.71	
7	"	$\frac{1}{8}$	1.349	0.012	1.337	0.385	1.08	1.16	
8	"	$\frac{1}{8}$	0.884	0.007	0.877	0.294	0.82	0.67	
9	"	$\frac{1}{4}$	4.908	0.202	4.706	1.624	0.45	20.60	14.73
10	"	$\frac{1}{4}$	4.345	0.180	4.165	1.529	4.27	18.20	
11	"	$\frac{1}{4}$	3.593	0.151	3.442	1.388	3.88	15.00	
12	"	$\frac{1}{4}$	3.011	0.129	2.882	1.272	3.55	12.60	
13	"	$\frac{1}{4}$	2.549	0.110	2.439	1.166	3.25	10.58	
14	"	$\frac{1}{4}$	1.760	0.077	1.683	0.963	2.69	7.25	
15	"	$\frac{1}{4}$	1.223	0.054	1.169	0.801	2.24	5.01	
16	"	$\frac{1}{4}$	0.632	0.028	0.604	0.573	1.60	2.56	
17	"	$\frac{1}{4}$	0.327	0.014	0.313	0.414	1.16	1.34	
18	"	$\frac{3}{8}$	5.048	0.512	4.536	2.683	7.50	56.20	5.18
19	"	$\frac{3}{8}$	4.370	0.449	3.921	2.501	6.98	48.70	
20	"	$\frac{3}{8}$	3.770	0.393	3.377	2.326	6.50	42.20	
21	"	$\frac{3}{8}$	3.191	0.337	2.854	2.137	5.97	35.60	
22	"	$\frac{3}{8}$	2.644	0.281	2.363	1.941	5.42	29.40	
23	"	$\frac{3}{8}$	1.995	0.216	1.779	1.689	4.72	22.25	
24	"	$\frac{3}{8}$	1.406	0.157	1.249	1.417	3.95	15.60	
25	"	$\frac{3}{8}$	0.780	0.090	0.690	1.038	2.90	8.40	
26	"	$\frac{3}{8}$	0.428	0.049	0.379	0.760	2.12	4.51	
27	July 7	$\frac{1}{2}$	4.916	0.950	3.966	3.700	10.32	106.80	2.376
28	"	$\frac{1}{2}$	4.340	0.852	3.488	3.489	9.75	95.00	
29	"	$\frac{1}{2}$	3.797	0.744	3.053	3.249	9.07	82.20	
30	"	$\frac{1}{2}$	3.224	0.640	2.584	3.005	8.39	70.30	
31	"	$\frac{1}{2}$	2.633	0.528	2.105	2.715	7.58	57.40	
32	"	$\frac{1}{2}$	1.937	0.391	1.546	2.326	6.50	42.20	
33	"	$\frac{1}{2}$	1.399	0.287	1.112	1.963	5.48	30.00	
34	"	$\frac{1}{2}$	0.817	0.175	0.642	1.505	4.20	17.62	
35	"	$\frac{1}{2}$	0.411	0.090	0.321	1.036	2.89	8.37	
36	"	$\frac{3}{4}$	5.058	2.781	2.277	6.146	17.15	294.50	0.4725
37	"	$\frac{3}{4}$	4.322	2.442	1.880	5.777	16.14	260.50	
38	"	$\frac{3}{4}$	3.730	2.090	1.640	5.391	15.05	226.00	
39	"	$\frac{3}{4}$	3.106	1.722	1.384	4.926	13.75	189.00	
40	"	$\frac{3}{4}$	2.578	1.425	1.153	4.530	12.66	160.00	
41	"	$\frac{3}{4}$	2.071	1.142	0.929	4.087	11.41	130.00	
42	"	$\frac{3}{4}$	1.466	0.819	0.647	3.405	9.50	90.10	
43	"	$\frac{3}{4}$	0.838	0.476	0.362	2.585	7.21	52.00	
44	"	$\frac{3}{4}$	0.431	0.252	0.179	1.831	5.11	26.10	
45	"	$\frac{3}{4}$	4.934	2.722	2.212	6.090	17.00	289.00	
46	"	$\frac{3}{4}$	2.368	1.307	1.061	4.346	12.13	147.50	
47	"	$\frac{3}{4}$	1.745	0.964	0.781	3.733	10.42	108.80	
48	"	$\frac{3}{4}$	1.215	0.675	0.540	3.094	8.64	74.40	

VALVE NO. 1—Continued

49	July 2	1	3.719	3.120	0.599	6.717	18.78	352.00	0.100
50	"	1	3.543	2.960	0.583	6.420	17.93	321.00	
51	"	1	3.030	2.618	0.412	5.970	16.68	278.00	
52	"	1	2.553	2.183	0.370	5.489	15.32	235.00	
53	"	1	1.965	1.703	0.262	4.897	13.68	187.00	
54	"	1	1.504	1.312	0.192	4.349	12.15	148.00	
55	"	1	1.026	0.885	0.141	3.564	9.95	99.00	
56	"	1	0.697	0.619	0.078	2.949	8.24	67.80	
57	"	1	0.328	0.296	0.032	1.993	5.57	31.00	
58	"	1	0.110	0.098	0.012	1.082	3.02	9.12	

VALVE NO. 2

Gage Length = 22 feet

Pipe Cross Section = 0.358 sq. ft.

	1919		feet	feet	feet				
1	July 9	1/8	5.064	0.061	5.003	0.857	2.39	5.74	56.65
2	"	1/8	4.325	0.053	4.272	0.792	2.21	4.90	
3	"	1/8	3.775	0.046	3.729	0.735	2.05	4.22	
4	"	1/8	3.199	0.038	3.161	0.674	1.88	3.54	12.78
5	"	1/8	2.533	0.030	2.503	0.599	1.67	2.80	
6	"	1/8	1.999	0.024	1.975	0.536	1.50	2.25	
7	"	1/8	1.398	0.017	1.381	0.450	1.26	1.58	
8	"	1/8	0.931	0.010	0.921	0.364	1.02	1.03	
9	"	1/8	0.416	0.005	0.411	0.248	0.69	0.48	
10	"	1/4	5.063	0.233	4.830	1.761	4.92	24.10	
11	"	1/4	4.398	0.206	4.192	1.648	4.60	21.20	
12	"	1/4	3.797	0.181	3.616	1.532	4.28	18.30	4.66
13	"	1/4	3.199	0.155	3.044	1.403	3.92	15.40	
14	"	1/4	2.640	0.130	2.510	1.276	3.54	12.75	
15	"	1/4	2.012	0.102	1.910	1.109	3.10	9.58	
16	"	1/4	1.384	0.071	1.313	0.920	2.57	6.60	
17	"	1/4	0.731	0.036	0.695	0.659	1.84	3.39	
18	"	3/8	0.412	0.021	0.391	0.498	1.39	1.94	
19	"	3/8	4.948	0.522	4.426	2.792	7.80	60.80	
20	"	3/8	4.313	0.489	3.824	2.608	7.28	53.00	1.996
21	"	3/8	3.765	0.427	3.338	2.437	6.80	46.20	
22	"	3/8	3.102	0.356	2.746	2.205	6.15	37.80	
23	"	3/8	2.538	0.296	2.242	1.994	5.57	31.00	
24	"	3/8	1.947	0.229	1.718	1.745	4.87	23.75	
25	"	3/8	1.311	0.160	1.151	1.431	3.99	15.95	
26	"	3/8	0.800	0.101	0.699	1.109	3.10	9.60	
27	"	3/8	0.425	0.054	0.371	0.802	2.24	5.01	
28	"	1/2	4.911	1.072	3.839	3.943	11.01	121.50	0.4535
29	"	1/2	4.268	0.942	3.326	3.682	10.28	106.00	
30	"	1/2	3.619	0.818	2.801	3.411	9.52	90.60	
31	"	1/2	3.024	0.692	2.332	3.121	8.71	76.00	
32	"	1/2	2.443	0.563	1.880	2.814	7.85	61.60	
33	"	1/2	1.826	0.426	1.400	2.428	6.78	46.00	
34	"	1/2	1.252	0.298	0.954	2.004	5.60	31.30	
35	"	1/2	0.760	0.187	0.573	1.559	4.35	18.90	
36	"	3/4	0.368	0.097	0.271	1.077	3.01	9.05	
37	July 10	3/4	4.813	2.780	2.033	6.145	17.15	294.50	0.4535
38	"	3/4	4.244	2.405	1.839	5.744	16.02	257.00	
39	"	3/4	3.543	2.000	1.543	5.277	14.73	217.00	
40	"	3/4	2.971	1.692	1.279	4.877	13.62	185.80	

VALVE NO. 2—Continued

41	July 10	$\frac{3}{4}$	2.423	1.340	1.083	4.396	12.27	150.60	0.1046
42	"	$\frac{3}{4}$	1.896	1.051	0.845	3.908	10.92	119.00	
43	"	$\frac{3}{4}$	1.365	0.769	0.596	3.302	9.22	84.90	
44	"	$\frac{3}{4}$	0.810	0.456	0.354	2.523	7.04	49.50	
45	"	$\frac{3}{4}$	0.329	0.192	0.137	1.578	4.41	19.40	
46	"	1	3.301	3.043	0.258	6.477	18.10	327.50	
47	"	1	2.853	2.600	0.253	5.950	16.62	276.00	
48	"	1	2.438	2.135	0.303	5.425	15.16	230.00	
49	"	1	1.958	1.665	0.293	4.845	13.52	183.00	
50	"	1	1.530	1.260	0.270	4.282	11.97	143.00	
51	"	1	1.168	0.983	0.185	3.770	10.53	111.00	
52	"	1	0.846	0.714	0.132	3.178	8.87	78.60	
53	"	1	0.523	0.440	0.083	2.478	6.91	47.80	
54	"	1	0.230	0.195	0.035	1.593	4.45	19.80	
55	"	1	3.944	3.170	0.774	6.805	19.02	362.50	
56	"	1	3.474	2.935	0.539	6.337	17.79	316.00	
57	"	1	3.024	2.600	0.424	5.948	16.60	276.00	
58	"	1	2.555	2.184	0.371	5.487	15.32	235.00	
59	"	1	2.021	1.717	0.304	4.910	13.73	188.20	
60	"	1	1.525	1.292	0.233	4.324	12.09	146.00	
61	"	1	1.009	0.860	0.149	3.519	9.83	96.40	
62	"	1	0.663	0.568	0.095	2.828	7.90	62.20	
63	"	1	3.778	3.077	0.701	6.630	18.52	344.00	
64	"	1	3.542	2.940	0.602	6.372	17.80	317.00	

TABLE 30

LOSS OF HEAD IN 12-INCH GATE VALVE

VALVE NO. 1

Gage Length = 33 feet

Pipe Cross Section = 0.7854 sq. ft.

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	k in $\frac{v^2}{2g}$
1	1919 June 26	$\frac{1}{8}$	feet	feet	feet				96.2
2	"	$\frac{1}{8}$	4.857	0.029	4.828	1.412	1.80	3.23	
3	"	$\frac{1}{8}$	4.197	0.026	4.171	1.312	1.67	2.79	
4	"	$\frac{1}{8}$	3.585	0.022	3.563	1.214	1.54	2.38	
5	"	$\frac{1}{8}$	2.867	0.018	2.849	1.087	1.38	1.91	
6	"	$\frac{1}{8}$	2.777	0.015	2.162	0.953	1.21	1.47	
7	"	$\frac{1}{8}$	1.522	0.012	1.510	0.791	1.01	1.01	
8	"	$\frac{1}{8}$	0.912	0.007	0.905	0.606	0.77	0.59	
9	June 19	$\frac{1}{4}$	0.320	0.002	0.318	0.358	0.46	0.21	17.4
10	"	$\frac{1}{4}$	4.663	0.142	4.521	3.213	4.09	16.70	
11	"	$\frac{1}{4}$	4.131	0.127	4.004	3.024	3.85	14.80	
12	"	$\frac{1}{4}$	3.435	0.106	3.329	2.759	3.51	12.30	
13	"	$\frac{1}{4}$	2.946	0.091	2.855	2.563	3.26	10.63	
14	"	$\frac{1}{4}$	2.351	0.073	2.278	2.285	2.91	8.43	
15	"	$\frac{1}{4}$	1.764	0.055	1.709	1.983	2.52	6.37	
16	"	$\frac{1}{4}$	1.238	0.039	1.199	1.657	2.11	4.45	
17	"	$\frac{3}{8}$	0.512	0.017	0.495	1.052	1.33	1.76	5.605
18	"	$\frac{3}{8}$	5.005	0.436	4.569	5.649	7.19	51.75	
			4.345	0.380	3.965	5.277	6.71	45.10	

VALVE NO. 1—Continued

19	June 19	$\frac{3}{8}$	3.714	0.329	3.385	4.909	6.25	39.00	
20	"	$\frac{3}{8}$	3.077	0.275	2.802	4.474	5.70	32.50	
21	"	$\frac{3}{8}$	2.469	0.221	2.248	4.015	5.11	26.10	
22	"	$\frac{3}{8}$	1.917	0.171	1.746	3.524	4.48	20.10	
23	"	$\frac{3}{8}$	1.239	0.112	1.127	2.851	3.63	13.18	
24	"	$\frac{3}{8}$	0.824	0.076	0.748	2.332	2.96	8.80	
25	"	$\frac{3}{8}$	0.357	0.033	0.324	1.511	1.92	3.70	
26	June 20	$\frac{1}{2}$	4.204	0.834	3.370	7.618	9.70	94.00	2.288
27	"	$\frac{1}{2}$	3.686	0.712	2.974	7.123	9.06	82.00	
28	"	$\frac{1}{2}$	3.195	0.613	2.582	6.655	8.47	71.80	
29	"	$\frac{1}{2}$	2.677	0.513	2.164	6.128	7.80	60.80	
30	"	$\frac{1}{2}$	2.173	0.420	1.753	5.550	7.07	49.90	
31	"	$\frac{1}{2}$	1.656	0.323	1.333	4.870	6.20	38.40	
32	"	$\frac{1}{2}$	1.164	0.231	0.933	4.103	5.23	27.30	
33	"	$\frac{1}{2}$	0.777	0.153	0.624	3.334	4.24	18.00	
34	"	$\frac{1}{2}$	0.294	0.060	0.234	2.049	2.61	6.80	
35	June 23	$\frac{1}{2}$	1.646	0.934	0.712	8.030	10.22	104.60	0.414
36	"	$\frac{3}{4}$	1.416	0.815	0.601	7.534	9.59	92.00	
37	"	$\frac{3}{4}$	1.260	0.719	0.541	7.141	9.09	82.60	
38	"	$\frac{3}{4}$	1.030	0.587	0.443	6.521	8.30	69.00	
39	"	$\frac{3}{4}$	0.830	0.473	0.357	5.895	7.50	56.20	
40	"	$\frac{3}{4}$	0.688	0.395	0.293	5.375	6.85	46.85	
41	"	$\frac{3}{4}$	0.481	0.282	0.199	4.540	5.78	33.40	
42	"	$\frac{3}{4}$	0.351	0.207	0.144	3.879	4.94	24.40	
43	"	$\frac{3}{4}$	0.145	0.094	0.051	2.597	3.30	10.90	
44	"	$\frac{3}{4}$	1.524	0.877	0.647	7.801	9.94	98.70	
45	"	$\frac{3}{4}$	1.216	0.702	0.514	7.066	9.00	81.00	
46	June 19	1	0.858	0.801	0.057	7.473	9.51	90.50	0.0468
47	"	1	0.667	0.615	0.052	6.677	8.50	72.10	
48	"	1	0.544	0.497	0.047	6.034	7.68	59.00	
49	"	1	0.415	0.384	0.031	5.310	6.76	45.60	
50	"	1	0.291	0.272	0.019	4.460	5.68	32.20	
51	"	1	0.170	0.158	0.012	3.391	4.32	18.60	
52	"	1	0.963	0.898	0.065	7.875	10.02	100.80	
53	"	1	0.868	0.800	0.068	7.474	9.51	90.30	
54	"	1	0.707	0.643	0.064	6.816	8.68	75.20	
55	"	1	0.835	0.772	0.063	7.354	9.36	87.50	
56	"	1	0.722	0.657	0.065	6.877	8.75	76.70	

TABLE 31

LOSS OF HEAD IN 1/2-INCH GLOBE VALVES

VALVE NO. 1

Gage Length = 10 feet

Pipe Cross Section = 0.00206 sq. ft.

Diameter of Opening through Seat = 0.501 inches

Full Opening Rise of Valve above Seat = 0.29 inches

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	k in $\frac{v^2}{2g}$ H = k — $\frac{v^2}{2g}$
	1919		feet	feet	feet				
1	May 16	Full	0.364	0.141	0.223	0.00226	1.09	1.20	12.56
2	"	"	0.903	0.324	0.582	0.00355	1.72	2.97	
3	"	"	1.261	0.433	0.828	0.00418	2.04	4.14	
4	"	"	1.799	0.576	1.223	0.00492	2.39	5.70	
5	"	"	2.133	0.205	1.928	0.00276	1.34	1.80	
6	"	"	3.448	0.457	2.991	0.00432	2.09	4.38	
7	"	"	3.678	1.215	2.463	0.00741	3.59	12.95	
8	"	"	4.576	1.494	3.082	0.00632	4.04	16.34	
9	"	"	5.208	1.750	3.458	0.00906	4.40	19.40	
10	"	"	0.768	0.263	0.505	0.00318	1.54	2.38	
11	"	"	0.962	0.332	0.630	0.00362	1.76	3.09	
12	"	"	1.551	0.540	1.011	0.00472	2.29	5.24	
13	"	"	1.963	0.660	1.303	0.00530	2.57	6.62	
14	"	"	2.175	0.750	1.425	0.00568	2.76	7.60	
15	"	"	2.818	0.930	1.888	0.00642	3.11	9.70	
16	"	"	3.682	1.185	2.497	0.00732	3.55	12.65	
17	"	"	4.041	1.312	2.729	0.00775	3.76	14.20	

VALVE NO. 2

Gage Length = 10 feet

Pipe Cross Section = 0.00206 sq. ft.

Diameter of Opening Through Seat = 0.50 inches

Full Opening Rise of Valve above Seat = 0.2655 inches

	1919		feet	feet	feet				
1	June 6	Full	0.172	0.060	0.112	0.00150	0.73	0.53	14.50
2	"	"	0.548	0.168	0.380	0.00269	1.30	1.70	
3	"	"	1.170	0.340	0.830	0.00395	1.92	3.67	
4	"	"	1.519	0.440	1.079	0.04520	2.19	4.82	
5	"	"	1.769	0.505	1.264	0.04870	2.36	5.60	
6	"	"	2.422	0.690	1.732	0.05680	2.76	7.61	
7	"	"	3.201	0.900	2.301	0.06510	3.16	10.00	
8	"	"	3.290	0.954	2.336	0.06710	3.26	10.60	
9	"	"	4.000	1.130	2.870	0.07330	3.56	12.68	
10	"	"	4.473	1.290	3.183	0.07790	3.78	14.30	
11	"	"	4.824	1.370	3.454	0.08050	3.90	15.25	

VALVE NO. 3

Gage Length = 10 feet

Pipe Cross Section = 0.00206 sq. ft.

Diameter of Opening through Seat = 0.50 inches

Full Opening Rise of Valve above Seat = 0.28 inches

	1919		feet	feet	feet				
1	June 7	Full	0.326	0.100	0.226	0.00213	1.03	1.07	13.60
2	"	"	0.661	0.201	0.460	0.00303	1.47	2.16	
3	"	"	0.965	0.295	0.670	0.00366	1.78	3.15	
4	"	"	1.448	0.460	0.988	0.00459	2.22	4.95	
5	"	"	1.850	0.595	1.255	0.00529	2.56	6.59	
6	"	"	2.565	0.822	1.743	0.00626	3.04	9.23	
7	"	"	2.995	0.940	2.055	0.00677	3.24	10.47	
8	"	"	3.532	1.150	2.382	0.00739	3.58	12.81	
9	"	"	4.015	1.320	2.695	0.00791	3.84	14.76	
10	"	"	4.700	1.525	3.175	0.00854	4.14	17.18	
11	"	"	5.049	1.660	3.389	0.00886	4.30	18.52	
12	"	"	0.964	0.300	0.664	0.00368	1.78	3.19	
13	"	"	1.719	0.515	1.204	0.00492	2.39	5.71	
14	"	"	2.919	0.900	2.019	0.00654	3.17	10.05	
15	"	"	3.092	0.903	2.189	0.00655	3.18	10.11	
16	"	"	3.710	1.100	2.610	0.00721	3.50	12.25	
17	"	"	4.268	1.255	3.013	0.00771	3.74	14.01	
18	"	"	2.420	0.740	1.680	0.00591	2.83	8.24	

TABLE 32

LOSS OF HEAD IN 3/4-INCH GLOBE VALVES

VALVE NO. 1

Gage Length = 10 feet

Pipe Cross Section = 0.00358 sq. ft.

Diameter of Opening through Seat = 0.75 inches

Full Opening Rise of Valve above Seat = 0.28 inches

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	k in $\frac{v^3}{2g}$
	1919		feet	feet	feet				
1	Aug. 22	Full	5.328	2.020	3.308	0.02025	5.66	32.00	6.76
2	"	"	4.238	1.620	2.618	0.01797	5.02	25.20	
3	"	"	3.656	1.380	2.276	0.01667	4.66	21.60	
4	"	"	2.872	1.080	1.792	0.01472	4.12	16.90	
5	"	"	2.482	0.940	1.542	0.01360	3.80	14.40	
6	"	"	1.899	0.735	1.164	0.01191	3.33	11.05	
7	"	"	1.258	0.495	0.763	0.00961	2.69	7.20	
8	"	"	0.561	0.210	0.351	0.00630	1.76	3.10	

VALVE NO. 2

Gage Length = 10 feet

Pipe Cross Section = 0.00358 sq. ft.

Diameter of Opening through Seat = 0.78 inches
 Full Opening Rise of Valve above Seat = 0.3125 inches

	1919		feet	feet	feet				
1	Aug. 22	Full	5.292	2.120	3.172	0.02080	5.81	33.70	6.19
2	"	"	4.571	1.810	2.761	0.01917	5.36	28.60	
3	"	"	3.510	1.420	2.090	0.01678	4.69	22.00	
4	"	"	2.889	1.150	1.739	0.01518	4.24	17.95	
5	"	"	2.478	0.990	1.488	0.01400	3.92	15.30	
6	"	"	1.757	0.720	1.037	0.01178	3.29	10.80	
7	"	"	1.132	0.465	0.667	0.00934	2.61	6.80	
8	"	"	0.629	0.250	0.379	0.00690	1.93	3.70	

VALVE NO. 3

Gage Length = 10 feet

Pipe Cross Section = 0.00358 sq. ft.

Diameter of Opening through Seat = 0.7656 inches
 Full Opening Rise of Valve above Seat = 0.3125 inches

	1919		feet	feet	feet				
1	Aug. 23	Full	5.333	2.040	3.293	0.02040	5.70	32.50	6.61
2	"	"	4.727	1.800	2.927	0.01913	5.35	28.50	
3	"	"	4.148	1.600	2.548	0.01792	5.01	25.00	
4	"	"	3.722	1.430	2.292	0.01691	4.72	22.30	
5	"	"	2.990	1.150	1.840	0.01513	4.23	17.85	
6	"	"	2.378	0.918	1.460	0.01345	3.76	14.10	
7	"	"	1.787	0.700	1.087	0.01161	3.24	10.51	
8	"	"	1.138	0.453	0.685	0.00921	2.58	6.60	
9	"	"	0.424	0.158	0.266	0.00547	1.53	2.33	

TABLE 33

LOSS OF HEAD IN 1-INCH GLOBE VALVES

VALVE NO. 1

Gage Length = 10 feet

Pipe Cross Section = 0.00604 sq. ft.

Diameter of Opening through Seat = 1.01562 inches

Full Opening Rise of Valve above Seat = 0.4375 inches

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	k in $\frac{v^2}{2g}$
	1919		feet	feet	feet				
1	July 22	Full	0.528	0.160	0.368	0.01160	1.92	3.68	5.90
2	"	"	1.014	0.310	0.704	0.01630	2.70	7.29	
3	"	"	1.547	0.480	1.067	0.02040	3.38	11.40	
4	"	"	2.052	0.640	1.412	0.02360	3.92	15.30	
5	"	"	2.596	0.810	1.786	0.02660	4.40	19.30	
6	"	"	3.086	0.960	2.126	0.02900	4.80	23.00	
7	"	"	3.508	1.100	2.408	0.03090	5.18	26.20	
8	"	"	3.940	1.230	2.710	0.03280	5.44	29.50	
9	"	"	4.722	1.470	3.252	0.03590	5.95	35.30	

VALVE NO. 2

Gage Length = 10 feet

Pipe Cross Section = 0.00604 sq. ft.

Diameter of Opening through Seat = 1.01562 inches

Full Opening Rise of Valve above Seat = 0.4375 inches

	1919		feet	feet	feet				
1	July 31	Full	0.320	0.100	0.220	0.00893	1.48	2.18	6.01
2	"	"	0.633	0.190	0.443	0.01274	2.11	4.46	
3	"	"	1.019	0.310	0.709	0.01633	2.71	7.32	
4	"	"	1.467	0.450	1.017	0.01975	3.28	10.65	
5	"	"	2.003	0.620	1.383	0.02320	3.84	14.75	
6	"	"	2.513	0.770	1.743	0.02600	4.31	18.50	
7	"	"	3.088	0.950	2.138	0.02880	4.78	22.80	
8	"	"	3.503	1.140	2.363	0.03170	5.25	27.50	
9	"	"	4.435	1.410	3.025	0.03510	5.82	33.70	
10	"	"	3.999	1.240	2.759	0.03300	5.47	29.80	
11	"	"	4.802	1.480	3.322	0.03600	5.97	35.50	

VALVE NO. 3

Gage Length = 10 feet

Pipe Cross Section = 0.00604 sq. ft.

Diameter of Opening through Seat = 1.01562 inches

Full Opening Rise of Valve above Seat = 0.4375 inches

	1919		feet	feet	feet				
1	Aug. 1	Full	0.650	0.190	0.460	0.01275	2.12	4.45	6.17
2	"	"	1.151	0.340	0.811	0.01711	2.84	8.02	
3	"	"	1.805	0.530	1.275	0.02162	3.58	12.80	
4	"	"	2.494	0.750	1.744	0.02560	4.25	17.95	
5	"	"	2.927	0.880	2.047	0.02762	4.58	20.95	
6	"	"	3.719	1.120	2.599	0.03140	5.20	27.00	
7	"	"	4.140	1.260	2.880	0.03310	5.49	30.05	
8	"	"	4.942	1.500	3.442	0.03610	6.00	36.00	

TABLE 34

LOSS OF HEAD IN 2-INCH GLOBE VALVES

VALVE NO. 1

Gage Length = 9.98 feet

Pipe Cross Section = 0.02338 sq. ft.

Diameter of Opening through Seat = 2.02 inches

Full Opening Rise of Valve above Seat = 1.00 inch

Run	Date of Test	Valve Opening	Losses in pipe and valve	Pipe Friction	Loss of Head thru valve only	Discharge cu. ft. per sec.	Velocity feet per second	Velocity squared	k in $\frac{v^2}{2g}$
	1919		feet	feet	feet				
1	May 7	Full	0.035	0.012	0.023	0.0115	0.50	0.25	6.26
2	"	"	0.057	0.016	0.041	0.0154	0.66	0.43	
3	"	"	0.105	0.024	0.081	0.0255	0.90	0.89	
4	"	"	0.212	0.042	0.170	0.0306	1.29	1.66	
5	"	"	0.500	0.096	0.404	0.0472	2.03	4.11	
6	"	"	0.718	0.137	0.581	0.0564	2.42	5.85	
7	"	"	1.106	0.205	0.901	0.0703	3.02	9.10	
8	"	"	1.579	0.290	1.289	0.0843	3.62	13.20	
9	"	"	3.041	0.543	2.498	0.1182	5.07	25.70	
10	"	"	3.540	0.622	2.918	0.1272	5.46	29.80	
11	"	"	3.845	0.675	3.170	0.1326	5.70	32.60	
12	"	"	4.517	0.788	3.729	0.1445	6.21	38.60	
13	"	"	5.005	0.880	4.125	0.1520	6.52	42.50	
14	"	"	2.131	0.383	1.748	0.0985	4.22	17.85	
15	"	"	2.440	0.440	2.000	0.1059	4.55	20.65	

VALVE NO. 2

Gage Length = 9.98 feet

Pipe Cross Section = 0.02338 sq. ft.

Diameter of Opening through Seat = 2.01 inches

Full Opening Rise of Valve above Seat = 1.00 inch

	1919		feet	feet	feet				
1	May 16	Full	0.087	0.017	0.070	0.0155	0.66	0.44	7.46
2	"	"	0.150	0.030	0.120	0.0240	1.03	1.06	
3	"	"	0.304	0.053	0.251	0.0341	1.46	2.13	
4	"	"	0.508	0.085	0.423	0.0442	1.90	3.59	
5	"	"	0.731	0.120	0.611	0.0530	2.28	5.18	
6	"	"	1.012	0.168	0.844	0.0627	2.68	7.22	
7	"	"	1.469	0.239	1.230	0.0760	3.25	10.00	
8	"	"	2.037	0.320	1.717	0.0893	3.83	14.70	
9	"	"	2.568	0.397	2.171	0.1002	4.30	18.50	
10	"	"	3.195	0.490	2.705	0.1120	4.81	23.00	
11	"	"	3.455	0.525	2.930	0.1160	4.98	24.75	
12	"	"	3.930	0.606	3.324	0.1255	5.39	29.00	
13	"	"	4.434	0.670	3.764	0.1323	5.69	32.40	

VALVE NO. 3

Gage Length = 9.98 feet

Pipe Cross Section = 0.02338 sq. ft.

Diameter of Opening through Seat = 1.875 inches

Full Opening Rise of Valve above Seat = 0.954 inches

	1919		feet	feet	feet				
1	May 19-20	Full	0.200	0.040	0.160	0.0287	1.23	1.52	6.69
2	"	"	0.400	0.077	0.323	0.0410	1.76	3.10	
3	"	"	0.645	0.098	0.547	0.0469	2.01	4.05	
4	"	"	0.996	0.183	0.813	0.0654	2.81	7.89	
5	"	"	1.500	0.265	1.235	0.0805	3.46	11.95	
6	"	"	2.001	0.347	1.654	0.0934	4.01	16.05	
7	"	"	2.500	0.430	2.070	0.1044	4.48	20.10	
8	"	"	3.015	0.510	2.505	0.1147	4.92	24.20	
9	"	"	3.543	0.595	2.948	0.1242	5.33	28.45	
10	"	"	3.998	0.670	3.328	0.1324	5.68	32.30	
11	"	"	4.571	0.762	3.809	0.1420	6.08	37.10	

VALVE NO. 4

Gage Length = 9.98 feet

Pipe Cross Section = 0.02338 sq. ft.

Diameter of Opening through Seat = 1.875 inches

Full Opening Rise of Valve above Seat = 0.882 inches

	1919		feet	feet	feet				
1	May 21	Full	0.206	0.038	0.168	0.0283	1.21	1.46	7.21
2	"	"	0.421	0.075	0.346	0.0407	1.75	3.06	
3	"	"	0.712	0.122	0.590	0.0531	2.28	5.20	
4	"	"	1.002	0.170	0.832	0.0633	2.72	7.39	
5	"	"	1.567	0.260	1.307	0.0797	3.41	11.65	
6	"	"	2.017	0.325	1.692	0.0903	3.87	15.00	
7	"	"	2.489	0.400	2.089	0.1007	4.33	18.70	
8	"	"	3.029	0.483	2.546	0.1110	4.77	22.70	
9	"	"	3.533	0.560	2.973	0.1200	4.17	26.60	
10	"	"	4.020	0.630	3.390	0.1280	5.50	30.30	
11	"	"	4.615	0.720	3.895	0.1378	5.90	35.00	

VALVE NO. 5

Gage Length = 9.98 feet

Pipe Cross Section = 0.02338 sq. ft.

Diameter of Opening through Seat = 2.04687 inches

Full Opening Rise of Valve above Seat = 0.9062 inches

	1919		feet	feet	feet				
1	May 22	Full	2.994	0.540	2.454	0.1131	4.85	23.60	6.74
2	"	"	3.483	0.620	2.863	0.1224	5.25	27.60	
3	"	"	3.982	0.700	3.282	0.1328	5.61	31.50	
4	"	"	4.583	0.795	3.788	0.1402	6.02	36.20	
5	"	"	0.240	0.040	0.200	0.0283	1.21	1.48	
6	"	"	0.423	0.080	0.343	0.0408	1.75	3.06	
7	"	"	0.641	0.125	0.516	0.0516	2.21	4.90	
8	"	"	1.033	0.198	0.835	0.0656	2.82	7.92	
9	"	"	1.487	0.280	1.207	0.0791	3.40	11.50	
10	"	"	2.000	0.382	1.618	0.0921	3.95	15.65	
11	"	"	2.507	0.460	2.047	0.1031	4.43	19.61	

VALVE NO. 6

Gage Length = 9.98 feet

Pipe Cross Section = 0.02338 sq. ft.

Diameter of Opening through Seat = 2.0468 inches

Full Opening Rise of Valve above Seat = 0.844 inches

	1919		feet	feet	feet				
1	May 23	Full	0.195	0.035	0.160	0.0253	1.09	1.18	8.45
2	"	"	0.373	0.062	0.311	0.0355	1.52	2.32	
3	"	"	0.732	0.122	0.610	0.0508	2.18	4.74	
4	"	"	1.076	0.175	0.901	0.0615	2.64	6.97	
5	"	"	1.446	0.232	1.214	0.0713	3.06	9.37	
6	"	"	1.968	0.301	1.667	0.0834	3.57	12.79	
7	"	"	2.508	0.395	2.113	0.0941	4.03	16.25	
8	"	"	2.921	0.446	2.475	0.1014	4.35	18.90	
9	"	"	3.486	0.520	2.966	0.1107	4.75	22.60	
10	"	"	3.766	0.550	3.216	0.1147	4.92	24.20	
11	"	"	4.680	0.670	4.010	0.1280	5.50	30.00	

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**PERMEABILITY TESTS ON BROKEN STONE
CONCRETE**

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PERMEABILITY TESTS ON BROKEN STONE CONCRETE

Introduction.—In the years 1912 to 1914 a series of tests was made at the University of Wisconsin to determine the effect of various influences on the permeability of gravel concrete to water. A paper containing the results of these tests appeared in the *Journal of the Western Society of Engineers* in November, 1914. This bulletin is a report on a similar and subsequent series of tests on the permeability of broken stone concrete to water.

The specimens and tests described in this bulletin were made during the years 1914 and 1915. A total of sixty-one batches of machine mixed concrete were prepared; each batch was cast into three or four permeability specimens and an equal number of compression cylinders.

More or less study was made of the influence of the following variables on the water-tightness of the concrete; the effect of age, thickness, richness of mix, grading of aggregate, time of mixing, consistency, curing treatment, and direction of flow.

Compression tests on 6x18-in. cylinders were also made to furnish data which would indicate whether or not water-tightness is related to the strength of the concrete.

The entire series of specimens is listed according to the batch numbers in Table 3. This table indicates the materials and proportions used, as well as the time of mixing and thickness of specimens. Tables 4 to 11 give further information regarding the differences in fabrication and treatment between some of the specimens.

Proportions of 1:1½:3 and 1:2:4 (by volume) and 1:9 (by weight) were selected for most of the mixes because such concretes are in the range of proportions which are used with similar materials in practice. Furthermore, it was found that the apparatus available for these tests would not detect leakage into concretes richer than 1:1½:3, and that mixes leaner than 1:9 were not dense enough to hold a pressure of 40 lb. per sq. in. in testing.

Acknowledgment is made to Mr. John Miller, former laboratory assistant, for careful work in preparing materials and

making and testing specimens. Messrs. F. D. Bickel, O. A. Bailey, Thomas Utegaard, and L. F. Nelson computed and tabulated the results of tests, and C. E. Ives drew and lettered most of the tables. Checking of the computations and the writing of this bulletin is largely the work of Mr. C. A. Wiepking.

The Universal Portland Cement Company donated the cement used in the tests.

MATERIALS

Cement.—All of the specimens described in this report were made with Universal Portland cement. Two different lots, P₂ and P₈, from separate storage bins, were used. These lots consisted of fifty barrels each, selected from a local supply, mixed separately and stored in bulk in galvanized iron grain bins. In order to make the storage bins as tight as possible, the seams were calked with tar and the door was edged with felt. These precautions were taken to prevent variations in the properties of the cement, for the making of the specimens necessarily extended over a long period of time.

TABLE NO. 1
Physical Properties of Universal Portland Cement

Physical Properties of Concrete Formulations												
Mix	Age in days	Tensile Str. ^{lb.}		Residue on sieves %		Time of set		Soundness			Specific Gravity	Remarks
		Mix				Initial	Final	Air	Water	Steam		
		Neat	13% standard sand	No. 100	No. 200	Hr.-Min	Hr.-Min.					
P ₂	7	680	283	30	21.8	1-40	5-40	O.K.	O.K.	O.K.	309	* Tested after storing in bulk 1 yr. and 11 mo. in a tight galv. iron bin.
	28	735	338									
	60	671	364									
	100	772	359									
	360	702	268									
	2 yrs	633	272									
*P ₂	7	646	260			1-36	4-0	O.K.	O.K.	O.K.		
	28	572	357									
	60	651	388									
	100	680	346									
	360	653	327									
	2 yrs	582	307									
P ₈	7	557	215	32	20.8	2-25	5-20	O.K.	O.K.	O.K.		
	28	610	329									
	60	756	389									
	100	610	377									
	360	756	324									

The physical properties of the two lots of cement are given in Table 1. The first test of P₂ was made a year before the first batch (No. 115) of this series. The second test (marked *) was made at the same time as batch 115 and is representative of the cement in batches 115 to 129, 132, 133, and 140. Cement from lot P₈ was used in batches 130 and 131, 134 to 139, and in all batches numbered above 145. The loose weight per cubic foot of cement was assumed to be 100 pounds.

Sand.—All but two of the batches were made with Janesville sand as the fine aggregate. Four successive shipments from the Janesville Sand and Gravel Company, Janesville, Wis-

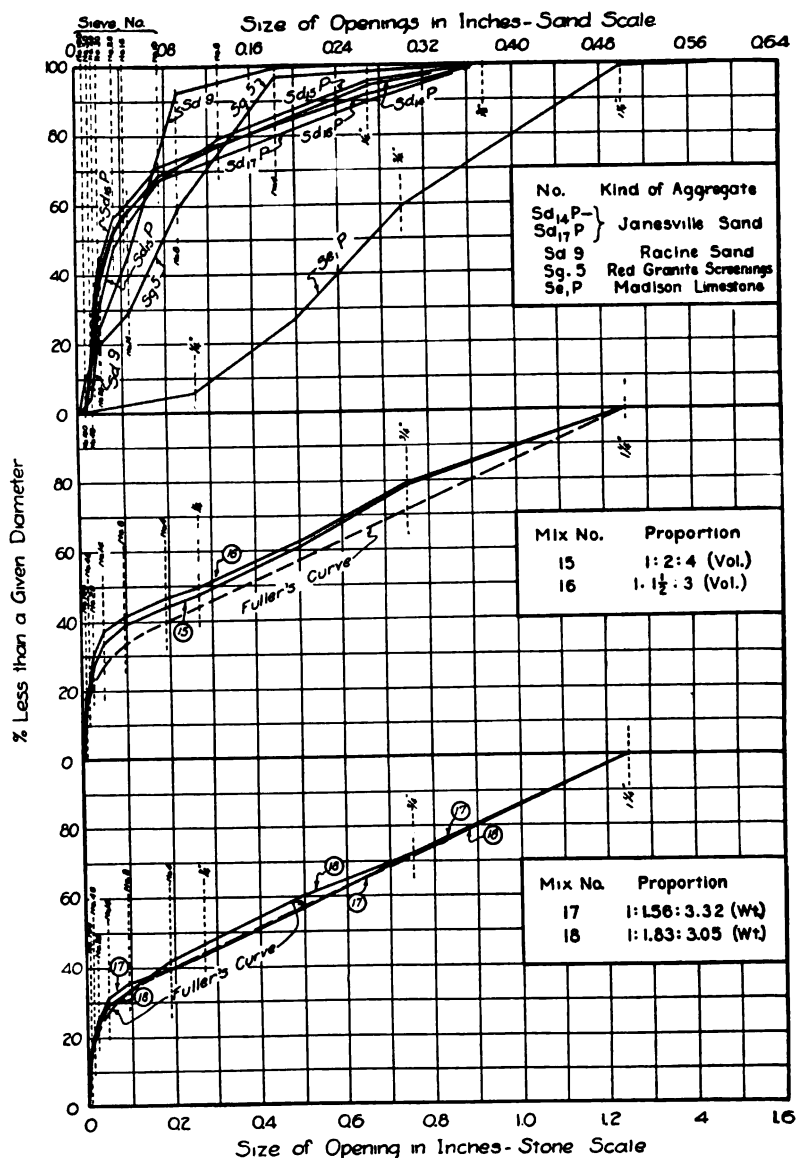
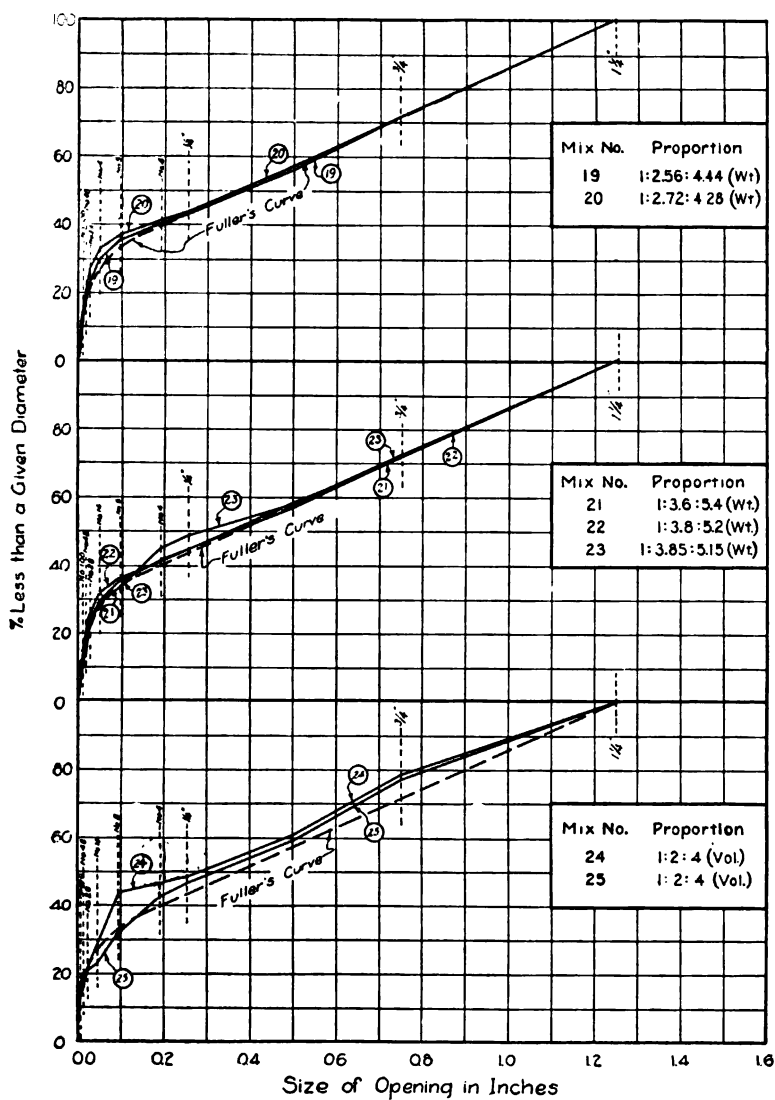


Fig. 1.- Mechanical Analysis Curves for Aggregates and Mixes No. 15 to 18



consin, were used; they were designated $Sd_{14}P$ to $Sd_{17}P$, respectively. These sands were similar in physical properties, as

TABLE NO 2
Properties of Aggregates

Aggregate	Weight per cu ft	Specific Gravity	Absorption	Mineral Analysis - %							Fineness Modulus	Remarks
				Igneous Rocks	Quartz Group	Dolomite Group	Heavy Minerals	Calcite	Impurities	Total		
$Sd_{14}P$	109.7	2.66	0.19	5.39	65.81	27.93	0.87			100	3.32	Janesville Sand
$Sd_{15}P$	107.8	"	"	5.39	65.81	27.93	0.87			100	3.46	"
$Sd_{16}P$	111.5	"	"	5.39	65.81	27.93	0.87			100	3.19	"
$Sd_{17}P$	109.2	"	"	5.39	65.81	27.93	0.87	Shale	Not Sorted	100	3.37	"
Sd_9	105.5	2.75	0.70	4.14	32.58	58.16	0.61	4.16	0.35	100	3.30	Racine Sand
Sg_5	92.3	2.62	—								3.70	Red Granite Screenings
Se_1P	92.4	2.74	0.45			66.70		30.52	2.78	100		Madison Stone

indicated in Table 2, which gives the weights per cubic foot, specific gravity, mineral analysis, and fineness moduli. Figure 1, which shows the mechanical analysis curves for these sands, also indicates that they are similar.

In determining the weight per cubic foot, the air-dry material was dumped into a suitable hopper from which it was poured at the rate of one cubic foot per minute into a cubic foot measure placed 3 feet below the hopper. The measure was then struck off even full with a straight edge and the material weighed.

In batch 188 Racine sand, Sd_9 , was used. This sand is coarser than the Janesville material. Red granite screenings, 80 per cent chips and 20 per cent dust, were used in batch 189. The physical properties of these materials are also given in Table 2, where they may be compared with the Janesville sands. The mechanical analysis curves for these sands will also be found in Fig. 1.

Stone.—Madison stone, Se_1P , was used in all of the concrete mixes, as the coarse aggregate. This material came from Stephens quarry at Madison, and is a soft lower magnesian limestone, crushed and screened through a $1\frac{1}{2}$ -inch screen. Its physical and mechanical properties are given in Table 2 and in Fig. 1.

FORMS OF SPECIMENS

PU Specimens.—All batches of concrete, except numbers 165 to 170, were made into the "PU" type of specimen. This type was developed after a number of other designs had been experimented upon, and had previously been used with success in the series of permeability tests on gravel concrete. Figures 3 and 4 show a sketch and a photograph of the PU type of specimen.

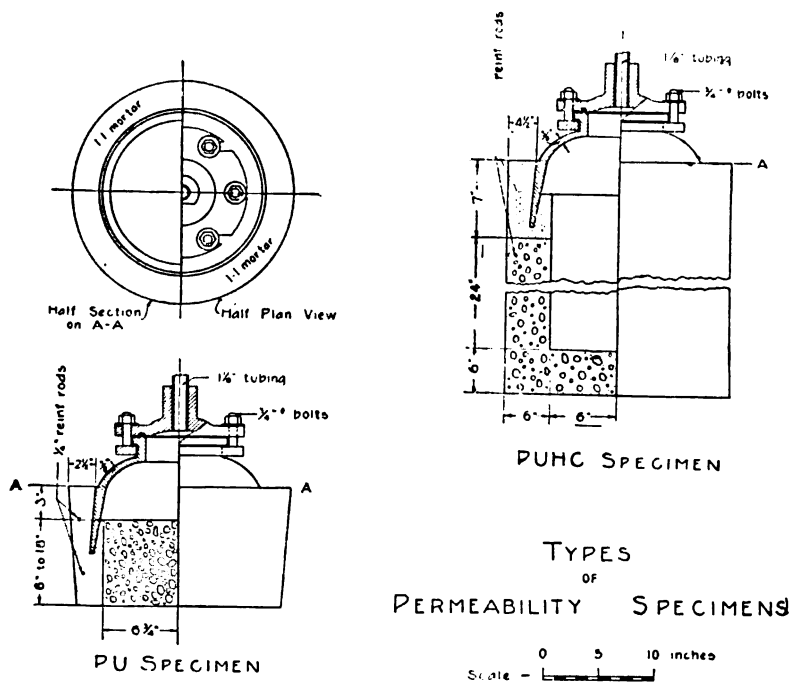


Fig. 3.

The area of the concrete core under test is one square foot, so that the leakages are determined directly in terms of this unit of area. Since the results of tests on this type of specimen depend on the imperviousness of the mortar shell it is important that a rich and properly mixed mortar be used. It is also imperative that proper bond be secured between the mortar, the iron casting,

and the concrete core to prevent any leakage at these places. The only difficulties in securing tight specimens were encountered with lean mixes of concrete, in which pockets frequently occurred at the edge of the core, and with specimens submitted to dry curing



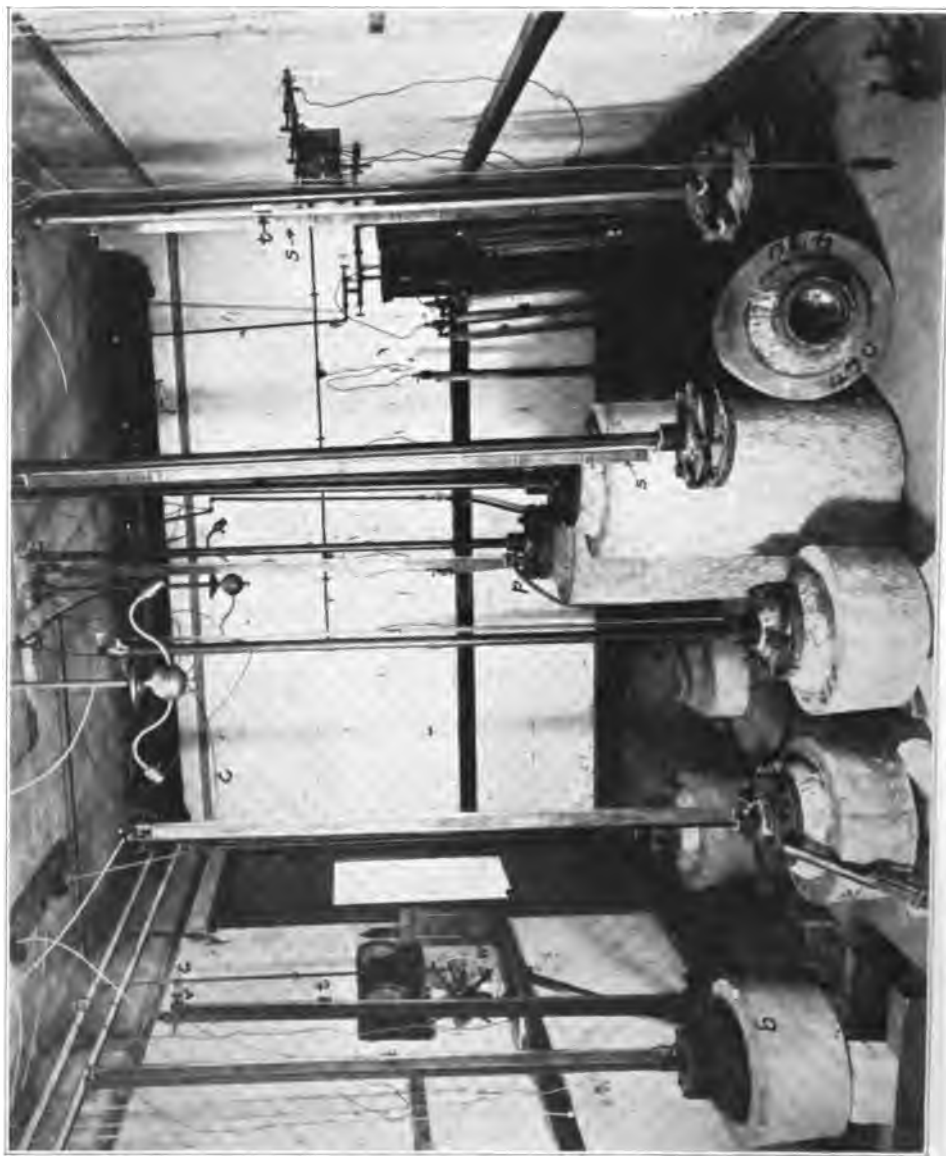
Fig. 4—LONGITUDINAL SECTIONS OF PU AND PUHC SPECIMENS

conditions, in which cracks sometimes occurred between the core and the shell.

Tests on specimens made entirely of the 1:1 mortar used for the shells have shown that such a mortar is practically impervious to water; the leakages obtained were so low that they would not affect any results determined from the concretes in this series.

Of two such specimens tested at the age of 21 days, one absorbed 0.0022 gallon in 304 hours and only 0.0003 gallon entered the other in the same period. A third specimen, tested when four months old, showed no leakage in 304 hours. The bottoms of these specimens were covered with impervious building paper, so that the leakages were through or into the sides only.

PUHC Specimens.—Since the PU specimen can be used only for observing the permeability of concrete when the water flow parallels the direction of placing the concrete, batches 165 to 170 were made into "PUHC" specimens, in order to give information regarding the flow of water perpendicular to the direction of placing the concrete. This type of specimen is similar to the PU, having the same general shape and the same kind of iron casting, but the side walls of the specimen to within 2 inches of the casting are made of concrete rather than of mortar. Thus nearly all of the water, under pressure, permeates through the side walls perpendicular to the direction of placing. It should also be noted that in the PUHC specimen the inside area of concrete exposed to the water is considerably less than that exposed to air.



TESTING APPARATUS

The apparatus for measuring the flow of water into permeability specimens of the PU or PUHC types is shown in Fig. 5. Measurement of the flow of water into the specimen is made by a water column or "permeability tube" attached to each specimen. Each water column consists of an iron pipe $1\frac{1}{8}$ inch in diameter, a half-inch glass water gage, and a scale (s) graduated to read the quantity of water in the tube. The smallest divisions of the water gage represent intervals of 0.001 gallon; and, by the use of a sliding target (t), readings may be interpolated to 0.0001 gallon. The water gage is connected to the pipe at the top and bottom.

The permeability tube is attached to the specimen by means of a heavy iron flange. A rubber ring is inserted in a groove between the flange and the bonnet to insure a tight joint when the connecting bolts are turned down. A small cock (p) is attached to the tube just above the flange so that water may be forced into the column before commencing the test. The air tube (c) from the pressure tank enters the top of the permeability tube, where a valve (v) is provided for cutting off the supply when the water column is not in use. The tubes from the battery of water columns are joined to a manifold which is supplied with air by a pipe from the pressure tank (r). The tank is of relatively large capacity, when compared to the volume of the pipes and tubes, so that leaks in the specimens causing a lowering of the water levels do not materially decrease the air pressure.

No provision was made for catching and measuring the water escaping from the free surfaces of specimens, but whenever such leakage was visible it was recorded and is noted in the tables of data herein.

PREPARING AND PROPORTIONING MATERIALS

Preliminary Treatment of Aggregates.—The sand for these specimens was air-dried before use unless otherwise specified. It was spread out in a thin layer in the mixer house and turned and raked for one or two days until dry. In the batches made with wet sand the air-dry sand was placed in the mixer and wetted with a given amount of water; the mixer was run until the sand was uniformly wet before the other materials were added. The surfaces of the particles of stone were usually dry at the time of making the concrete. Occasionally specimens of stone were selected from the storage pile and tested for moisture content so that the total per cent of water in the mix could be estimated.

Grading the Aggregates.—A Converse rotary sieve, set at a slope of $\frac{1}{2}$ -inch per foot and run at a speed of 11 r.p.m., was used in grading the stone. This sieve separated the stone into the following sizes: 0 to $\frac{1}{4}$, $\frac{1}{4}$ to $\frac{1}{2}$, $\frac{1}{2}$ to $\frac{3}{4}$, and $\frac{3}{4}$ to $1\frac{1}{4}$ inches. These sizes will hereafter be referred to as the No. 0, 1, 2, and 3 sizes, respectively, and the material from Madison stone greater than $1\frac{1}{4}$ inch in size will be called the No. 4 stone. Ordinarily the Janesville sands were used as received, but in a few batches graded sands were used. These were graded by hand-sieving on riddles.

Mechanically graded materials are indicated in the tables accompanying this report by the letter "g" placed after the mix number. Thus 1:2:4g means that one part of cement, two parts of sand, and four parts of graded stone were used. Similarly 1:2g:4 would indicate that the sand in the mix was graded. Graded coarse aggregate was used in all of the concretes made, and the sand was graded for batches 146, 172, and 189.

Proportioning.—Mixes in this series were based on proportions by weight and also by volume. In most of the tests 1:2:4 and 1:1 $\frac{1}{2}$:3 (by volume) and 1:9 (by weight) concretes were used. A few 1:5 and 1:7 (by weight) batches were also made. In determining the amounts of sand and stone in mixes proportioned by weight, the attempt was made to combine the mechanical analysis curves of the cement, sand, and gravel so that the curve of the total materials would approach the empirical curve of Fuller and Thompson.¹

¹Concrete, Plain and Reinforced, P. 202.

Weighing the Materials.—All measurements of quantities of materials were made by weighing on a triple beam scale. In order to facilitate handling of materials a hopper cart was used. The cart was run onto the scale, balanced, and the fine aggregate, cement, and coarse aggregate added in succession. Then the cart was moved over to the mixer and emptied into it.

MAKING THE SPECIMENS

Mixing the Materials.—A No. 0 Smith mixer, run at a speed of 28 to 30 r.p.m., was used in mixing the concrete batches. The mixer was wetted with sufficient water to cover the inside surface before the materials were dumped into it from the hopper cart. In most of the batches all the materials were charged into the mixer at the same time; in the "wet sand" batches the sand was first mixed with about 7 per cent of water, after which the other materials, dry, were added. The water was added to the batch after the given dry mixing period of $\frac{1}{4}$ or $\frac{1}{2}$ minute. It was admitted to the mixer at the rate of 350 pounds per minute through both spout and hopper of the machine. A gage, calibrated in pounds and attached to the supply tank, indicated the amount of water used. After the admission of the water the concrete was mixed for a period of from $\frac{1}{2}$ to $4\frac{1}{2}$ minutes, after which the machine was discharged into wheelbarrows. In emptying the mixer, care was taken to allow mortar clinging to the inner surface to drop out into the wheelbarrows, so that there would be a minimum loss in richness of the mix.

The 1:1 mortar for the shells of the specimens was mixed with a hoe in a metal tray. Air-dried sand and cement were first mixed dry until a uniform color obtained, after which the proper amount of water was worked into the batch. Care was taken to make this mortar uniform and of a soft consistency, so that no difficulty would be encountered in molding the water-tight shells. The consistency of the mortar was such that when formed into a cone 6 in. high and 3 in. in diameter it flattened on removal of the mold to a height of approximately 2 inches. The mortar was mixed at the same time that the concrete was being made, so that the specimen could be molded as soon as the concrete was ready.

Setting-up and Filling the Molds.—The molds for the PU specimens were assembled as shown in Fig. 6 (a). The dome-shaped bonnets (b) were set neck down into the cast iron pedestals (c). The bonnets were tamped full of damp sand to a level three inches below the upper rim and a circular piece of waxed paper was placed on the sand to keep the water in the concrete from soaking into it. A sheet-iron form of the proper height (for specimens 6, 12, or 18 inches thick) was then clamped onto the iron pedestal as in (a). Just before placing the concrete

and mortar, a sheet-iron cylinder, wetted to prevent sticking, was set into the mold on the wax paper. This cylinder has a cross-



Fig. 6. MOLDS FOR PERMEABILITY SPECIMENS

sectional area of one square foot; the concrete under test was placed inside it, and the mortar outside, between the cylinder and the outer form. The cylinder served merely as a division wall while the materials were being placed; after placing it was pulled out so that the mortar and concrete came into close contact.

The molds were filled as soon as possible after the concrete was mixed. A layer of 1:1 shell mortar was first tamped into the space between the cylinder and the outer mold. Equal portions from each wheelbarrow of concrete were then taken to make up a 3-inch layer inside the cylinder. Thus successive layers of mortar and concrete were placed and tamped or puddled until the required thickness was obtained. Dry mixes were tamped with an iron tamper having a face 3 inches in diameter. Medium and wet mixes were puddled with a half-inch iron rod and soupy mixes were poured into place without tamping or puddling. Two reinforcing rings were placed in the mortar shell to take up stresses developed in curing. Care was taken to lift the inner cylinder vertically to avoid distorting the concrete core, and the entire specimen was tamped and jarred in order to bring the core and the shell into good union. The top was troweled off even with the rim of the outer mold, care being taken not to brush any of the shell mortar over the concrete core. To prevent drying of the surface, a cement sack soaked in water was placed over each specimen. Figure 7 shows how the concrete cores and mortar shells appeared in cross-section fractures of PU specimens.

PUHC specimens were also molded upside down. The outside of each test piece was formed by a galvanized iron cylin-

der 37 in. high and 24 in. in diameter of construction similar to the frustrum used on the PU specimens. The inside mold consisted of a cylindrical core of damp sand held in shape by



Fig. 7—CROSS SECTIONAL FRACTURES OF PU SPECIMENS SHOWING CONCRETE CORES AND MORTAR SHELLS

heavy building paper which in turn was covered by a layer of waxed paper.

These specimens were molded one by one with a good deal of care. In filling the molds concrete was introduced in layers about 6 in. thick and very carefully puddled to prevent segregation and the formation of air pockets. Before pouring the bottom, which was on top during molding, the concrete in the side walls was allowed to settle for a half hour or more.

Three compression cylinders 6 in. in diameter and 18 in. long were made with each batch of concrete.

Removing Molds.—On the day after molding, the tops of PU specimens and the outer surfaces of PUHC specimens were scrubbed with wire brushes to remove laitance and rich mortar. Usually the specimens were removed from the molds on the second day after pouring. The sand was washed out of the bonnets and the inner surfaces of the specimens were chipped with a heavy steel bar provided with a chisel end. Figure 8

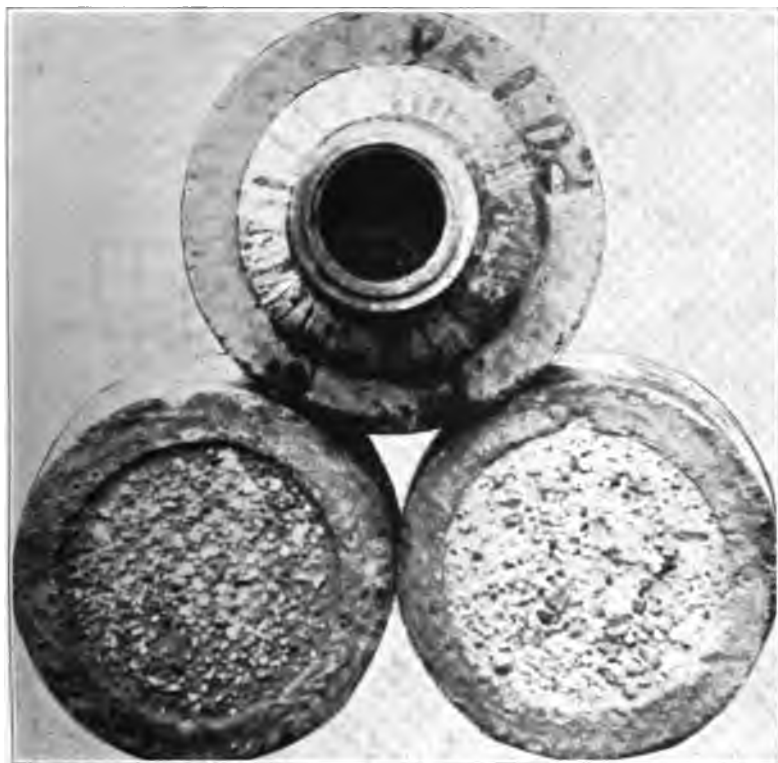


FIG. 8.—APPEARANCE OF SURFACES OF PU SPECIMENS
AFTER CLEANING

shows the appearance of the surfaces of PU specimens after cleaning. The cleaned specimens were usually filled with water, covered with sacks and sprinkled twice daily until tested.

The outer mold of each PUHC specimen was stripped on the day following fabrication and the outside of the specimen was swathed in wet sacks. Four or five days after molding, the core was removed and the interior surface of the specimen was chipped. The test pieces were then inverted, filled with water, and covered

with sacks which were kept damp throughout the curing period.

Curing and Storage Changes.—Some of the specimens, as indicated in Table 9, were cured in the hallway of the laboratory for various periods of time, while others were left in the molds for a period longer than two days. In all cases the specimens were given the same cleaning process. After passing through the sprinkling process, the "oven dried" specimens listed in Table 9 were heated in a gas oven at relatively low temperatures. These specimens were heated only during the daytime, and the rate of heating was such that the maximum temperature of 150 to 170 degrees Fahrenheit was reached in about three hours. The test-pieces were weighed whenever the storage or curing condition was changed.

The compression cylinders were usually given the same curing treatment as the permeability specimens of the same batch of concrete.

TESTING

Attachment of Specimens to Apparatus.—After the specimens had been passed through their prescribed curing process they were taken to the testing room and washed out to remove the scum which usually formed on the inner surface while they were filled with water. About 5 pounds of damp Janesville sand was placed in each PU specimen to serve as a filter for the water from the permeability tubes, and the specimens were filled with city water. They were allowed to stand for several hours to allow the water to permeate the sand and air bubbles to rise to the top of the specimen.

The water columns were attached to the specimens by means of the flange coupling, a rubber gasket being used to make a tight joint. The tubes were filled nearly to the top by forcing city water through the cocks at the bottoms of the tubes, as shown at (p) Fig. 5. As soon as a group of specimens was ready for testing, the air from the pressure tank was admitted to the distributing system, and the valves (v) at the tops of the permeability tubes were opened to transmit the pressure to the top of the water of each individual specimen. The supply was then cut off at the tank and the air pressure gage was observed in order to detect any leaks that might occur in the distributing system. If the pressure recorded by the gage did not drop appreciably in two or three minutes, the system was considered tight, and the valve at the tank was again opened.

Methods of Taking Data.—The pressure of air in the tank was maintained as nearly as possible at 40 lb. per sq. in. Initial readings of the water columns were now taken and recorded along with the time and date. The scales (s, Fig. 5) on the tubes were read by means of sliding targets (t), estimating to tenths of the smallest divisions to obtain readings to 0.0001 gallon. For specimens of low leakage a set of short tubes was sometimes used, the readings being in terms of an arbitrary scale on which the smallest divisions represented intervals of about $1\frac{1}{4}$ cubic centimeters. The readings from these tubes were later reduced to gallons by means of a conversion factor determined by calibration.

During the first half-hour the progress of the test was noted occasionally in order to make sure that the apparatus was working properly and to detect any specimens that were defective or

that leaked very rapidly. The air supply for such specimens was shut off before their tubes were emptied, in order to prevent material reduction of the air pressure on the rest of the system. Readings for the tight specimens were taken half an hour after the start of the test, then within two or three hours, and twice daily thereafter, until a period of 100 to 200 hours had elapsed. Specimens that were empty within 100 hours but that were not obviously defective or too pervious, were refilled before the water level sank below the bottom of the scale.

At the time of each successive reading the appearance of the bottom of the specimens was observed, and the condition was noted whenever dampness, wet spots, dripping, or discoloration due to moisture could be distinguished. Particular attention was given to indications that water leaked through between the core and the mortar shell.

In order to correct for fluctuations in the readings due to temperature and pressure changes, a blank specimen was made. This blank consisted of the usual bonnet set in 1:1 mortar, but the bottom of the casting was tightly sealed by means of a welded steel plate. It was attached to a permeability tube and filled with water in the usual manner, (b, Fig. 5). Since there could be no leakage through the blank, the changes in its scale readings were due entirely to temperature and pressure changes or other conditions in the permeability room. In computing leakages the fluctuations registered by the blank tube were applied to correct the readings of the specimens, for they were of the same design as the blank specimen, and would undergo the same changes due to external conditions. It was found by experiment that the reading of the blank tube increased by 0.0003 gallons when the room temperature was increased 1 degree Fahr., due to the fact that the expansion of the iron bonnet was less than the expansion of the water. Readings of the blank were taken whenever the scales for the other specimens were read.

At the time readings were taken the air pressure of the system was raised to offset any decrease which had occurred due to leakage. Usually a difference of not more than three pounds occurred in the interval overnight,—a period of about 16 hours.

The humidity of the atmosphere in the testing room was also determined at the time of each reading, since this factor affected the rate of evaporation of moisture from the free surface of the specimens and thus influenced the visible leakage from the

free surface. Before a reading was taken, an electric fan (w, Fig. 5) was run for several minutes in order to circulate the air in the testing room; the current was directed against the wick of a wet bulb thermometer and readings of both wet and dry-bulb mercury columns were taken. The humidity of the atmosphere in the room varied considerably between winter and summer; during cold weather the room was heated by steam and humidity varied from 25 to 50 per cent; in summer it was usually over 70 per cent.

COMPUTATIONS

After reducing the information on data cards to give elapsed periods of time in hours from the start of the test to the time of taking each reading and also the corresponding increments in tube readings for each individual specimen, corrected for re-filling of the tube when necessary, the elapsed time-leakage curves for each specimen were plotted. The corresponding readings for specimens of the same batch of concrete and like curing treatment were then averaged and corrected for blank readings. Curves were plotted showing the changes in the blank readings and the average leakages for each group of specimens, as time elapsed.

The average water pressure for each group of specimens was computed by averaging gage readings and adding the pressure due to the height of the water column above the top surface of the specimen. In finding the average pressure indicated by the gage, a numerical mean was taken between all readings recorded, including both the low and the high pressure whenever the pressure was raised.

From the wet-and-dry bulb thermometer readings the humidity at the time of each observation was determined by consulting a humidity table; the numerical mean of the values up to 50 hours was computed.

In order to give numerical data for the comparison of different groups of specimens and also to indicate the effect of continuation of flow, the average rates of leakage for each group of like specimens for periods of 0 to 50, 20 to 50 and 40 to 50 hours were computed. The difference between the average corrected tube reading at the beginning of the interval and at 50 hours was divided by the number of hours in the interval to give the average flow per hour. The results are the average rates of leakage in gallons per square foot per hour, since the area of the concrete core under test was one square foot.

The results for the PUHC specimens were computed in the same manner. Since the inner area (exposed to water) of these specimens is much less than the outer (exposed to air), the mean of these areas was used in reducing total leakage to leakages per square foot.

RESULTS OF TESTS

Evidence of Leakage in "Dry" Specimens.—In considering the results of these permeability tests it should be borne in mind that the tube readings represent seepage of water into the concrete and necessarily include both absorption and flow through it.

In specimens exhibiting considerable leakage at the free surface the absorption is relatively small, but in specimens appearing dry on the free surface its effect is of more importance. However, since all specimens were filled with water several hours before they were connected to the apparatus, the absorption during testing was made small. Also from the shapes of the time-leakage curves it appears that the rate of flow into the specimens was most rapid during the first ten or twenty hours of the test and became nearly constant during the 40-50 hour period. This decrease in rate of flow was undoubtedly due to the saturation of the pores and to plugging of the capillary ducts or tubes by silt and bacteria from the water. To determine the effect of bacteria in the water in retarding the rate of flow two pairs of specimens which had exhibited approximately equal permeability were re-tested, one pair being filled with water from the regular supply, the other with a solution of formaldehyde. There was, however, no appreciable difference in leakage in the second test. Apparently the bacterial action was negligible. Tests made with this apparatus, while the experiments on gravel concrete were in progress², demonstrated that evaporation from the free surface of specimens exhibiting no visible signs of leakage would account for the continuous and rather uniform rate of flow which such specimens exhibit after being under pressure twenty or more hours. Furthermore, it was demonstrated that the rate of flow was more rapid for low humidities than for high humidities; and it was also shown that leakage at the free surface was more readily detected by the eye when the humidity was high than when it was low.

From the foregoing evidence it is believed that this method of testing the permeability of concrete to water furnishes a safer index of water-tightness than methods in which leakage is measured by the amount of water escaping from the free surface or by

²See Jour. W. S. E. XIX, 846.

TABLE NO. 3
Proportions, Methods of Mixing, and Thickness of Specimen

Batch No	Specimen No	Materials			Proportions of Stone by Weight						Nominal Proportions			Machine Mixing	Mortar Analysis Cure No	Time (min) Dry Wet	Thickness of Specimen (inches)
		Cement	Sand	Stone	Proportions						Cement	Sand	Stone				
					0-1/4	1/4-1/2	1/2-3/4	3/4-1 1/2	Proportion By								
115	485-488	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V	15	X	1 1/2	6	
116	489-492	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	12	
117	493-496	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	18	
118-9	497-503	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
120	505-508	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
121	509-511	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V	16	X	1 1/2	6	
122-3	513-519	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V		X	1 1/2	6	
124	521-524	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V		X	1 1/2	6	
125	525-528	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V	15	X	1 1/2	6	
126	529-532	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
127	533-535	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
128	537-539	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
129	541-543	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V	16	X	1 1/2	6	
130	545-547	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V		X	1 1/2	6	
131	549-551	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V		X	1 1/2	6	
132	553-555	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V	15	X	1 1/2	6	
133	557-560	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
134	561-564	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
135	565-568	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
136	569-572	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V		X	1 1/2	6	
137	573-576	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V	16	X	1 1/2	6	
138-9	577-584	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V		X	1 1/2	6	
140	585-588	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09*	1	1 1/2	3g	V		X	1 1/2	6	
145	589-591	P ₂	S ₄ P	S ₄ P	0.80	0.85	1.68		1.56	332g	W	17	X	1 1/2	6		
146	592-594	P ₂	S ₄ P	S ₄ P	0.74	0.62	1.71		1.83	305g	W	18	X	1 1/2	6		
147	595-597	P ₂	S ₄ P	S ₄ P	0.92	1.24	2.28		2.56	444g	W	19	X	1 1/2	6		
148	598-600	P ₂	S ₄ P	S ₄ P	1.10	1.40	2.90		3.60	540g	W	21	X	1 1/2	6		
149	601-603	P ₂	S ₄ P	S ₄ P	1.10	1.40	2.90		3.60	540g	W		X	1 1/2	12		
150	604-606	P ₂	S ₄ P	S ₄ P	1.10	1.40	2.90		3.60	540g	W		X	1 1/2	18		
151-3	607-615	P ₂	S ₄ P	S ₄ P	1.10	1.40	2.90		3.60	540g	W		X	1 1/2	6		
154	616-618	P ₂	S ₄ P	S ₄ P	1.10	1.40	2.90		3.60	540g	W		X	1 1/2	6		
155	619-621	P ₂	S ₄ P	S ₄ P	1.10	1.40	2.90		3.60	540g	W		X	1 1/2	6		
156	622-624	P ₂	S ₄ P	S ₄ P	1.10	1.40	2.90		3.60	540g	W		X	1 1/2	6		
163	647-650	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V	15	X	1 1/2	6	
164	651-654	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V	16	X	1 1/2	6	
165	PUIHC 13	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V		X	1 1/2	6	
166	PUIHC 14	P ₂	S ₄ P	S ₄ P	0.14	0.61	0.91	1.09	1	1 1/2	3g	V		X	1 1/2	6	
167-8	PUIHC 15-16	P ₂	S ₄ P	S ₄ P	1.00	0.86	2.30		1	2	4g	V	15	X	1 1/2	6	
169	PUIHC 17-18	P ₂	S ₄ P	S ₄ P	0.19	0.83	1.25	1.50	1	2	4g	V		X	1 1/2	6	
171	PUI635-637	P ₂	S ₄ P	S ₄ P	0.84	1.16	2.28		2.72	418g	W	20	X	1 1/2	6		
172	PUI638-660	P ₂	S ₄ P	S ₄ P	0.90	1.40	2.85		3.85	515g	W	23	X	1 1/2	6		
176	PUI670-672	P ₂	S ₄ P	S ₄ P	1.00	1.33	2.90		3.8	52g	W	22	X	1 1/2	6		
177	PUI673-675	P ₂	S ₄ P	S ₄ P	1.00	1.30	2.89		3.8	52g	W		X	1 1/2	12		
178	PUI676-678	P ₂	S ₄ P	S ₄ P	1.00	1.30	2.90		3.8	52g	W		X	1 1/2	18		
179	PUI679-681	P ₂	S ₄ P	S ₄ P	1.00	1.33	2.90		3.8	52g	W		X	1 1/2	6		
180	PUI682-684	P ₂	S ₄ P	S ₄ P	1.00	1.33	2.90		3.8	52g	W		X	1 1/2	6		
181	PUI685-687	P ₂	S ₄ P	S ₄ P	1.00	1.33	2.90		3.8	52g	W		X	1 1/2	6		
182	PUI688-690	P ₂	S ₄ P	S ₄ P	1.00	1.33	2.90		3.8	52g	W		X	1 1/2	6		
183	PUI691-693	P ₂	S ₄ P	S ₄ P	1.00	1.33	2.90		3.8	52g	W		X	1 1/2	6		
184	PUI694-697	P ₂	S ₄ P	S ₄ P	1.00	1.30	2.90		3.8	52g	W		X	1 1/2	6		
185	PUI698-701	P ₂	S ₄ P	S ₄ P	1.00	1.30	2.90		3.8	52g	W		X	1 1/2	6		
186	PUI702-705	P ₂	S ₄ P	S ₄ P	1.00	1.30	2.90		3.8	52g	W		X	1 1/2	6		
187	PUI706-708	P ₂	S ₄ P	S ₄ P	1.00	1.30	2.90		3.8	52g	W		X	1 1/2	6		
188	710-712	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V	24	X	1 1/2	6	
189	713-715	P ₂	S ₄ P	S ₄ P	0.19	0.81	1.22	1.46	1	2	4g	V	25	X	1 1/2	6	

*Note:--Specimens No. 485 to 508 and 710 to 715, inclusive, contain 0.02 parts of material larger than 1 1/4 inch

visual examination of that surface. It also appears probable that the proportion of the flow into the concrete which is absorbed by it, is relatively small after the first twenty hours of the test period. Therefore, it seems evident that, for specimens exhibiting little if any visible signs of leakage, the average rates of flow for the 20-50 and 40-50 hour periods are better indices of true leakage than the values of the 0-50 hour period.

Effect of Age on Permeability.—One batch of 1:1½:3 and one batch of 1:2:4 concrete were tested to show the

TABLE NO. 4

Effect of Age on the Permeability of Concrete

Materials:— *Universal Portland Cement (P); Jamesville Sand (S₄); and Madison Stone (S₆, P)*

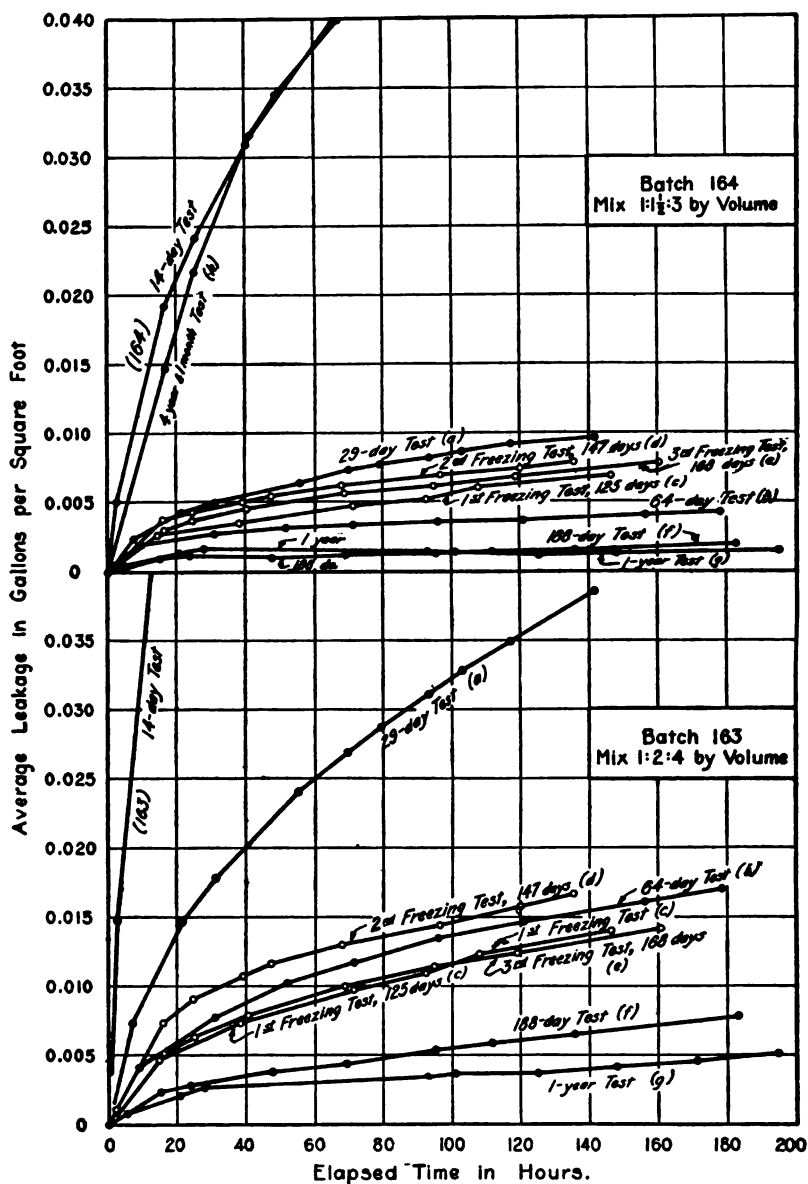
All specimens were 6" thick.

All concrete was machine mixed by a No. 0 Smith Mixer, ½ minute dry and 1½ minutes wet. Specimens were cured in the molds for two days, then filled with water and sprinkled twice a day until the first tests were made, thereafter, between testing periods, they were allowed to stand in the permeability room filled with water. The inside surfaces of the specimens were cleaned before each test. Date Made:—Dec. 24, 1914.

Batch No.	Specimen No.	Mix	Wt. Cement	Wt. Sand	Wt. Stone	Date Tested	Age in Days	Avg. Rate of Absorption per Sq. Ft. for 30 Hr.	Water Pressure lb. / sq. in.	Avg. Rate of Leakage in Gal. per Sq. Ft. per Hr. for the Interval in Hours			Remarks
										0-50	20-50	40-50	
163	647-650	1:2:4	94	96	1915	Jan 7	14	54	43.9	0001847	0001194	0001112	649 not included. Leak in base of glass tube.
163a	647-650					Jan 22	29	37	44.6	000452	000232	000253	
163b	647-650					Feb 26	64	42	45.0	000199	000133	000112	
163c	647-650					Apr 28	125	70	43.6	000164	000096	000070	First Freezing
163d	647-650					May 20	147	80	45.2	000235	000121	000097	Second Freezing
163e	647-650					Jun 10	168	80	44.9	000172	000093	000073	Third Freezing
163f	648-649					Jul 1	188	75	45.0	000077	000045	000039	
163g	648-649					Dec 23	364	65	45.0	000058	000030	000014	
164	651-654	1:1½:3	96	98	1915	Jan 7	14	54	44.5	000696	000455	000391	
164a	651-654					Jan 22	29	37	44.7	000121	000064	000041	
164b	651-654					Feb 26	64	42	45.2	000061	000035	000024	
164c	651-652					Apr 28	125	70	43.6	000078	000037	000036	First Freezing
164d	651-652					May 20	147	80	44.0	000110	000052	000045	Second Freezing
164e	651-652					Jun 10	168	80	45.0	000097	000052	000040	Third Freezing
164f	653-654					Jul 1	188	75	45.1	000020	000016	000016	
164g	653-654					Dec 23	364	65	45.0	000035	000017	000008	
164h	651-652					Feb 3	152	55	40.7	000700	000583	000410	4 yr. 1 month test

*Cured Outdoors for 4½ Years.

effect of age on permeability. The results of the tests are given in Table 4. Figure 9 shows the average time-leakage curve for each group of specimens at each test period. None of the specimens showed any discoloration or dampness of the bottom during the 14-day or subsequent tests. The concrete of both batches was so dense that the rate of flow of water into the



specimens was not high enough to cause visible dampening of the free surface.

The table and the curves show that the rate of flow, at any given age, was consistently less for the 1:1½:3 concrete than for the 1:2:4 mix. The computed rates of leakage for periods of 0-50, 20-50, and 40-50 hours generally are less for each successive test. However, the decrease is not large after 64 days for the 1:2:4 and after 29 days for the 1:1½:3 mix. Thus it appears that the imperviousness of a medium or rich mix is not increased materially with age after it has been given a proper curing period. A comparison of batches 115, 133, and 163 and 122, 137 and 164 in Table 6 shows that the rate of flow over a given period is about the same for specimens that have been previously subjected to water pressure as for similar specimens of the same age that are tested for the first time.

Effect of Freezing and Outdoor Curing.—Some of the specimens of batches 163 and 164 were subjected to freezing and weather conditions. The results of tests on these specimens are also given in Table 4 and Fig. 9. In submitting the specimens to freezing conditions they were placed in the chamber of a refrigerator which was operated by means of an ammonia compressor. The test pieces were stored in the permeability room and kept full of water until the start of the freezing period, when they were emptied. The first run in the freezer extended over four days; the average temperature for the period was 3°F., the second and third days being at 0 degrees. The lowest temperature obtained was -10° on the last day of the freezing process. After removal from the refrigerator the specimens were allowed to thaw out in the laboratory; they were submitted to water pressure and tested one day after removal from the freezer. A second and third run in the freezer followed, the treatment of the specimens being the same as before. The length of the second run was 6 days, the lowest temperature being 10°F. The average temperature recorded was 12°F. The third freezing period lasted only three days at an average temperature of 5°F., the lowest temperature was 2°F.

The rates of flow for specimens from both batches after successive freezing treatments are given opposite (c), (d), and (e), in Table 4. The results do not indicate any notable change in the permeability of the specimens due to freezing. However, comparing the leakages obtained after freezing with values that

might be expected at the same age for specimens not frozen by interpolating between the results of the 64 and 188 day tests, it is apparent that the effect of freezing was to increase slightly the rate of flow.

After the last freezing test specimens No. 651 and 652 of the 1:1½:3 batch were placed out of doors resting on their curved surfaces where they were subject to all weather conditions. They remained outside for three and one-half years. They were taken inside in February 1919, and were thawed out and tested. The rates of leakage for these specimens were about the same as for the same set at 14 days, showing that the effect of the weathering agencies was to increase considerably the permeability.

TABLE NO.5
Effect of Thickness on the Permeability of Concrete

Materials:—Universal Portland Cement (P_3), Janesville Sand ($S_{d,14}$, $P-S_{d,17}$, P) and Madison Stone ($S_{d,17}$).
All concrete was machine mixed ½ min. dry and 1½ min. wet.
Specimens were cured in the molds for 2 days, then filled with water and sprinkled twice a day until tested. Age at test was 28 days.

Batch No	Specimen No	Mix by Vol. or Wt.	Percent Water		Compressive Strength, P.S.I.	Ave. % Humidity for 50 hr.	Water Press. lb./sq. in.	Thickness-in.	Ave. Rate of Leakage in Gal. per Sq. Ft. per Hr. for the Interval in Hours			Remarks
			Nominal	Moisture					0-50	20-50	40-50	
115	485-488	1:2:4g	9.4	10.2	2547	68	42.8	6	.000218	.000117	.000080	Cement - P_3 Sand - $S_{d,14}$ P
116	489-492	1:2:4g	9.4	10.4	2656	70	43.6	12	.000178	.000113	.000082	
117	493-496	1:2:4g	9.4	10.4	2643	70	43.6	18	.000334	.000200	.000170	
148	598-	1:3:5:4g	7.4	7.6	1218	41	44.4	6	.001364	.001060	.000901	One spec. only, other two leaked too fast to run. Pockets and air holes
149	601-603	1:3:5:4g	7.4	7.8	1366	37	44.2	12	.001544	.001286	.001065	
150	604-606	1:3:5:4g	7.4	7.8	1086	37	43.9	18	.001588	.001343	.001260	
176	670-672	1:3:5:2g	7.4	8.2	1633	67	44.3	6	.002033	.001420	.001635	All discolored Leaked badly Leaked badly
177	673-675	1:3:5:2g	8.23	8.9	1367	63	42.7	12	.004448	.003970	.003860	
178a	676-678a	1:3:5:2g	8.11	8.7	1266	72	43.1	18	.004620	.003888	.003570	

* Proportioned by weight

Effect of Thickness on Permeability.—Specimens 12 and 18 inches thick were made of 1:2:4 (by volume) and 1:9 (by weight) concrete to compare with similar 6-inch specimens. The results of these tests are recorded in Table 5. The data available is not complete enough to lead to definite conclusions, but it is apparent that increasing the thickness of the concrete did not materially affect the rate of flow into the specimens. Numerically the average leakage rates are somewhat greater for the 12 and the 18-inch specimens than for those made 6 inches thick. Probably this is due to the greater amount of pore space in the thicker specimens and to compression of the pores

in the exposed surface of the 6-inch specimens due to bending under testing pressures.

Effect of Proportion of Cement on Permeability.—Table 6 summarizes the results of tests on batches of concrete of medium consistency containing different proportions of cement. In these batches the cement varied from 10 to 18.5 per cent of the total weight of dry materials. No visible indications of leakage were noted for any of the specimens from batches

TABLE NO. 6
Effect of Proportion of Cement and Grading the Aggregate
on the Permeability of Concrete

Materials:—Universal Portland Cement (P in batches 115 to 133; P in others), Jamesville Sand (S₄P in batches 115 to 133; S₂ P in batch 188; S₃ P in batch 189; S₄P in batches 171 & 172; S₄P in batches 176 & 184; S₄P in others), and Madison Stone (S₄P).

All concrete was machine mixed $\frac{1}{2}$ min. dry and $\frac{1}{2}$ min. wet.

Specimens were cured in the molds for 2 days, then filled with water and sprinkled twice a day until tested. All specimens were 6" thick. Age at test was 28 days.

Fig. 2. Specimens tested at 15 days.										Fig. 3. Test results at 15 days.									
Batch No.	Specimen No.	Mix		Percent Cement by weight	Cement No.	Fineness Modulus	Percent Water		Compressive Strength (lb./sq. in.)	* Density	Ave. Rate of Leakage, gal. per sq. ft. per hour for 50 hr.	Ave. Rate of Leakage in Gal per Sq Ft per Hour for the Interval in Hours			Remarks				
		By Vol	By Weight				Moisture	0-50				20-50	40-50						
148	538		130-540	11	21	5.84	74	76	1218	0.750	41	4.44	.001364	.001060	.000901	500-500 tested to test to be tested			
176	670-672		138-532	11	22	5.73	74	82	1633	0.807	67	4.43	.002033	.001419	.001035	Air escaping			
184	634-635		138-532	11	22	5.73	78	81	1508	0.784	71	42.2	.001728	.001257	.001060				
172	658-660		130-535	11	23	5.78	73	69	1419	0.803	60	43.8	.001329	.000357	.000887	635 would not hold water			
147	535-537		125-544	14.3	19	5.98	77	79	1882	0.787	41	44.5	.000684	.000341	.000241	Air holes in cores			
171	655-657		127-548	14.3	20	5.78	76	93	2104	0.820	68	43.8	.000534	.000249	.000239				
115	485-488	1-2-4	122-532	16.9	15	5.76	94	102	2547	0.79	68	42.8	.000218	.000117	.000030				
133	557-558	1-2-4	122-537	16.9	15	5.76	94	96	1797	0.793	80	41.6	.000340	.000205	.000152				
183	647-650	1-2-4	122-537	16.9	15	5.81	94	96	2047	0.792	37	44.6	.000452	.000292	.000253	* Specimens were previously tested at 15 days			
188	710-712	1-2-4	121-530	17.2	24	5.80	68	94	2478	---	71.5	44.6	.000264	.000178	.000120				
189	712-715	1-2-4	119-539	18.1	25	6.03	75	79	2101	---	71.5	44.6	.000357	.000203	.000165				
145	589-591		115-532	20.5	17	6.13	83	---	1876	0.770	41	43.7	.000243	.000141	.000101				
146	592-594		118-535	20.5	18	6.08	83	---	2763	0.790	65	40.3	.000339	.000254	.000297	Air holes			
122	513-516	1-1-3	1105-277	22.6	16	5.76	96	103	3046	0.789	75	42.5	.000132	.000074	.000055				
137	573-574	1-1-3	1165-277	22.6	16	5.81	95	---	2730	0.786	41	44.2	.000140	.000080	.000062				
164	657-654	1-1-3	1165-277	22.6	16	5.80	96	98	2994	---	791	54	0.000121	.000064	.000041	* See above			

* Density based on volume of set cylinders.

with 12.5 per cent of cement or richer, but all of the specimens with 10 per cent were moist in spots, discolored, or actually dripping. These data are represented by the heavy lines and points in Fig. 10, which show that the proportion of cement greatly influenced the rate of leakage. It will be observed that any increase in the cement content above 15 or 16 per cent of the total weight of dry materials did not affect a corresponding increase in water-tightness, for these materials and methods of mixing and curing. The fine-lined curves of Fig. 10 are taken from Fig. 14 of the report in the *Jour. W.S.E.* By comparing the two sets of curves it is observed that the broken stone con-

creted exhibited more than twice the rate of flow of the gravel concretes having like ratios of cement to aggregate.

The lowest rate of leakage for which there was any visible indication of flow was 0.00011 gallons per sq. ft. per hr. This result was observed in the gravel concrete tests. If we consider concrete exhibiting 0.0001 gal. (about 3 drops) per sq. ft. per hr. to be water-tight, then the mixes of stone concrete having a cement-aggregate ratio of 0.17 by weight, were water-tight, whereas with the gravel concrete mixes a ratio of 0.14 gave the same result.

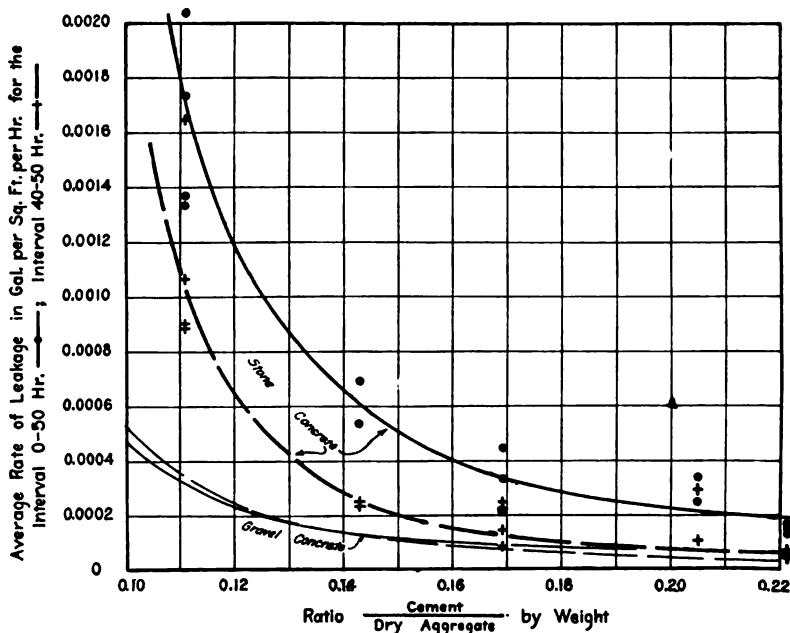


Fig. 10.—Effect of the Proportion of Cement on the Permeability of Stone Concrete

Effect of Grading the Aggregate on Permeability.—

The importance of so combining the aggregate in a concrete mix as to secure the highest density possible has been recognized for some time, and many tests have been made with graded sands and coarse materials to study the influence of density. In this series of tests the stone was screened into five sizes and recombined in proportions that would give a mechanical analysis curve approximating a straight line. The different sizes of stone

were kept separate until parts were weighed out for each batch at the time of mixing; this was done to avoid the segregation which would occur in a mixed stock pile of different sizes.

The results of tests on concretes with graded aggregates are given in Table 6. Figures 1 and 2 show the mechanical analysis curves for the different mixes. Fuller's empirical curve,³ for stone concrete with particles ranging in size up to $1\frac{1}{4}$ inch is drawn, for comparison, on the diagrams. Graded coarse aggregate was used in all of the batches; the sand was not graded except in batches 146, 172 and 189. Racine sand, which is coarser than the Janesville material, was used in batch 188. A test of a concrete deficient in the medium sizes of sand was made on batch 189, which contained red granite screenings as the fine aggregate.

At the time these tests were made, the value of the fineness modulus in estimating the character of the grading in an aggregate was not known, but the values for the mixed aggregates used have been calculated and inserted in Table 6. Inspection of these fineness modulus values shows that these mixes were, in nearly all cases, more coarsely graded than Abrams⁴ recommends.

The mechanical analysis curves for the 1:2:4 and the 1:1½:3 mixes are given in Fig. 1, No. 15 and 16. The latter is the higher of the two since the 1:1½:3 mix contains the higher percentage of cement. The ratio of sand to the different sizes of stone is the same in both mixes, as indicated by the similar shapes of the upper parts of the curves. Since these curves are fairly straight in the upper portion and higher than the Fuller curve due to high cement content, the concretes would be expected to be relatively impervious; inspection of the results from batches 115, 113, 163, 122, 137 and 164 shows that the rates of flow for the 40-50 hour interval are generally less than 0.0002. Both of these mixes of broken stone concrete exhibited considerably greater rates of flow than similarly proportioned and similarly graded mixes of gravel concrete previously reported in the *Jour. W.S.E.*

Analysis curves 24 and 25, Fig. 2, show the percentages of sizes in 1:2:4 mixes when Racine sand and red granite screenings, respectively, replaced the Janesville sand. The rates of flow obtained from specimens of batches 188 and 189 with these

³Concrete, Plain and Reinforced, p. 202.

⁴See p. 12, Bulletin No. 1, Structural Materials Research Laboratory, Lewis Institute, Chicago.

materials were not much different from the results given by batches 115, 133 and 163.

Analysis curves 17 and 18 correspond to the 1:5 (by weight) broken stone mixes which were graded to approach the Fuller curves. These mixes, although they exhibited no visible evidences of leakage from the free surface, gave three times the rate of flow shown by the gravel concrete mixes of like proportions and grading, previously reported.

Specimens of 1:9 broken-stone concrete, see analysis curves 21, 22 and 23, showed far greater variations in leakage than similarly graded and proportioned specimens of gravel concrete in previous tests.

From a comparison of the data from these tests with those from gravel concrete mixes it appears that the superiority of the gravel concrete was largely due to the greater density which resulted from the use of aggregate having rounded particles.*

Experience in these two series of tests shows that mixes graded to conform to the Fuller curve are generally tight and dense. Mixes having curves with the lower portions below the Fuller curve are too harsh to be worked into a water-tight concrete. More recent tests at this University of mixes varying in grading but having a constant fineness modulus gave widely divergent rates of flow indicating that the fineness modulus is a less satisfactory index of water-tightness than of strength. Although the number of tests is insufficient to establish definitely the best method of proportioning to secure a water-tight concrete, it appears from what has been done that lean mixes graded to Fuller's curve and sufficiently mobile to insure against pocketing during placement can be made water-tight. Whether true economy lies in using a lean mix containing a carefully graded aggregate or in the use of an arbitrarily proportioned mix richer in cement will depend upon the quantity of concrete to be placed, the cost of cement, and the extra cost of securing aggregate graded to meet the requirements of Fuller's curve.

The plotted points and heavy-lined curves in Fig. 11 show

*If one compares the densities in the two series of tests it should be noted that the densities of the gravel concretes were based on measurement prior to setting and were, therefore, less than if measured on set concrete.

the relation between the 0-50 and 40-50 hour leakage rates and the ratio,⁶

$$\frac{\text{absolute volume of cement}}{\text{volume of air + water voids}}$$
 per unit volume of concrete
for the broken stone concretes. A decrease of rate of flow was generally obtained by raising the value of the ratio, but the

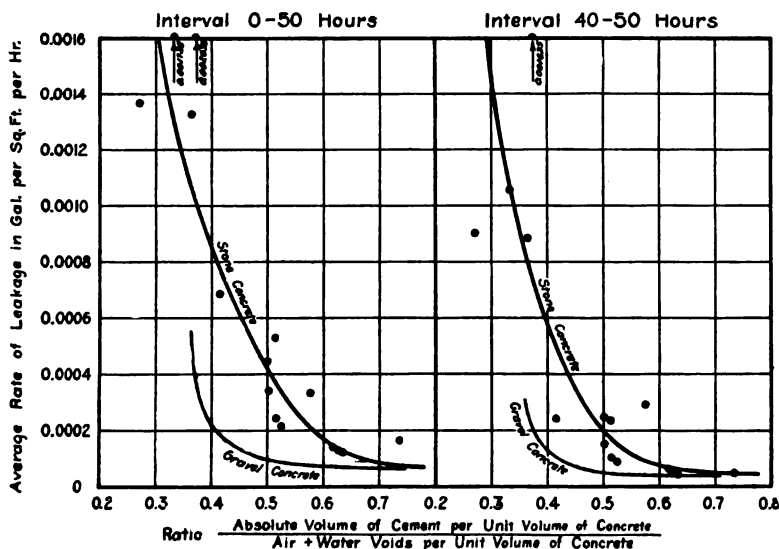


Fig. 11.— Curves Showing the Relation between the Permeability and the Ratio of the Volume of Cement to the Volume of Voids in Stone Concrete

curves indicate that it would probably not be necessary, for ordinary purposes, to make this ratio greater than 0.6 or 0.7 for broken stone concrete. By comparing the heavy-lined curves with the light-lined curves reproduced from the paper on gravel concrete it will be observed that the gravel concrete of like consistency to the stone concrete having a cement-voids ratio of 0.5 gave equally tight concrete.

The relation between the cement-voids ratio and the compressive strength of stone concrete is shown by the points and heavy

⁶Talbot has recently studied the effect on the compressive strengths of mortars and concretes due to changes in the cement-voids and "cement-space" ratios and devised methods of proportioning based upon these relations. See Proc. A. S. T. M., Vol. 21, p. 940.

curved line in Fig. 12. The fine-lined curve gives the relation for gravel concrete. These curves indicate that the compressive strength increases approximately in direct ratio to the cement-voids ratio.

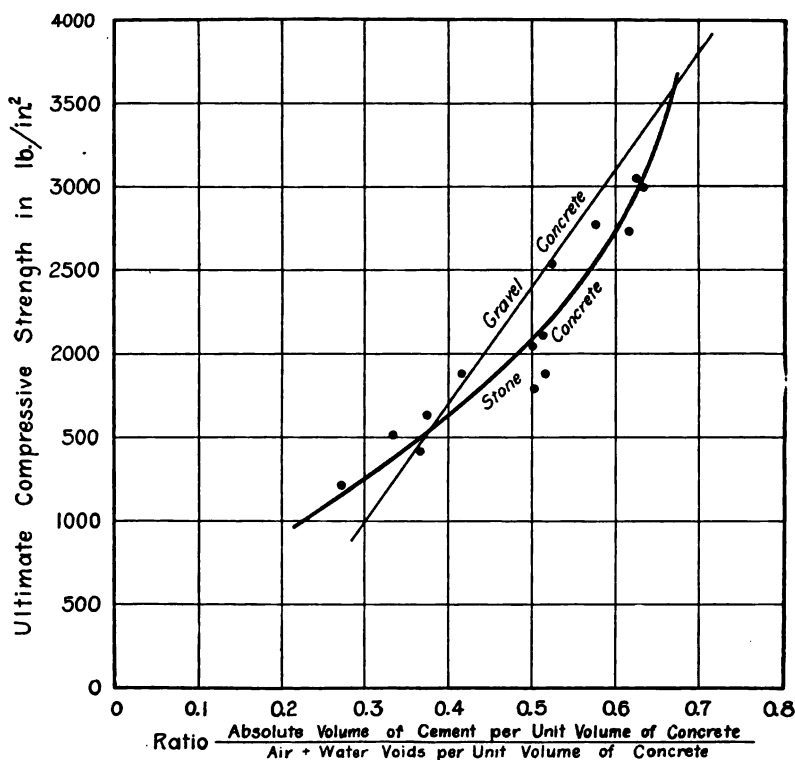


Fig. 12.— Relation between the Compressive Strength and the Ratio of the Volume of Cement to the Volume of Voids in Stone Concrete

Effect of Time of Mixing on Permeability.—To determine the effect of length of wet and dry mixing period and of the use of wet sand, tests were made on batches of $1:1\frac{1}{2}:3$, $1:2:4$, and $1:9$ proportions. A summary of the results of tests is given in Table 7, and the average time-leakage curves for each batch are plotted in Fig. 13. The tabulated results show that, for $1:1\frac{1}{2}:3$ proportions and a mixing period of $\frac{3}{4}$ to 5 minutes, the

rate of flow of water into the concrete was not affected by total time of mixing. The batches made with wet sand, however, were somewhat more permeable than those made with dry sand.

For leaner mixes the length of the mixing period had a marked effect. It is seen from the data of the 1:2:4 batches that concrete

TABLE NO. 7.

Effect of Time of Mixing on the Permeability of Concrete

Materials:—Universal Portland Cement (P in Batches 115, 122, 125, 129, 132, 133, & others), Janesville Sand (S₄₀ P in batches 115 to 133; S₄₀ P in batches 176-179-180; S₄₀ P in others) Madison stone (S₆₀ P) All concrete was machine mixed. In all wet sand mixes dry sand was put into machine and thoroughly mixed with about 7% of water before the cement and gravel were added.

All specimens were 6 in. thick.

Specimens were cured in the molds for 2 days, then filled with water and sprinkled twice a day until tested. Age at test was 28 days.

Batch No	Specimen No	Mix Vol. of Wt.	Percent Water	Time of Mixing (Min)		Condition of Sand	Compressive Strength (psi)	Avg. Humidity per 30 Hr.	Water Pess. Wt. in	Avg. Rate of Leakage in Gal per Sq Ft per Hr for the Interval in Hours			Remarks
				Dry	Wet					0-50	20-50	40-50	
129	541-543	1 1/8-3g	95	1/2	1/2	*	2915	71	412	.000132	.000099	.000089	
131	549-551	1 1/8-3g	95	1/2	1/2	Wet	2702	70	424	.000141	.000085	.000069	
122	507-516	1 1/8-3g	96	103	1/2	*	3046	75	42.5	.000132	.000074	.000053	
137	573-574	1 1/8-3g	9.5	1/2	1/2	*	2730	41	44.2	.000140	.000080	.000062	
164	651-654	1 1/8-3g	96	89	1/2	*	2994	37	44.7	.000121	.000064	.000041	Specimens previously tested at 14 days
130	545-547	1 1/8-3g	95	1/2	4 1/2	*	3206	70	42.4	.000117	.000074	.000060	
132	553-555	1 1/8-3g	96	1/2	4 1/2	Wet	2492	80	41.5	.000145	.000107	.000094	
125	525-528	1 2/4-3g	94	1/2	1/2	*	1792	77	42.8	.000528	.000327	.000270	
127	533-535	1 2/4-3g	94	1/2	1/2	Wet	1746	85	42.6	.000628	.000347	.000296	Floor damp No blank correction
115	485-488	1 2/4-3g	93.8	102	1/2	1/2	2547	68	42.8	.000218	.000117	.000090	
133	557-558	1 2/4-3g	93.4	1/2	1/2	*	1797	80	41.6	.000340	.000205	.000152	
163	647-650	1 2/4-3g	94	95	1/2	1/2	2047	37	44.6	.000452	.000292	.000253	Specimens previously tested at 14 days
126	529-532	1 2/4-3g	94	1/2	4 1/2	*	2111	85	42.9	.000290	.000195	.000175	Floor damp
128	537-539	1 2/4-3g	94	1/2	4 1/2	Wet	1952	71	41.4	.000403	.000297	.000256	
154	616-618	1 3/8-5 1/2	73.8	783	1/2	1/2	744	41	43.2	.016860			From 0 - 160 mm Test only over 160 min
156	622-624	1 3/8-5 1/2	73.8	783	1/2	1/2	1093	54	44.0	.012380	.012470	.018210	Air Holes; Pockets
148	598	1 3/8-5 1/2	74	76	1/2	1/2	1218	41	44.4	.001364	.001060	.000901	
151	507	1 3/8-5 1/2	70	7.5	1/2	1/2	1086	38	43.1	.002237	.002174	.002203	See Pockets
155	619-621	1 3/8-5 1/2	74	76.5	1/2	4 1/2	1432	53	44.0	.000627	.000390	.000336	Full of Air Holes
179	679-681	1 3/8-5 1/2	74	794	1/2	1/2	1398	66	42.8	.007090	.006250	.008170	All Bottoms Wet
176	670-672	1 3/8-5 1/2	74	824	1/2	1/2	1633	67	44.3	.002030	.001419	.001635	Discolored & Damp
184	694-695	1 3/8-5 1/2	74	805	1/2	1/2	1508	71	42.2	.001728	.001257	.001060	Discolored
180	682-684	1 3/8-5 1/2	82	922	1/2	4 1/2	1748	73	45.0	.001126	.000826	.000645	All "

* Mix by weight * Air-dry sand used

mixed two minutes leaked less rapidly than the specimens of the three-quarter minute batch. Similarly there was a gain of water-tightness when the mixing period was increased to five minutes. The effect of wet sand in increasing permeability was also more notable in this set of specimens than in the 1:1 1/2:3 batches.

The 1:9 batches proportioned 1:3.6:5.4 show, uniformly, that lengthening the mixing period decreased the permeability.

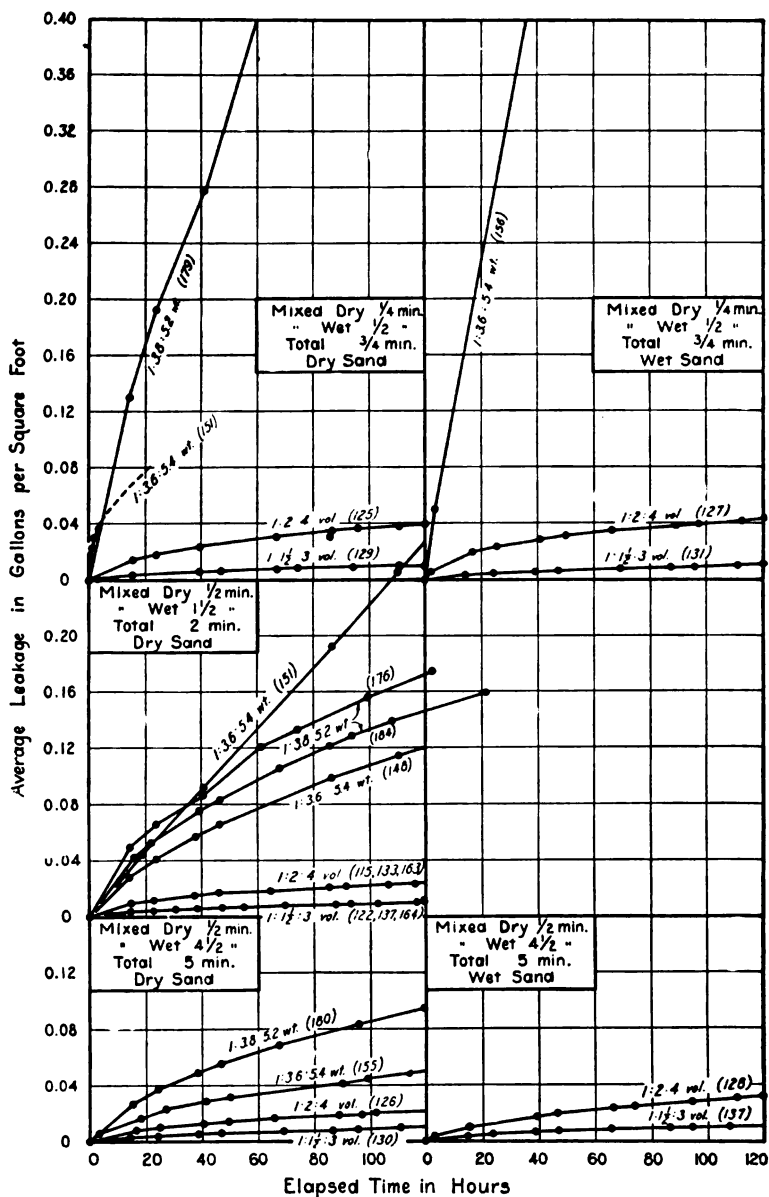


Fig.13.—Effect of Time of Mixing on the Permeability of Stone Concrete Made from Wet and Dry Sand

The same effect is shown by the data from the 1:3.8:5.2 specimens.

Effect of Consistency on Permeability.—The results from sixteen batches of concrete, as recorded in Table 8, show the variation of the rates of flow of water into specimens of dry, medium, wet, and soupy consistency and of four different pro-

TABLE NO. 8.

Effect of Consistency on the Permeability of Concrete

Materials—Universal Portland Cement (P); Janesville Sand (Sd_4P in batches 148-151-152, Sd_7P in batches 181-182, $Sd_{10}P$ in all others); Madison Stone (Se, P).

All concrete was machine mixed $\frac{1}{2}$ min. dry and $\frac{1}{2}$ min. wet

All specimens were 6 in. thick

Specimens were cured in the molds for 8 days, then filled with water and sprinkled twice a day until tested. Age at test was 1 mo.

Batch No.	Specimen No.	Mix by Vol or Wt.	Percent Water	Compressive Strength, lb./sq. in.	Age at Test, Days	Wetted Surface, sq. in.	Density, lb./cu. ft.	Ave. Rate of Leakage in Gal. per Sq. Ft. per Hr. for the Interval in Hours			Remarks
								0-50	20-50	40-50	
121	509-511	1:1½-3g	75	864	5497	79	42.5	.000165	.000114	.000047	
122	513-516	1:1½-3g	97	103	3046	75	42.5	.000132	.000074	.000035	
123	517-519	1:1½-3g	110	118	2858	75	42.5	.000216	.000135	.000104	
124	521-524	1:1½-3g	125	129	1686	77	42.6	.001156	.000977	.000960	Floor wet
118	497-499	1:2-4g	772	930	2961	68	44.7	.000328	.000173	.000160	
115	485-488	1:2-4g	938	102	2547	68	42.8	.000218	.000117	.000090	
119	501-503	1:2-4g	108	125	1333	68	43.8	.000896	.000580	.000510	
120	505-508	1:2-4g	121	131	1011	78	42.2	.009764	.009730	.009210	505-506, Bottoms wet 507, Discolored
151	607	1:36-54g	700	747	1086	38	43.1	.002237	.002174	.002203	Full of bad pockets 608-610, Cells not so tested
148	598	1:36-54g	737	757	1218	41	44.4	.001364	.001060	.000901	Air holes and pockets 599 & 600, Tested for first All damp
152	610-612	1:36-54g	925	972	872	38	43.3	.013960	.013600	.011790	
153	613-615	1:36-54g	113	117	638	40	42.2	.074940	Run for	115 min. only	All wet on bottoms
181	686-687	1:38-52g	675	761	888	73	43.1	.007640	.006687	.006190	683 would not hold water Bottoms all wet
176	670-672	1:38-52g	744	824	1633	67	44.3	.002033	.001419	.001635	Discolored and Damp
182	688-690	1:38-52g	975	103	1002	71	42.0	.037208	.033980	.031700	Bottoms all wet
183	691-693	1:38-52g	110	116	636	—	44.6	.341200	Run for	35 min. only	Specimens dripping

* Proportioned by weight

portions. The dry concrete test pieces (batches 121, 118, and 151) were tight enough to hold the pressure of water in testing, but those of 1:9 proportions (batch 181) leaked very rapidly. Difficulties were encountered in maintaining the pressure on the batches of soupy consistency, because the rate of flow for these specimens was much more rapid than for the drier mixes of the same richness. The 1:9 mixes of soupy consistency (batches 153 and 183) leaked so rapidly that constant pressure could not be maintained, and therefore they were not tested for a full period of 50 hours.

In all cases the specimens of medium consistency (batches 122, 115, 148, and 176) were more impervious than the others of the same richness.

From Table 8 one may compute the ratio of the volume of water (w) to the volume of cement (c) for each mix. Using nominal percentages of water and considering mixes which exhibited an average rate of flow less than 0.0001 gal. per sq. ft. per hr. during the 40-50 hr. period to be water-tight, it will be found that the 1:1½:3 mixes were water-tight for values of $\frac{w}{c}$ between 0.62 and 0.91. The 1:2:4 mixes were tight for $\frac{w}{c} = 0.99$; and the 1:9 mixes leaked least for $\frac{w}{c} = 1.12$. It is probable that the 1:2:4 mixes would have been water-tight for values of $\frac{w}{c}$ somewhat below and somewhat above 0.99, say from 0.90 to 1.05. From these tests and others not reported herein it appears that the water-cement ratio⁷ $\frac{w}{c}$ has an important bearing on the water-tightness of concrete, the maximum water-tightness for a given mix being obtained for a limited range in $\frac{w}{c}$. For values of $\frac{w}{c}$ above or below this range the water-tightness is reduced. For rich mixes, like 1:1½:3, the range in $\frac{w}{c}$ corresponding to water-tightness is wider than for leaner mixes, like 1:2:4.

Inasmuch as these tests were conducted before the slump or flow table tests had been devised no measurements of the consistency were made. However, from the behavior of these mixes and of other mixes of similar aggregates it is believed that a consistency corresponding to a slump between one and three inches, as measured by a 6x12-in. cylinder, or a relative flowability between 170 and 185, as determined on the Bureau of Standards flow table, gives the most impervious concrete with the least cement, other factors remaining constant. In the field, owing to conditions under which the concrete must be placed, a more fluid consistency may be required. For such constructions it is believed that the requisite fluidity may be obtained and the proper degree of water-tightness secured by increasing both the proportion of water and the proportion of cement in the same ratio.

⁷This factor was so named by Abrams who discovered the important relation it bears to the mechanical properties of concrete. See *Bulletin No. 1, Structural Materials Research Laboratory*.

For example:—Assume tests have indicated that a 1:7 mix (by weight) made with certain materials and mixed with 8.5 per cent of water gives a concrete which is sufficiently water-tight and strong for a given purpose, but not sufficiently fluid. The next step would be to make a series of trial mixtures each containing an increasing amount of cement and water but each having the same water-cement ratio as the previously-cited mix until a concrete having the desired mobility is obtained.

TABLE NO. 9.

Effect of Curing Conditions on the Permeability of Concrete
Materials:—Universal Portland Cement (P_2 in batches 133 and 140; others P_3); Janesville Sand ($S_{d_2}P$ in batches 133 and 140, $S_{d_1}P$ in batches 184 to 187; others $S_{d_2}P$), and Madison Stone ($S_{d_2}P$).
All concrete was machine mixed. All specimens were 6 in. thick.
Specimens were cured 2 days in molds, then cured in accordance with the schedule in columns headed "Days cured." Read from left to right.

Batch No.	Specimen No.	Mix by Vol.	Percent Water		Compressive Strength lb./sq. in.	Avg. Humidity for 30 Hr.	Water Press. lb./in. ²	Days Cured			Avg. Rate of Leakage in Gal. per sq. ft. per hr. for the interval in hours.			Remarks
			Nominal	Molded				Molds	Spring-leak	Over	0-50	20-50	40-50	
133	557-558	1:2.4g	934		1797	80	41.6	2	26		000340	000205	000152	
133	559-560	1:2.4g	934		1608	80	41.1	2	26		007534	006060	005110	Beads hanging
134	561-562	1:2.4g	938		1533	46	41.7	7	21		006139	004275	003941	Discolored
134	563-564	1:2.4g	938		2020	46	42.6	14	14		001496	000907	000718	Discolored
135	565-566	1:2.4g	938		3118	41	42.9	2	25	29	000922	000523	000394	
135	567-568	1:2.4g	938		1964	41	42.2	2	51	3	006129	004969	004441	Damp on bottom
136	569-570	1:2.4g	938		2024	41	43.3	2	53		008855	006315	005956	Bottoms wet
136	571-572	1:2.4g	938		2388	41	44.4	2	23	3	001115	000679	000634	
137	573-574	1:1½:3	946		2730	41	44.2	2	26		000140	000080	000062	
137	575-576	1:1½:3	946		2961	41	43.8	2	26		002314	001264	001033	
138	577-578	1:1½:3	946		3409	41	44.2	7	21		001189	000572	000419	
138	579-580	1:1½:3	946		3489	41	44.4	14	14		000636	000292	000217	
139	581-582	1:1½:3	946		3572	35	44.7	2	25	29	000383	000191	000139	
139	583-584	1:1½:3	946		3036	35	44.4	2	51	3	001023	001012	000541	
140	585-586	1:1½:3	946		2793	41	43.5	2	54		003548	002144	001653	
140	587-588	1:1½:3	946		2103	39	45.1	2	23	3	001755	001058	000907	
184	694-695	1:38:52g	780	805	1508	71	42.2	2	26		001728	001257	001060	Discolored
184	696-697	1:38:52g	780	805	1123	71	42.2	2	26		052255	020300	014240	Leaked badly
185	698-699	1:38:52g	818	843	1418	72	43.0	7	21		037800	038000	031200	Leaked badly
185	700-701	1:38:52g	818	843	1433	72	43.0	14	14		013720	012870	012600	Leaked badly
186	702-703	1:38:52g	818	843	1744	72	45.0	2	25	28	002162	001344	001080	
186	704-705	1:38:52g	818	843	1670	72	43.5	2	57	4	015948	012977	011650	Bottoms wet and beads hanging
187	706-707	1:38:52g	818	843	1716	74	42.6	2	61		000834	000477	000334	
187	708-709	1:38:52g	818	843	1566	72	43.9	2	22	4	006452	005170	005090	Bottoms wet and beads hanging

[†] Proportioned by weight

Effect of Curing Conditions on Permeability.—

Specimens of 1:1½:3, 1:2:4 and 1:9 concretes were submitted to the various curing treatments, listed in Table 9. The four specimens of each batch were grouped in pairs, and each pair was passed through a different curing process. Thus the four batches of each mix provided eight variations in curing.

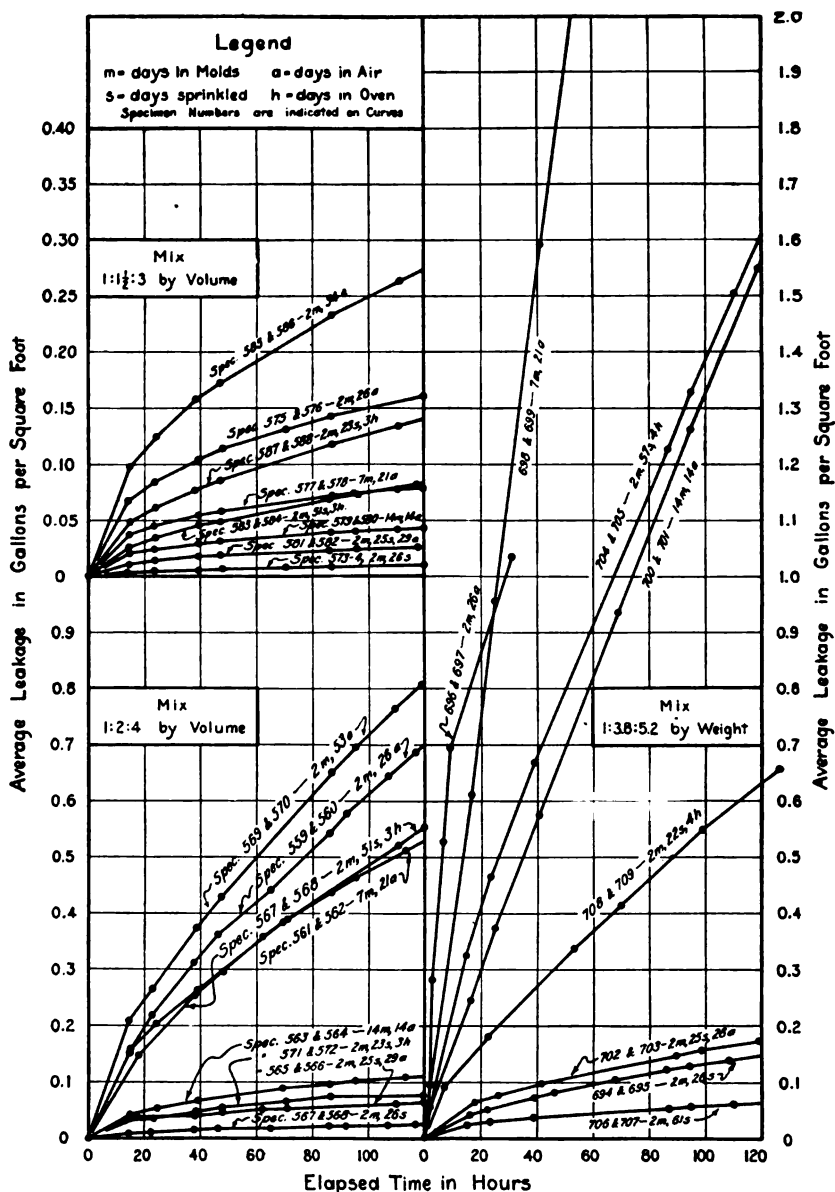


Fig 14.—Effect of Different Methods of Curing on the Permeability of Stone Concrete

PERMEABILITY TESTS—BROKEN STONE CONCRETE

Comparison of the curves of Fig. 14 shows that the specimens of 1:9 concrete were affected much more seriously by adverse curing conditions than the richer mixes. From the data in the table (batch 184) it is seen that removal from the molds after two days followed by dry curing at room temperature increased the rate of flow 10 to 30 times over the rate obtained when the concrete was sprinkled twice daily for a month. With batch 185 the specimens were left in the molds for 7 or 14 days, followed by air curing. The leakages obtained were correspondingly less than for the specimens removed from molds after 2 days, but still considerably higher than for the concrete sprinkled a month. Specimens 694-695 (sprinkled after removal from molds) were discolored on the free surface, whereas specimens 696-701 were actually dripping. A comparison of data from specimens 702 and 703 with that from 694 and 695, indicates that the permeability of 1:9 concrete which had been sprinkled for one month was not materially affected by subsequent drying in air at room temperature for an equal period. The results show that the 0-50 hour leakage was greater for 702 and 703 and that the 40-50 hour rates of the two pairs were about equal. This indicates that the permeability of the two sets was the same except that the drier concrete absorbed water more rapidly during the first part of the test. More water actually flowed into the air-dried specimens, 702 and 703, but the bottoms of the drier specimens were not dampened.

Comparing 1:9 concrete specimens 706 and 707 with 694 and 695, it is seen that the leakage was, as expected, considerably reduced by doubling the sprinkling period. Concrete in the older specimens appeared to be tight; no discoloration was noted. However, when specimens 704 and 705, which were treated similarly to specimens 706 and 707, were heated for four days at a temperature of 150° F., the effect of wet curing was lost and the concrete leaked visibly at the free surface. The results from specimens 708, 709, 694 and 695, which were tested when one month old, show that heating and drying after 22 days of sprinkling also increased the rate of flow, but to a less extent.

The results for 1:2:4 and 1:1½:3 concrete lead to conclusions similar to those derived from the 1:9 mixes. Specimens that were sprinkled until tested were in all cases most nearly impervious. Drying in air at room temperature had no particular effect on the rate of flow after the concrete had been properly

cured by sprinkling for 28 days, but early removal of the molds and curing in air for an equal period, without sprinkling, notably increased the leakage. Late removal of the molds, which prevented ready evaporation of moisture from the concrete, was effective in reducing the undesirable effect of air-curing. Heating of 1:2:4 and 1:1½:3 concretes, after 23 or 51 days of sprinkling, reduced the rate of flow markedly.

TABLE NO. 10.

Effect of Loss of Weight Due to Curing Conditions on the Permeability of Concrete.

For information concerning the method of making and curing specimens refer to Table 10

Batch No.	Specimen No.	Mix by Vol.	Normal Weight	Weight lost by Cylinders	Compressive Strength, lbs. per sq. in.	Ave. of Humidity, %	Water Phase, %	Weight lost by Air Spec.	Ave. Rate of Leakage in Gal. per sq. ft. per Hr. for the Interval in Hours			Remarks
									0-50	20-50	40-50	
133	557	1-2-4-g	934	168	1779	80	41.6	-0.66	.002380	.002293	.002200	Gained weight
133	558	1-2-4-g	934		1815	80	41.6	-0.57	.002300	.002200	.002200	Gained weight
135	565	1-2-4-g	938	169	3240	41	42.9	1.25	.002980	.002633	.002600	
136	572	1-2-4-g	938	253	2430	41	44.4	1.50	.002460	.002233	.002200	
135	566	1-2-4-g	938	140	2997	41	42.9	1.53	.002840	.002533	.002500	
135	568	1-2-4-g	938	140	1704	41	42.2	1.55	.001160	.002900	.002800	
135	567	1-2-4-g	938	110	2223	41	42.2	1.61	.011100	.009200	.008300	Damp on bottom
134	564	1-2-4-g	938	210	2461	46	42.6	1.77	.001800	.001200	.002800	
134	563	1-2-4-g	938	223	1579	46	42.6	1.83	.001200	.002733	.002400	
134	562	1-2-4-g	938	281	1411	46	41.7	2.28	.006660	.005200	.005000	Discolored
134	561	1-2-4-g	938	266	1656	46	41.7	2.40	.005460	.003567	.003200	Discolored
136	571	1-2-4-g	938	2685	2346	41	44.4	2.62	.001720	.001067	.002900	Bottom wet
133	560	1-2-4-g	934	323	1660	80	41.1	2.83	.006880	.005533	.005000	Beads hanging from bottom
133	559	1-2-4-g	934		1556	80	41.1	2.96	.008240	.006733	.005800	Beads hanging from bottom
136	570	1-2-4-g	938		2023	39	43.3	3.56	.008360	.005700	.005000	Bottom wet
136	569	1-2-4-g	938		2025	39	43.3	3.72	.009660	.007933	.006800	Bottom wet
137	573	1-1½-3-g	946	113	2600	41	44.2	-0.54	.002120	.002040	.002100	Gained weight
137	574	1-1½-3-g	946	126	2861	41	44.2	-0.44	.002148	.002057	.002120	Gained weight
138	580	1-1½-3-g	946		3489	41	44.4	1.80	.002598	.002283	.002210	
138	579	1-1½-3-g	946		3489	41	44.4	1.84	.002660	.002313	.002230	
140	587	1-1½-3-g	946	084	1977	39	45.1	1.87	.002812	.002370	.002310	
139	581	1-1½-3-g	946	254	3783	35	44.7	2.07	.002372	.002187	.002160	
139	582	1-1½-3-g	946	263	3360	35	44.7	2.07	.002430	.002200	.002150	
139	583	1-1½-3-g	946	239	3030	35	44.4	2.26	.002460	.002267	.002200	
138	577	1-1½-3-g	946		3345	41	44.2	2.43	.002120	.002603	.002450	
138	578	1-1½-3-g	946		3473	41	44.2	2.50	.002150	.002550	.002420	
137	576	1-1½-3-g	946	308	2943	41	43.8	2.86	.002502	.002203	.002130	
140	588	1-1½-3-g	946	126	2228	39	45.1	2.93	.002654	.002683	.002510	
137	575	1-1½-3-g	946	295	2980	41	43.8	2.97	.002212	.002277	.002090	
139	584	1-1½-3-g	946	363	3042	35	44.4	3.54	.001554	.001077	.002910	

Inspection of the remarks noted in Table 10 will show that, in general, the rate of flow into the 1:2:4 or 1:1½:3 concrete increases with the per cent of weight lost in curing. The results for the richer mix are in good agreement, but there are some irregularities in the data from the 1:2:4 specimens. It will also be noted that none of the 1:1½:3 mixes under any of the conditions tested gave evidence of leakage on the free surface.

Effect of Direction of Flow on Permeability.—The PUHC specimens of 1:1½:3 and 1:2:4 mixes were made to compare with similar PU specimens. As previously noted, the PUHC specimens are designed so that most of the leakage is in a direction perpendicular to the direction of placing, whereas the flow in PU specimens is entirely in the direction of placing, see Fig. 3. The results are listed in Table 11.

Whereas the number of tests is too small to furnish conclusive evidence, it appears that the rate of flow into the 1:1½:3

TABLE NO. 11.

Effect of Direction of Flow on the Permeability of Concrete
Materials - Universal Portland Cement (P in batches 115, 122, and 133, P in others)
Janesville Sand (S₄₅, P in batches 115 to 133, S₄₅, P in others) and Madison Stone (S₄₅, P)
All concrete was machine mixed by a No. 3 Smith Mixer, ½ minute dry & ½ minutes wet
All specimens were 6 inches thick
All specimens cured 2 days in molds and then sprinkled 26 days, and tested at age of 28 days.

Batch No.	Specimen No.	Mix		Percent Water		Compressive strength (psi)	Air Humidity (for 50 hours)	Water Pressure (lb./sq. in.)	Direction of Flow		Ave Rate of Leakage in Gal per Sq. Ft. per hr for the Interval in Hours			Remarks
		By Volume	By Weight	Normal	Moisture				Parallel to Bed	Perpendicular to Bed	0-50	20-50	40-50	
122	PU50-516	1:1½:3	1165271	937	103	3046	75	425	✓	✓	000132	000074	000055	
137	PU573574	1:1½:3	1165271	946		2730	41	442	✓	✓	000140	000080	000062	
164a	PU651654	1:1½:3	1165271	957	964	2994	54	445	✓	✓	000121	000069	000041	
165	PUHC.13	1:1½:3	1165271	937		2744	41	440	✓	✓	000069	000049	000052	
166	PUHC.14	1:1½:3	1165271	957		2369	57	431	✓	✓	000092	000067	000066	
115	PU465466	1:2:4	1220373	936	102	2547	68	428	✓	✓	000210	000117	000090	
133	PU557556	1:2:4	1220373	944	96	1797	80	416	✓	✓	000340	000205	000152	
163a	PU647650	1:2:4	1220373	940	96	2047	37	446	✓	✓	000452	000292	000253	
167	PUHC.15	1:2:4	1226416	787		2166			✓	✓				Full of holes. Could not be tested * (2.8.3. Stone only in these batches)
168	PUHC.16	1:2:4	1226416	787		1773			✓	✓				
169	PUHC.17	1:2:4	1227379	819		2210	45	427	✓	✓	000514	000513	000480	
169	PUHC.18	1:2:4	1227379	757		2492	44	420	✓	✓	000400	000250	000236	

mixes was approximately the same whether the direction was parallel or perpendicular to bed. With the leaner 1:2:4 mix, great variations in water-tightness of the PUHC specimens were encountered. These tests indicate the importance of homogeneity in structure and the necessity for avoiding pockets and horizontal seams in pouring walls to be subjected to water pressure. Where segregation is likely to occur thick sections and rich mixes should be used if water-tightness is to be secured.

The Relation Between Permeability and Compressive Strength.—Figure 15 shows a curve in which the average leakage rates for the interval of 40 to 50 hours of certain batches of concrete are plotted against the corresponding compressive strengths. In this chart only the specimens of normal

curing (sprinkled for 26 days) were considered. The batches differed in consistency, time of mixing, grading of aggregate and proportion of cement.

This curve leads to a conclusion similar to that derived from the tests on gravel concretes and may be stated as follows: If concrete which has an average rate of leakage less than 0.0001

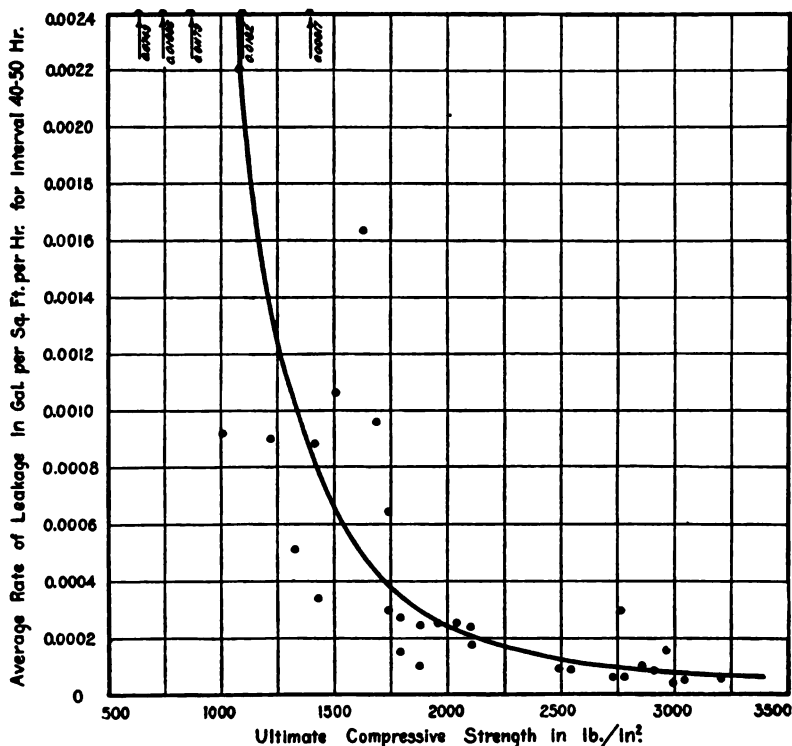


Fig. 15.—Relation between the Compressive Strength and the Permeability of Well-Cured Stone Concrete

gallon per square foot per hour for the 40-50 hr. period under 40 lb. per sq. in. pressure be considered impervious, then it appears that concretes having a compressive strength above 2,500 lb. per sq. in. may be so considered. This conclusion applies only when proper curing conditions obtain, for the results of Table 9 show that improper curing conditions considerably increase the permeability of concrete without affecting a corresponding decrease of compressive strength.

Effect of Duration of Flow on the Rate of Leakage.—

Except in a few irregular cases the rate of leakage into stone concrete specimens of this series decreased with the continuation of the test. The average leakage rates for periods up to 50 hours, as given in the tables, support this statement. The greatest reduction in rate of flow was noted for specimens which were dried out before the start of the test. It is logical to assume that these specimens absorbed water rapidly from the permeability tubes during the first hours, so that the 0-50 hour leakage would be considerably higher than the rate for 20 to 50 or 40 to 50 hour periods. In some cases high rates of flow were obtained without any visible leakage at the free surface.

The continual reduction in rate of flow, even with specimens that have been water-soaked before the start of the test, leads to the conclusion that the pores in the concrete were sealed to some extent by the impurities carried in the water.

CONCLUSIONS

From the permeability tests on gravel and stone concretes which have been made at this University the following conclusions are drawn:

1. None of the concretes tested were *absolutely* water-tight, if continuous flow *into* the specimen is considered proof of permeability. The specimens of rich mixes, when submitted to proper curing conditions, were so impervious that no leakage was apparent at the free surface. Such concretes can be considered water-tight for ordinary purposes.

2. The cement content of a given mix appears to be the most important factor in determining the degree of impermeability. Other factors remaining constant, the water-tightness of concrete made with well-graded aggregate increases with the proportion of cement in the mix. In the mixes tested, this increase in water-tightness was very marked as the cement content was raised until the ratio of cement to aggregate, by weight, was 0.15. Further increase in cement content was attended by a relatively small increase in water-tightness. For the broken stone concretes of these tests a ratio of cement to aggregate, by weight, of 0.17 or more gave a water-tight mix, whereas a cement-aggregate ratio of 0.14 was sufficient for the previously-reported gravel concrete mixes. Mixes containing high proportions of cement were much less affected by variations in consistency, time of mixing, and grading of aggregate than lean mixes.

3. The ratio of the volume of cement to the volume of air plus water voids, or cement-voids ratio, is a valuable measure of both strength and water-tightness of concrete. The strength increases nearly in direct proportion to the cement-voids ratio for the mixes tested. Water-tightness in the mixes of broken stone concrete was obtained when the cement-voids ratio was 0.55, or above. With the previously reported gravel-concrete mixes a cement-voids ratio of 0.45 gave similar results.

4. The water-tightness of mixes containing less than 14 per cent of cement to total dry materials, by weight, is much affected by variations in consistency, curing conditions, placement, and grading of the aggregate. It is also affected, but to lesser extent, by variations in the time of mixing, and by the moisture

content of the fine aggregate. Lean concretes made of angular broken stone are more sensitive to the effects of these variables than similar mixes containing round pebbles.

5. Maximum water-tightness obtains for a limited range of the water-cement ratio $\frac{w}{c}$. For the rich mixes this range is much wider than for lean mixes.

6. A plastic consistency which gives a slump of one to three inches when tested in a 6x12-in. cylinder appears to be best suited to produce an impervious concrete with a minimum of cement. In order to secure maximum density it is better to use somewhat more water than above indicated rather than a lesser quantity; but the use of excessive amounts of water, which is usually accompanied by segregation of the aggregate, greatly increases the permeability of lean mixes.

7. When conditions of placement require the use of a more fluid mix than recommended above, increased fluidity may be obtained, without sacrifice of imperviousness, by raising both the cement and water contents in the same proportion.

8. To secure a high degree of imperviousness, concrete should be cured in a moist atmosphere or under water. At temperatures of 60° to 75° F., the moist-curing period should be maintained for two weeks to a month after molding. Lean mixes or thin sections require longer wet-curing periods than rich mixes or thick sections. When it is impracticable to keep the work moist after removal of the molds, and the humidity is low, provision should be made to keep the molds in place in order to avoid evaporation. The effect of dry curing appears to be much more detrimental to water-tightness than to compressive strength.

9. Thorough mixing is of great importance in securing the homogeneity necessary in water-tight concrete. Considering only the consistencies more commonly used, slumps one to 10 inches, it appears that lean mixes made with damp sand require a somewhat longer mixing period than when made with dry sand. With a mixer of the type used, a mixing period of one and one-half to two minutes after all materials have been placed in the machine is required to secure thorough mixing of lean concretes. Materials for such concrete should be mixed dry from 15 to 30 seconds before water is added.

10. With lean mixes very much greater care must be exercised during placement to secure sufficient homogeneity so that they will be impervious when the direction of flow is parallel to the bed. The rich mixes of $1:1\frac{1}{2}:3$ proportions exhibited practically the same degree of water-tightness for flow normal to or parallel to the bed.

11. The results show that lean mixes in which the materials have been proportioned in accordance with Fuller's mechanical analysis curve can be made very impervious provided it is possible to use a plastic consistency and to secure proper methods of placement and curing. Therefore, when a construction involving a large yardage of concrete is being planned, the properties and the costs of lean mixes made of specially graded aggregates should receive due consideration.

12. For making water-tight concrete, well graded gravel of good quality is superior to broken stone of like quality and grading.

13. For properly placed and cured broken-stone or gravel concrete the permeability decreased as the compressive strength increased until the latter reached 2500 lb. per sq. in. Concretes of this strength were water-tight.

14. For the concretes tested, increasing the thickness above six inches did not materially affect the leakage at pressures of 40 lb. per. sq. in. With the more variable conditions of mixing, placing, and curing encountered in the field, sections thicker than six inches would undoubtedly be required for water-tightness.

15. The permeability of rich concrete cured in moist air for one month and subsequently protected from the weather decreases with age but the decrease is negligible after the concrete becomes one month old.

16. When concrete is subjected to freezing its permeability appears to be increased slightly. Weathering agencies, such as combined freezing and thawing, wetting and drying, and variations in temperature seem to produce a marked increase in permeability. The effect of these agencies must be more thoroughly investigated, however, before their magnitude can be stated.

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**COMPARATIVE TESTS OF NEW BILLET STEEL
AND REROLLED STEEL REINFORCING BARS**

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RESEARCHES IN APPLIED MECHANICS

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COMPARATIVE TESTS OF NEW BILLET STEEL AND REROLLED STEEL REINFORCING BARS

INTRODUCTION

Inception of the Tests.—In the autumn of 1921 Robert C. Johnson, Milwaukee representative of the Corrugated Bar Company of Buffalo, New York, suggested these tests as a possible thesis to James W. Johnson, a senior in the civil engineering course at the University of Wisconsin. The Corrugated Bar Company had made some tests during the summer of 1921 which showed that rerolled steel bars lost a considerable percentage of their strength in tension after being bent cold. It was suggested that a series of tests, similar to those of the Corrugated Bar Company, comparing the physical properties of new billet and rerolled reinforcing bars, would be of value to engineers and other users of reinforcing steel.

Such a series of tests was carried out during the school year 1921-22 by James W. Johnson, with the assistance of Leland L. Karas, a senior in the mechanical engineering course.

A second series of tests, amplifying the first, was carried out during the summer of 1922 by Honore C. Hubbard, a graduate of the chemical engineering course at the University of Wisconsin. The results of these two series of tests are described in this bulletin.

Outline of Tests.—For the first series of tests new billet steel of intermediate grade was obtained from one company, rerolled steel from a second company, and both types of steel from a third company. Two different sizes of bar were used for each type of steel. The tests made upon these steels were the commercial tension test, the cold bend test, and the tension test of the bar after it had been cold bent into a U shape and then placed in a template.

In the second series of tests the material consisted of intermediate grade and hard grade new billet steel from one company, rerolled steel from a second company, and both grades of new billet steel and also rerolled steel from a third company. The tests carried out were the commercial tension test, the cold bend test, and the tension test of the bar after it had been bent cold into a V shape and then placed in a template. Some of the steel which remained from the first series of tests was tested at this time in the V-bend tension test.

The tension test and the cold bend test were chosen for this investigation because they are the tests now specified in the Standard Specifications of the American Society for Testing Materials for new billet steel and rerolled steel reinforcing bars.

The U-bend test was chosen in order to determine whether either type of steel lost any percentage of its tensile strength after it had been bent cold. The V-bend test was chosen for the second series of

tests because it was thought that the V-bend represented better than the U-bend the actual bending to which reinforcing bars are subjected in practice.

The tests already mentioned were supplemented by some chemical analyses and metallographic examinations.

Acknowledgments.—Thanks are due the Calumet Steel Company, the Inland Steel Company, and the Corrugated Bar Company for donating the steel used in this investigation. The Corrugated Bar Company also supplied the U-bend template, and the bending apparatus which was used in bending the bars into the U shape.

William C. Muehlstein and James A. Schad, engineers for the Industrial Commission of Wisconsin, cooperated in this investigation in planning the work and in securing the necessary steel.

Credit is due James W. Johnson and Leland L. Karas for their conscientious work in carrying out the first series of tests, and Honore C. Hubbard for carrying out the second series of tests.

Professor M. O. Withey, of the University of Wisconsin, was especially helpful with suggestions in the planning and carrying out of the tests. He kept in close touch with the work throughout the investigation.

MATERIALS, TESTS, TESTING APPARATUS, AND TEST SPECIMENS

Material for First Series of Tests.—The new billet steel bars were deformed, open hearth bars, of intermediate grade, as follows:

From Company No. 3 { 10 one-half inch squares
10 seven-eighths inch rounds

From Company No. 4 { 10 five-eighths inch rounds
10 seven-eighths inch rounds

The rerolled bars were deformed bars, as follows:

From Company No. 5 and 55 { 40 one-half inch squares
40 three-quarter inch squares

From Company No. 6 { 10 five-eighths inch rounds
10 seven-eighths inch rounds

The rerolled bars from Company No. 5 and 55, and the new billet bars from Company No. 4 were selected by James A. Schad of the Industrial Commission, and marked by him before they were shipped to the University. The purpose of this selection was to secure material which was known to be representative of average quality. The other bars of Companies No. 3 and No. 6 were selected by the company, but are assumed to be representative of average quality.

Material for Second Series of Tests.—The new billet steel bars were deformed, open hearth bars, of intermediate grade and of hard grade, as follows:

From Company No. 8 { 10 one-half inch squares
Intermediate Grade { 10 three-quarter inch squares

From Company No. 9 { 10 one-half inch squares
Intermediate Grade { 10 seven-eighths inch rounds

From Company No. 10 { 10 one-half inch squares
Hard Grade { 10 three-quarter inch squares

From Company No. 11 { 10 one-half inch squares
Hard Grade { 10 seven-eighths inch squares

The rerolled bars were deformed bars, as follows:

From Company No. 7 { 10 five-eighths inch rounds
10 seven-eighths inch rounds

All of these bars were selected by James A. Schad of the Industrial Commission, and marked by him before they were shipped to the University.

It should be stated here that in the foregoing, Company No. 3, No. 8 and No. 10 are the same company; Company No. 4, No. 6, No. 7,

No. 9 and No. 11 are the same; and Company No. 5 and 55 are the same. The different numbers were used merely for convenience in identification and marking.

The material for the first series of tests was received from the companies in February, 1922; and the material for the second series was received about two months later.

Figure 1 makes clear the types of deformed bars which were tested.

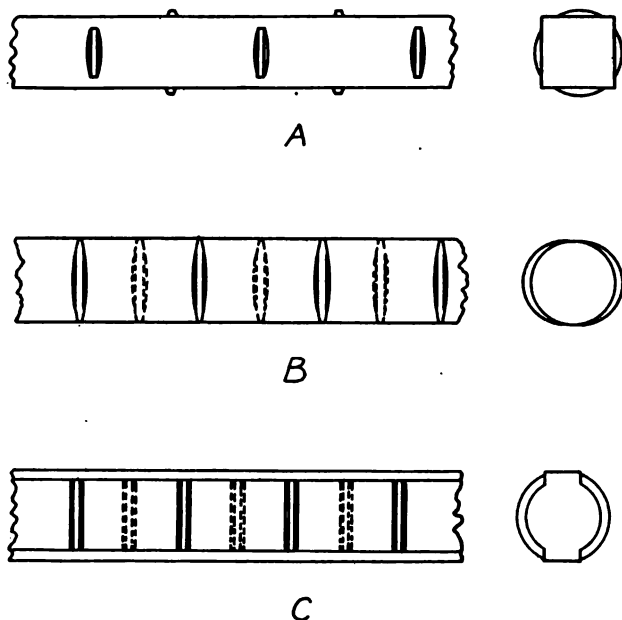


Figure 1.—Types of Deformed Bars Used in the Tests

- (A) 5 A-T, $\frac{1}{2}$ in. and $\frac{3}{4}$ in.; 55 A-T, $\frac{1}{2}$ in. and $\frac{3}{4}$ in.; 3 A-J, $\frac{1}{2}$ in.; 8 A-J, $\frac{1}{2}$ in.; 10 A-K, $\frac{1}{2}$ in. and $\frac{3}{4}$ in.; 8 L-U, $\frac{3}{4}$ in.; 9 A-K, $\frac{1}{2}$ in.; 11 A-J, $\frac{1}{2}$ in.
- (B) 4 A-J, $\frac{5}{8}$ in. and $\frac{7}{8}$ in.; 9 L-U, $\frac{7}{8}$ in.; 7 A-J, $\frac{5}{8}$ in.; 11 L-U, $\frac{7}{8}$ in.; 6 A-J, $\frac{5}{8}$ in. and $\frac{7}{8}$ in.; 7 L-U, $\frac{7}{8}$ in.
- (C) 3 A-J, $\frac{7}{8}$ in.

Marking of Specimens.—Each bar was cut into 4 pieces and each piece was stamped on both ends with a number representing the company which supplied the bar, and also with a letter which designated each particular bar. Thus the first 10 bars of Company No. 3 were marked 3A, 3B, etc. The second 10 bars from this company were also marked 3A, 3B, etc., but could be distinguished from the first 10 because of difference in size. The 4 specimens for the different tests cut from each bar, such as 3A, were all marked 3A.

Bars No. 5 and No. 55 came in the same shipment from one company, but for convenience in marking they were divided into two lots, each composed of 20 bars of each size. One lot was numbered 5

and the other 55, the bars in each lot being lettered from A to T for each size.

Tests and Apparatus.—The tension tests were made in accordance with the specifications of the American Society for Testing Materials, some of the smaller sizes being tested in a 50,000 lb. Riehle machine, and the larger sizes in a 100,000 lb. Riehle machine.

The seven-eighths inch round bars from Company No. 3 could not be measured easily to determine their average cross-section, because of their shape. (See Fig. 1.) These specimens were, therefore, measured for length, weighed, and their average cross-section area computed from the following formula:

$$\text{Area} = \frac{\text{Weight}}{\text{length} \times \text{weight per cu. in.}}$$

The specimens used for the tension tests were 22 inches long. They were lightly prick punched at each inch for at least 10 inches. The yield point and ultimate strength were determined, and the reduced diameter after rupture, the elongation in 2 inches, and the elongation in 8 inches were measured. From these data the unit stress at yield point and ultimate, the per cent of reduction of area, and the per cent of elongation in 2 inches and in 8 inches were computed. The character of fracture and any irregularities were recorded.

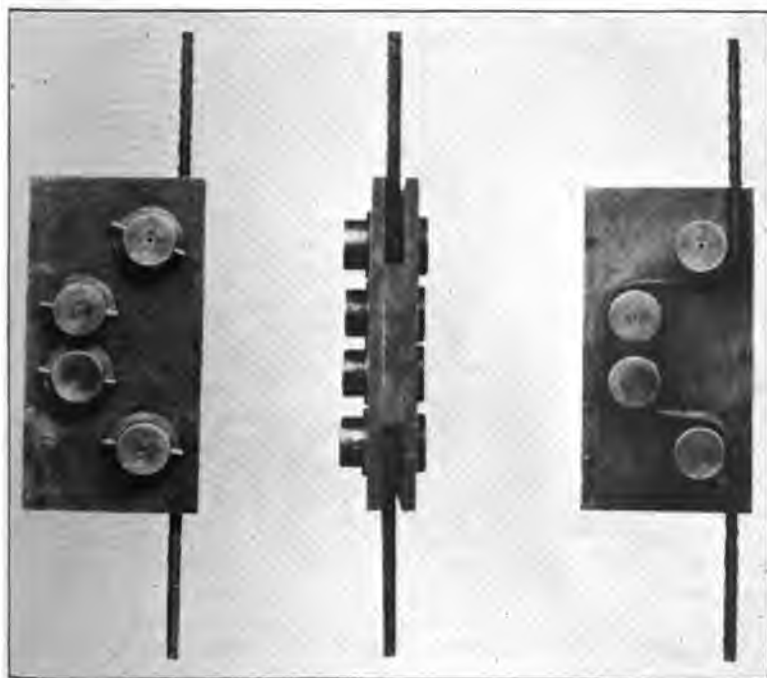


Figure 2.—Apparatus Employed in the U-bend Tension Test

The specimens used in the U-bend tension test were $4\frac{1}{2}$ feet long. They were bent at four places through a 90° angle to form a U-shaped specimen which would fit as nearly as possible to the template pins, and have the two extending ends colinear. The bent specimen was placed in the template and tested in a 100,000 lb. Riehle machine.

Figure 2 is a photograph of the U-bend template. Figure 3A is a sketch giving dimensions. The pins used in this apparatus were 3 inches in diameter, and in bending bars into the U shape, pins were used which were 3 inches in diameter for all sizes except the $\frac{7}{8}$ inch bars, for which $3\frac{1}{2}$ inch pins were used. None of these pins, therefore, were smaller than those required in the specifications of the American Society for Testing Materials for the cold bend test of new billet and rail steel reinforcing bars.

The template used for the V-bend test was similar to that used for the U-bend test. Figure 3B is a sketch giving the dimensions. The pins used were 3 inches in diameter, and the maximum angle of

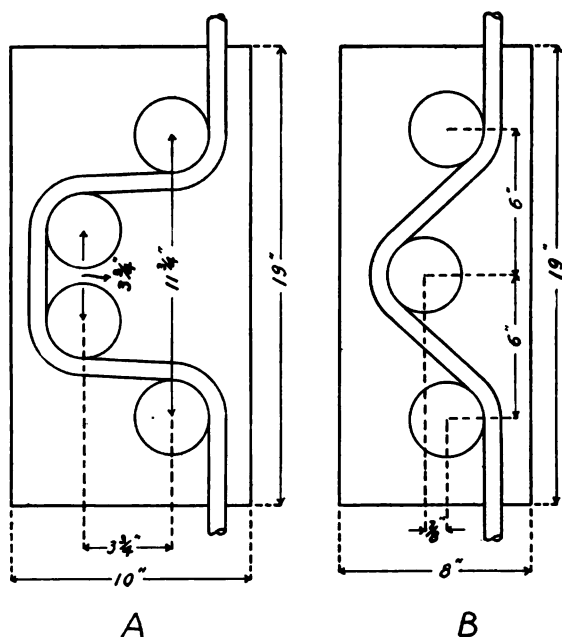


Figure 3.—A—Diagram of U-bend Tension Test
Figure 3.—B—Diagram of V-bend Tension Test

bend with this apparatus was approximately 90° . It was found convenient to bend the V-bend specimens while in the apparatus, so that all the V-bend specimens were bent using 3 inch pins.

The cold bend tests were made according to the specifications of the American Society for Testing Materials. The specimens were 14

inches long, and a Riehle bending tool was used. The bends were made around pins of the sizes required by the specifications, as follows:

TYPE OF BAR	SIZE OF BAR	DIAMETER OF PIN	ANGLE OF BEND TO PASS TEST
Intermediate Grade			
New Billet Steel	$\frac{1}{2}$ inch square	$1\frac{1}{2}$ inch	180°
" " "	$\frac{5}{8}$ inch round	$1\frac{1}{8}$ inch	180°
" " "	$\frac{3}{4}$ inch square	$2\frac{1}{4}$ inch	90°
" " "	$\frac{7}{8}$ inch round	$2\frac{5}{8}$ inch	90°
Hard Grade			
New Billet Steel	$\frac{1}{2}$ inch square	2 inch	180°
" " "	$\frac{3}{4}$ inch square	3 inch	90°
" " "	$\frac{7}{8}$ inch round	$3\frac{1}{2}$ inch	90°
Rerolled Steel	$\frac{1}{2}$ inch square	2 inch	180°
" "	$\frac{5}{8}$ inch round	$2\frac{1}{2}$ inch	180°
" "	$\frac{3}{4}$ inch square	3 inch	90°
" "	$\frac{7}{8}$ inch round	$3\frac{1}{2}$ inch	90°

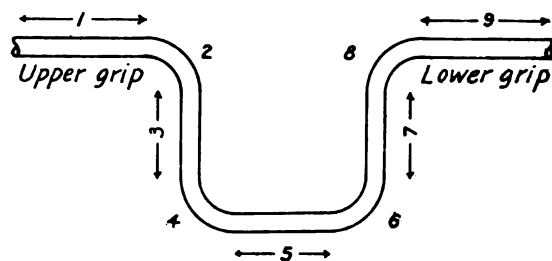
TEST DATA

Explanation of Tables.—Tables No. 1 to 11 give the results obtained in the various tests. An explanation of Table No. 1 will help to make all of them clear. Where the character of fracture is given by number, the following schedule is referred to:

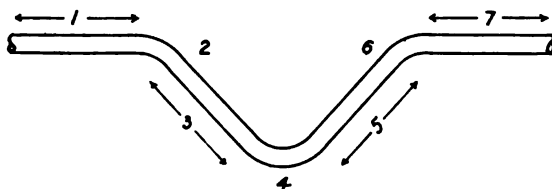
1. Truncated pyramid
2. Fine grain
3. Dull gray
4. Bright crystalline
5. Square and flat
6. Dull gray, rough center with bright crystalline sides
7. Bright crystalline surface with dull spot at punch mark
8. Showing flaws on longitudinal cracks
9. Cup and cone
10. Diagonal
11. Notched or V shaped
12. Silky

Under the heading "U-bend Tension Test", the "% change" refers to the increase or decrease in the yield point and ultimate of the U-bend test as compared with the same values in the ordinary tension test.

Under the heading "Failed at" is recorded the point on the specimen at which failure took place as based on the numbers shown in Figure 4A for the U-bend test, and in Figure 4B for the V-bend test.



A



B

Figure 4.—A—Position of Failure in U-bend Tension Test
Figure 4.—B—Position of Failure in V-bend Tension Test

Under the heading "Cold-Bend Test" the record is marked O. K. if the specimen met the requirements of the American Society for Testing Materials. If it did not meet these requirements, the angle of failure is recorded in the appropriate column. This angle represents the arc through which the bar was bent, and not the interior angle formed by the specimen at failure.

In Tables No. 1 to 11 the results recorded in each table are for bars which were all from the same company. In Table No. 1 the first 10 specimens were from the first series of tests, and the second 10 from the second series. For the ordinary tension test the results of all 20 specimens are averaged, but for the U-bend and the V-bend tests the results are averaged separately because of the fact that the two tests were not the same.

The items labeled "Ave.", "Max.", "Min.", and "% Var." are respectively the average, maximum, minimum and mean variation from the average in per cent, of the results in the column above these items. The mean variation from the average in per cent was found by finding first the difference between each result and the average result, then dividing by the number of results to determine the mean variation from the average, and then dividing by the average result to determine the mean variation in per cent of the average.

The average results which are recorded in the tables are given for obvious reasons, while the items for maximum, minimum, and mean variation from the average, give an idea of the uniformity of the material.

DISCUSSION OF RESULTS

New Billet Intermediate Grade Bars.—Tables No. 1, 2, 3, and 4, give the results for $\frac{1}{2}$ inch, $\frac{3}{8}$ inch, $\frac{1}{4}$ inch, and $\frac{3}{16}$ inch intermediate grade new billet deformed bars. Tables No. 1 and 2 are for the bars from one company, and Tables No. 3 and 4 for those from a second company.

Referring to Tables No. 1 and 2 it is seen that all the bars satisfied the requirements of the American Society for Testing Materials as regards ductility, both in percentage of elongation in 8 inches and in the cold bend test. The first 10 bars in each of these tables met the requirements as to yield point and ultimate strength, but the second 10 bars in each table did not. There were 3 bars out of 20 in Table No. 1 and 10 bars out of 20 in Table No. 2 that fell below the yield point requirement. There were 8 bars out of 20 in Table No. 1 and 10 bars out of 20 in Table No. 2 that fell below the ultimate strength requirement.

It may be noted here that in Table No. 2 there are 10 bars whose ultimate strength was higher than the maximum called for in the

TABLE NO. 1
TESTS OF $\frac{1}{2}$ SQUARE
INTERMEDIATE GRADE NEW BILLET DEFORMED BARS

Spec. No.	Diam. of Bar	TENSION TEST		U BEND TENSION TEST		COLD BEND TEST				
		% Elongation 2 in.	Character of Fracture	Unit Stress - Lbs. per Sq. In. Yield Point Ultimate	Unit Stress - Lbs. per Sq. In. Yield Point Ultimate	% Change Y.P. Ult.	Failed at 180°			
3A	47.5	31.5	23.2 All	81600	82800	54900	83000	+ 62 + 02	1	O.K.
3B	50.0	32.5	22.8 Fractures	80300	83500	58100	88500	+157 + 24	1	O.K.
3C	55.5	35.0	22.5 were V-	47600	74500	58600	85900	+89 + 19.3	1	O.K.
3D	48.0	37.5	21.4 Shaped	52500	81700	54800	83000	+44 + 16	1	O.K.
3E	44.7	35.0	21.4 with fine,	46200	74500	56000	84300	+212 + 13.1	1	O.K.
3F	45.0	35.0	21.4 light gray,	52500	81900	55600	84000	+39 + 16	1	O.K.
3G	56.2	31.0	23.8 silky texture	54200	83600	52900	75300	- 24 - 9.8	1	O.K.
3H	42.6	38.5	18.4	45400	73800	56600	84400	+252 + 14.4	1	O.K.
3I	38.0	37.5	20.6	51500	83400	53900	84000	+ 47 + 07	1	O.K.
3J	35.8	30.0	20.6	81000	81900	53300	82700	+ 45 + 10	1	O.K.
Ave.						85300	89200	+103 + 42		
Max.						96100	89900	+252 + 15.3		
Min.						52900	75300	- 24 - 02		
Stdev.						25	21	+79.3	45.0	
1/2 INCH SQUARE BARS										
8A	33.0	30.0	23.1 2-10	40400	66800	45700	68200	+151 + 2.1	1	O.K.
8B	49.0	35.0	21.3 2-10	42700	70000	42800	71400	+02 + 2.0	1	O.K.
8C	28.6	32.5	21.6 2-10	41900	69300	43500	71900	+38 + 3.7	1	O.K.
8D	51.3	35.0	21.4 2-1	42400	70800	42300	72600	-02 + 25	7	O.K.
8E	48.0	35.5	23.4 2-1	40600	66400	42100	71800	+37 + 3.5	1	O.K.
8F	54.5	40.0	26.4 2-5	39800	67700	42300	72400	+69 + 7.0	1	O.K.
8G	48.0	37.5	20.0 2-5	40500	66600	41200	66400	+17 - 1.7	1	O.K.
8H	38.2	36.0	23.1 2-10	39800	66500	42800	73700	+73 + 10.8	1	O.K.
8I	56.0	36.5	23.1 2-5	39300	66300	41200	69500	+77 + 4.5	1	O.K.
8J	50.4	38.0	25.0 2-5	40800	68800	41200	68300	+10 - 0.4	1	O.K.
Ave.	44.3	34.7	22.3	45500	74300	42800	70800	+48 + 3.4		
Max.	56.0	40.0	26.4	54300	83500	46700	73700	+131 + 10.8		
Min.	28.6	30.0	19.4	39200	66500	41200	66200	-02 - 0.4		
Stdev.	14.9	6.4	6.3	10.4	7.9	2.9	2.5	73.4	75.6	

TABLE NO. 2
TESTS OF $\frac{3}{8}$ ROUND AND $\frac{1}{4}$ SQUARE
INTERMEDIATE GRADE NEW BILLET DEFORMED BARS

Spec No.	TENSION TEST				U BEND TENSION TEST				COLD BEND TEST		
	% Elongation 2 in. of Area	2 in.	8 in.	Character of Fracture	Unit Stress - Lbs per Sq. in. Yield Point	Ultimate	Unit Stress - Lbs per Sq. in. Yield Point	Ultimate	% Change Y.P. Ult.	Failed At	Bent 90° Angle of Fracture
3 A	42.5	25.0		All	55400	90600	55100	91000	-03 + 0.4	1	O.K.
3 B	57.5	18.7		Fractures	96100	96600	57300	96200	+21 - 0.4	9	O.K.
3 C	32.5	30.0		were Cup	57700	97200	Didn't fit Template				O.K.
3 D	30.0	18.7		and Cone	56000	96300	59000	96800	+54 + 0.5	1	O.K.
3 E	30.0	20.0		or else	56000	97400	58400	97300	+43 - 0.1	1	O.K.
3 F	38.5	22.5		V Shaped	57900	97500	61600	97200	+64 - 0.3	1	O.K.
3 G	37.5	21.9		with fine,	57300	96400	59400	94500	+37 - 2.0	1	O.K.
3 H	30.0	21.9		light gray,	96200	96400	Didn't fit Template				O.K.
3 I	40.0	23.7		silky texture	54500	90000	55700	89400	+22 - 0.7	1	O.K.
3 J	37.5	23.7			54000	89500	55400	89600	+26 + 0.1	1	O.K.
Ave.							57700	94000	+33 - 0.3		
Max.							61600	97300	+64 - 2.0		
Min.							55100	89400	-03 - 0.1		
Stdev.							3.2	3.2	51.5	180	
3/8 INCH SQUARE BARS											
8 L	57.4	57.5	32.0	10-2	35000	55000	36900	55500	+54 + 0.9	1	O.K.
8 M	56.7	53.0	33.0	1-2	34100	57300	36200	57200	+62 - 0.2	7	O.K.
8 N	54.2	52.0	32.0	1-2	35200	56800	35900	57400	+09 + 1.1	7	O.K.
8 O	55.5	57.5	33.7	10-2	34700	55400	35600	55200	+26 - 0.4	1	O.K.
8 P	55.3	55.0	33.7	10-2	33900	55300	36700	56000	+63 + 0.9	1	O.K.
8 Q	58.3	57.5	32.5	1-2	34400	55200	36300	55400	+35 + 0.4	1	O.K.
8 R	57.7	54.5	33.0	1-2	33600	55200	36800	55300	+95 + 0.2	7	O.K.
8 S	57.7	53.0	32.6	1-2	33400	55100	37500	54800	+23 + 0.5	1	O.K.
8 T	55.9	51.0	35.0	1-2	34300	56300	37300	56400	+88 - 0.2	1	O.K.
8 U	56.7	51.5	34.0	5-2	33600	55600	37300	55700	+10 + 0.2	1	O.K.
Ave.	56.6	44.1	27.4		45200	73300	36600	55900	+70 + 0.3		
Max.	58.3	57.5	35.0		57800	97800	37500	57400	+23 + 1.1		
Min.	54.2	47.5	18.7		33400	55000	35900	55200	+09 + 0.2		
Stdev.	2.0	23.0	21.1		24.3	25.9	1.6	1.2	41.8	140	

TABLE NO. 3
TESTS OF $\frac{1}{2}$ " ROUND AND $\frac{1}{2}$ " SQUARE
INTERMEDIATE GRADE NEW BILLET DEFORMED BARS

TENSION TEST												U BEND TENSION TEST				COLD BEND TEST	
Spec. No.	Bar Size	% Elongation		Character of Fracture	Unit Stress - Lbs. per Sq. In.		Unit Stress - Lbs. per Sq. In.		% Change		Failed At	Bent 180°	Angle of Failure				
		2 in.	8 in.		Yield Point	Ultimate	Yield Point	Ultimate	Y.P. Ult.	Y.P. Ult.							
4A	30.3	37.5	31.2	11-2	41 100	60100	43400	61800	+3.3	+2.0	1	O.K.					
4B	62.6	42.5	31.2	9-2	43 000	69800	43500	61900	+5.6	+3.5	1	O.K.					
4C	67.0	37.5	30.0	9-2	42 900	60500	43300	61400	+0.9	+1.5	1	O.K.					
4D	66.2	42.5	29.4	9-2	43 800	59600	48400	61 900	+3.6	+3.9	1	O.K.					
4E	66.0	40.0	38.8	9-2	42 700	60 000	42300	61 600	-0.9	+2.5	1	O.K.					
4F	69.5	42.5	26.1	9-2	42 500	59 900	43400	60100	+2.1	+0.3	1	O.K.					
4G	64.0	40.0	29.4	9-2	44 500	60 600	43 800	62 100	+2.9	+2.5	1	O.K.					
4H	62.5	48.0	26.1	9-2	42 800	60 600	46600	62 100	+6.9	+2.5	1	O.K.					
4I	63.0	42.5	26.6	9-2	42 400	60 200	46 300	61 500	+9.2	+2.2	1	O.K.					
4J	64.9	42.5	29.4	9-2	43 100	59 100	44 700	61 000	+5.7	+3.2	1	O.K.					
Ave.							44 700	61 800	+4.2	+2.5							
Max.							46 600	62 100	+9.2	+3.9							
Min.							42 300	60 100	-0.9	+0.3							
Std.							2.8	0.7	60.7	27.6							
1/2 INCH SQUARE BARS																	
9A	37.1	32.5	19.8	10-2	46400	72400	42600	74300	-7.8	+1.2	7	O.K.					
9B	34.5	31.5	20.4	10-2	45800	70700	42700	71700	-6.8	+1.4	7	O.K.					
9C	37.5	30.0	19.5	5-2	46900	71800	44 100	74300	-6.0	+3.5	7	O.K.					
9D	38.2	30.0	22.6	10-2	47000	71 000	45 200	74300	-3.8	+4.6	7	O.K.					
9E	37.0	36.5	21.9	10-2	47600	73400	46 900	75 000	-1.5	+1.9	7	O.K.					
9F	41.6	31.5	23.2	10-2	46400	74000	45 700	74 700	-1.5	+0.9	7	O.K.					
9G	37.6	34.0	26.4	5-2	46700	69800	41 300	71 200	-11.5	+2.0	7	O.K.					
9H	42.4	32.5	23.1	5-2	47200	69900	44 100	70300	-6.8	+0.6	7	O.K.					
9J	45.8	37.0	22.5	5-2	46000	70900	43 000	72 300	-6.5	+2.8	7	O.K.					
9K	41.8	36.5	23.9	10-2	47800	70500	46 700	72 100	-1.9	+2.1	1	O.K.					
Ave.	51.9	37.5	25.8		46800	66700	44300	73000	-5.4	+2.1							
Max.	67.0	45.0	31.2		47800	74000	46900	75000	-11.5	+4.6							
Min.	34.5	30.0	19.5		41100	59 100	41300	70300	-1.8	+0.6							
Std.	25.6	10.1	14.2		4.3	6.7	3.4	2.0	42.9	44.8							

TABLE NO. 4
TESTS OF $\frac{1}{2}$ " ROUND
INTERMEDIATE GRADE NEW BILLET DEFORMED BARS

INTERMEDIATE GRADE STEEL													
TENSION TEST				U BEND TENSION TEST						COLD BEND TEST			
Spec. No.	Yield Point Stk.	% Elongation		Character of Fracture	Unit Stress - Lbs. per Sq. in.		Unit Stress - Lbs. per Sq. in.		% Change		Failed At	Bent 90°	Angle of Failure
		1 in.	8 in.		Yield Point	Ultimate	Yield Point	Ultimate	Y.P. Ult.	Ult.			
4A	42.8	32.5	25.0	2-9	42 300	66400	40900	67100	-3.3	+0.8	1	O.K.	
4B	40.0	36.0	23.1	2-9	43 200	64400	35800	67700	-17.1	+2.0	2	O.K.	
4C	44.5	40.0	20.4	2-11	43 500	66000	36600	66200	-15.9	+1.9	1	O.K.	
4D	46.2	42.5	26.2	2-11	44 000	67500	39200	67400	-10.9	-0.1	1	O.K.	
4E	40.8	40.0	26.1	2-9	43 800	66200	42600	66400	-2.1	+0.3	1	O.K.	
4F	39.4	37.5	25.0	2-9	42 600	66800	40000	67100	-6.1	+0.4	1	O.K.	
4G	55.0	45.0	30.0	2-9	44 300	66100	42500	66600	-4.5	+0.8	1	O.K.	
4H	40.9	37.5	26.9	2-11	43 500	67300	42100	67900	-3.2	+0.9	1	O.K.	
4I	45.5	40.0	26.6	2-11	44 100	67200	41800	67600	-5.2	+0.6	1	O.K.	
4J	46.2	37.5	26.9	2-9	42 300	67600	42300	67300	0.0	-0.1	1	O.K.	
Ave.							40400	67100	-6.6	+0.8			
Max.							42600	67900	-17.1	+2.0			
Min.							35800	66200	0.0	-0.1			
Std.							4.9	0.7	66.7	66.2			
1/2" INCH ROUND BARS													
TENSION TEST				U BEND TENSION TEST						COLD BEND TEST			
Spec. No.	Yield Point Stk.	% Elongation		Character of Fracture	Unit Stress - Lbs. per Sq. in.		Unit Stress - Lbs. per Sq. in.		% Change		Failed At	Bent 90°	Angle of Failure
		1 in.	8 in.		Yield Point	Ultimate	Yield Point	Ultimate	Y.P. Ult.	Ult.			
9L	43.2	37.5	24.5	2-9	44 000	76800	46300	76900	+3.2	+0.1	1	O.K.	
9M	37.2	32.5	23.6	2-9	46700	83500	40300	84300	+3.4	+1.0	1	O.K.	
9N	36.4	36.0	24.0	2-9	46400	82100	46000	82400	+3.4	+0.4	1	O.K.	
9O	36.0	36.0	22.5	2-11	44 300	77500	44 100	76600	+0.5	+1.4	1	O.K.	
9P	36.6	37.5	25.2	2-10	47 600	79500	48400	80100	+1.7	+0.6	1	O.K.	
9Q	34.7	36.0	24.2	2-11	47 000	76900	48300	79200	+2.6	+0.4	1	O.K.	
9R	41.5	37.5	24.5	2-9	47 100	81400	49000	82200	+4.0	+1.0	1	O.K.	
9S	34.4	33.5	25.4	2-11	46700	76300	46500	79200	+3.9	+1.1	1	O.K.	
9T	35.3	36.0	21.8	2-9	44500	80100	49200	81000	+10.5	+1.1	1	O.K.	
9U	46.0	34.5	23.1	2-9	46200	77400	47700	77900	+3.3	+0.6	1	O.K.	
Ave.	41.1	37.0	25.4		44700	73200	47800	80400	+3.6	+0.8			
Max.	56.0	45.0	30.0		47600	83500	49200	84300	+10.5	+1.4			
Min.	34.4	32.5	21.8		42300	65000	44100	77900	-0.5	-0.1			
Std.	9.9	7.0	7.2		3.3	8.0	2.2	2.1	45.0	41.3			

specifications. Considering the excellent ductility of these bars, the above fact would probably not be considered an objection.

Referring to Tables No. 3 and 4, it is seen that all bars met the requirements of ductility as regards percentage of elongation and cold bend test. All the bars also met the yield point requirement. However, in Table No. 3 there were 12 bars out of 20, and in Table No. 4 there were 10 bars out of 20 which fell below the ultimate strength requirement.

Therefore, for Tables No. 1 to 4 the results show that 16.3% of the bars fell below the yield point requirement of 40,000 pounds per square inch, with an average for the unsatisfactory bars of 11.4% below that requirement. Also, 50% of the bars fell below the minimum ultimate strength requirement of 70,000 pounds per square inch, with an average for the unsatisfactory bars of 10.4% below that requirement.

It may be stated in passing that all of these bars would satisfy the requirements for new billet bars of structural steel grade, except 38 bars whose ultimate strength was higher than the maximum called for in the specifications.

All of the bars in Tables No. 1 to 4 satisfied the ductility requirements.

The results of the U-bend tests and the V-bend tests in Tables No. 1 to 4 may be summarized as follows: In the U-bend test the bars showed an average increase in yield point of 2.8%, and an average increase in ultimate strength of 1.9%. In the V-bend test they showed an average increase in yield point of 2.5% and an average increase in ultimate strength of 1.7%. These results indicate that these new billet bars had not been injured because of the cold bending which they had undergone, and that they had the necessary ductility to withstand the cold bending to which they are subjected in practice. The results indicate further that so far as the new billet bars are concerned there was no essential difference in severity of test as between the U-bend and the V-bend test.

The uniformity of these bars may be judged by the fact that the mean variation from the average for all the bars for the items of per cent elongation in 8 inches, yield point, and ultimate strength was 12.2%, 10.6%, and 12.9%, respectively.

New Billet Hard Grade Bars.—Tables No. 5 and 6 give the results of the tests on the new billet hard grade deformed bars. Table No. 5 is for the bars from one company, and Table No. 6 for those from another. All the bars of Table No. 5, except 2, met the requirements of the American Society for Testing Materials as regards ductility, both in percentage of elongation in 8 inches and in the cold bend test. There were 17 bars out of 20 which failed to meet the yield point requirement, and 14 bars out of 20 which failed to meet the ultimate strength requirement.

TABLE NO. 5
TESTS OF $\frac{1}{2}$ AND $\frac{3}{4}$ SQUARE
HARD GRADE NEW BILLET DEFORMED BARS

Spec. No.	TENSION TEST				V BEND TENSION TEST				Failed At	Bent 180°	Angle of Failure
	% Change in Area	% Change in Dia.	Character of Fracture	Unit Stress—Lbs. per Sq. In. Yield Point Ultimate	Unit Stress—Lbs. per Sq. In. Yield Point Ultimate	% Change in Y.P.	% Change in Ult.				
10A	34.5	21.5	1-2	49000	75700	48300	76800	-1.4	+1.8	1	O.K.
10B	34.3	21.8	11-2	46700	75000	46500	76300	+9.2	+1.7	2	O.K.
10C	35.1	21.0	11-2	46900	74000	46400	75500	-1.1	+2.0	1	O.K.
10D	34.6	22.5	1-2	48400	74500	46800	76800	-3.9	+3.1	2	O.K.
10E	34.3	22.6	10-2	46300	74100	46200	75400	-0.2	+1.8	1	O.K.
10F	41.5	21.0	10-2	48100	73000	47300	74800	-1.7	+2.6	1	O.K.
10G	34.5	21.0	10-2	46600	76000	49400	77700	+1.6	+2.5	1	O.K.
10H	30.6	21.0	1-2	48000	75900	47200	74500	-1.7	+0.8	1	O.K.
10J	41.6	21.8	1-2	48100	76500	47700	75500	-0.8	-1.3	1	O.K.
10K	32.1	21.5	11-2	46800	75300	46700	75900	-4.3	+0.3	1	O.K.
Ave.	36.0	21.7		48100	74000	47500	75900	-1.3	+1.5		
Min.	41.6	22.6		49000	76500	49400	77700	-4.3	+3.1		
Max.	30.1	21.0		46700	73000	46400	74800	+0.2	+0.3		
Stdev.	6.8	2.1						16.0	63.3		

¾ INCH SQUARE BARS												Bent 60°
10A	30.5	22.0	5-2-4	52300	78200	45000	77800	-14.0	-0.3	6	O.K.	
10B	25.3	18.5	5-2-4	46800	88600	47700	86800	+1.9	-2.6	6	O.K.	
10C	31.6	24.0	5-2-4	43700	78100	45200	79600	+3.4	+1.9	2	O.K.	
10D	27.5	18.0	5-2-4	52700	92400	50400	73800	-4.0	-20.5	2	O.K.	
10E	22.5	16.8	5-2-4	44600	82300	46200	81900	+3.8	-0.7	2	O.K.	
10F	28.2	21.0	5-2-4	43600	76800	46200	80300	+6.0	+1.9	2	O.K.	
10G	23.9	20.2	5-2-4	51100	80300	46800	82300	-8.4	-3.3	6	O.K.	
10H	23.5	21.0	5-2-4	44700	83000	47000	75600	+3.2	-8.9	6	75°	
10I	25.6	21.0	5-2-4	42200	63900	44000	85500	+6.3	+2.4	1	65°	
10J	34.0	22.8	5-2-4	44700	76700	47700	78400	+6.7	-0.1	6	O.K.	
Ave.	26.4	20.5		46600	82900	46600	80200	+0.5	-3.0			
Min.	31.6	22.6		52700	92400	50400	86800	-14.0	-20.5			
Max.	22.8	18.0		42200	76100	44000	75300	+1.9	-0.1			
Stdev.	10.0	6.3		7.0	4.4	2.0	3.9	11.0	17.0			

TABLE NO. 6
TESTS OF $\frac{1}{2}$ SQUARE AND $\frac{3}{4}$ ROUND
HARD GRADE NEW BILLET DEFORMED BARS

Spec. No.	TENSION TEST				V BEND TENSION TEST				COLD BEND TEST		
	% Area	% Elongation 2 in. 8 in.	Character of Failure	Unit Stress ~ Lbs. per sq. in. Yield Point Ultimate	Unit Stress ~ Lbs. per sq. in. Yield Point Ultimate	% Change Y. P. Ult.	Failed At	Bent 180°	Angle of Failure		
11A	23.8	28.0 22.0	10-2	50200	75100	49400	76700	-1.6	+2.1	1	O.K.
11B	38.5	35.0 23.8	1-2	52400	76700	48500	78500	-7.4	-0.3	1	O.K.
11C	47.8	36.5 23.2	1-2	50600	77700	47500	77300	-6.1	-0.5	1	O.K.
11D	45.3	38.0 22.5	1-2	50300	76000	50000	77000	-0.6	-1.3	1	O.K.
11E	34.4	32.5 21.9	10-2	51000	77000	50300	77400	-1.4	+0.5	1	O.K.
11F	43.6	36.5 23.2	5-2	51400	76800	49200	79800	-4.3	+1.3	1	O.K.
11G	36.9	38.5 23.8	5-2	52400	77700	50600	79000	-3.1	+1.7	1	O.K.
11H	52.3	32.5 21.0	10-2	50300	76800	48000	76800	-4.6	-0.0	1	O.K.
11I	33.1	31.5 21.9	10-2	50000	77400	47800	77200	-4.4	-0.3	1	O.K.
11J	45.0	31.5 22.5	1-2	52200	77900	50900	76600	-2.5	+0.9	1	O.K.
Ave.	40.3	33.9 22.6		51100	77500	49200	77800	-3.6	+0.4		
Min.	52.3	38.5 23.8		52400	76800	50900	79800	-7.4	+2.1		
Max.	23.8	28.0 21.0		50000	75100	47500	76700	-0.6	0.0		
Stdev.	16.2	7.2 3.3		1.7	1.0	2.1	1.8	48.9	22.3		
¾" ROUND BARS											
11L	34.6	45.0 27.2	9-2	45500	78800	48100	79200	+3.7	+0.5	1	O.K.
11M	31.0	37.5 21.2	9-2	50400	86700	50500	88400	+0.2	+2.0	1	O.K.
11N	31.2	35.0 21.2	11-2	49000	84200	50000	82700	+2.0	-1.8	1	O.K.
11O	45.4	42.5 24.0	9-2	47600	78000	49000	79600	+2.9	+2.5	7	O.K.
11P	37.7	37.0 22.4	9-2	47000	85100	48600	86300	+3.4	+1.4	1	O.K.
11Q	38.4	35.0 22.2	9-2	48500	82200	46900	83300	+0.8	+1.3	7	O.K.
11R	33.9	34.0 23.7	9-2	49100	85400	49700	87000	+1.2	+1.9	1	O.K.
11S	32.6	31.5 21.2	5-4-2	50600	91900	51400	92400	+1.6	+0.5	1	O.K.
11T	41.8	33.5 20.6	9-2	49500	83500	50500	84500	+2.0	+1.2	1	O.K.
11U	36.0	31.5 19.4	9-2	48700	82800	48700	83300	0.0	+0.6	1	O.K.
Ave.	38.0	36.3 22.3		48600	83900	49500	84700	+2.0	+1.0		
Min.	31.6	45.0 27.2		50600	91900	51400	92400	+1.7	+2.3		
Max.	31.0	31.5 19.4		45500	78000	48100	79200	0.0	+0.5		
Stdev.	13.9	9.6 7.1		2.4	3.3	1.6	3.6	61	83		

The bars in Table No. 6 all met the requirements for ductility as represented by the percentage of elongation in 8 inches and the cold bend test. However, 8 bars out of 20 failed to meet the yield point requirement, and 12 bars out of 20 failed to meet the ultimate strength requirement.

These results, therefore, show that 5% of the bars failed to meet the ductility requirements; 62.5% failed to meet the yield point requirement of 50,000 pounds per square inch, with an average for the unsatisfactory bars of 5.8% below this requirement; and 65% failed to meet the minimum ultimate strength requirement of 80,000 pounds per square inch, with an average for the unsatisfactory bars of 4.2% below this requirement.

Again it may be noted that all of these bars would satisfy the requirements for new billet steel of intermediate grade, except one bar whose ultimate strength was too high for this grade.

The results of the V-bend tests may be summarized as follows: The bars showed an average decrease in yield point of 0.6%, and an average decrease in ultimate strength of zero. These results indicate that the bars had not been injured because of the cold bending which they had undergone. The results also indicate that these bars had the necessary ductility to withstand the cold bending to which they are subjected in practice.

The uniformity of the material may be judged by the mean variation from the average for the items of percentage of elongation in 8 inches, yield point, and ultimate strength, which was 4.7%, 3.1% and 2.5%, respectively.

Comparison of Intermediate Grade and Hard Grade New Billet Bars.—Comparing the results of the new billet intermediate grade and the new billet hard grade bars, it may be said that the intermediate grade met the ductility requirements somewhat better than the hard grade, but that in both grades there was too large a percentage of bars which fell below the minimum requirements of strength, both in yield point and in ultimate strength.

The uniformity of the hard grade seemed to be somewhat better than that of the intermediate grade. This, however, must be discounted because the hard grade material represented only half as many bars as the intermediate grade, and furthermore the hard grade was all part of one shipment, while the intermediate grade was obtained at two different times.

The results indicate that both the intermediate grade and the hard grade new billet bars were not injured by the cold bending which these bars have to undergo in practice.

Rerolled Bars.—Tables No. 7 to 10 give the results obtained from the rerolled deformed bars. Tables No. 7 and 8 are for the bars from one company, and Tables No. 9 and 10 are for the bars from a second company. In these tables the yield point values in the U-bend test were omitted for most of the bars because the specimens showed no well defined yield point.

TABLE NO. 7
TESTS OF $\frac{3}{8}$ " ROUND
REROLLED DEFORMED BARS

Spec. No.	TENSION TEST				U BEND TENSION TEST				COLD BEND TEST				
	Spec. Size	% Elongation 2 in.	% Elongation 8 in.	Character of Fracture	Unit Stress - Lbs. per Sq. In. Yield Point	Unit Stress - Lbs. per Sq. In. Ultimate	Unit Stress - Lbs. per Sq. In. Yield Point	Unit Stress - Lbs. per Sq. In. Ultimate	% Change Y.P. Ult.	Failed At	Bend 180°	Angle of Rupture	
6A	24.3	12.5	10.0	5-6	72 000	132 000	Broke in Bending	—	119 500	—	-9.8	8	O.K.
6B	17.2	15.0	11.2	5-6	71 000	129 000	Broke in Bending	—	—	—	—	—	O.K.
6C	24.3	15.0	9.3	5-6	69 500	132 500	—	—	—	—	—	—	O.K.
6D	27.3	17.5	11.2	5-6	63 100	113 000	Did not fit Template	—	—	—	—	—	O.K.
6E	20.7	15.0	9.3	5-6	63 000	130 000	—	—	123 800	—	-4.6	2	O.K.
6F	34.0	22.5	13.1	5-6	70 200	114 000	—	—	114 600	—	+0.5	1	O.K.
6G	25.6	17.5	10.6	5-6	65 000	116 200	—	—	110 700	—	-6.3	6	O.K.
6H	34.2	22.5	14.4	5-6	69 600	113 500	—	—	43 400	—	-61.7	6	O.K.
6I	25.6	15.0	12.5	5-6	71 000	128 900	—	—	96 000	—	-25.5	2	45°
6J	25.8	15.0	10.0	5-6	71 000	121 500	—	—	124 300	—	+2.3	8	
Ave.									104 600		-15.0		
Max.									124 300		-61.7		
Min.									43 400		+0.5		
Stdev.									19.1		108		
3/8" ROUND BARS													
7A	31.3	18.5	12.5	5-4	75 000	120 000	86 600	126 700	+13.5	+5.2	2	O.K.	
7B	5.8	9.0	6.6	5-4	74 500	124 500	Broke in Bending	—	—	—	—	—	O.K.
7C	15.1	12.5	9.4	5-4	77 600	136 500	89 900	129 800	+18.9	-4.9	1	O.K.	
7D	8.1	5.5	4.4	5-4	75 700	123 100	—	74 100	—	-40.0	6	O.K.	65°
7E	19.7	20.0	15.0	5-4	78 200	123 000	75 400	127 200	-3.6	+3.4	1	O.K.	
7F	24.9	20.5	12.6	5-4	76 600	126 600	87 300	134 300	+14.0	-1.6	6	O.K.	
7G	16.8	21.0	14.3	5-4	79 000	124 100	75 400	127 000	-4.6	+2.3	1	O.K.	
7H	6.7	2.5	9.0	5-4	76 000	137 700	—	83 300	—	-59.6	6	O.K.	52°
7I	33.1	17.5	11.5	5-4	72 600	125 300	84 500	122 800	+6.4	-1.8	6	O.K.	
7J	22.4	21.0	14.5	5-4	71 300	114 300	71 700	119 700	+0.6	+4.8	1	O.K.	78°
Ave.	22.7	15.8	11.1		72 100	124 400	81 900	114 900	+7.1	-8.1			
Max.	34.2	22.5	15.0		79 000	137 700	89 000	129 800	+14.0	-40.0			
Min.	5.8	2.5	4.4		63 000	113 000	71 700	74 100	+0.6	-1.8			
Stdev.	32.3	26.1	18.0		3.0	4.8	7.8	14.0	14.0	175.0			

TABLE NO. 8
TESTS OF $\frac{3}{8}$ " ROUND
REROLLED DEFORMED BARS

Spec. No.	TENSION TEST						U BEND TENSION TEST						COLD BEND TEST	
	Spec. Length in.	% Elongation		Character of Failure	Unit Stress—Lbs. per Sq. In.		Unit Stress—Lbs. per Sq. In.		% Change		Failed At	Bend 180°	Angle of Rupture	
		2 in.	8 in.		Yield Point	Ultimate	Yield Point	Ultimate	Y.P.	Ult.				
6A	19.0	20.0	13.7	7-2-5	67 200	116 400	25 700	32 400	-61.8	-72.2	4	O.K.		
6B	25.0	22.5	16.2	7-2-5	67 400	112 300	67 500	113 800	+0.1	+1.3	2	O.K.		
6C	17.0	17.5	16.2	7-2-5	63 400	103 900	Didn't fit Template						O.K.	
6D	25.6	25.0	17.5	7-2-5-8	62 700	102 300	66 100	107 700	+8.6	+5.3	2	O.K.		
6E	18.9	20.0	15.6	7-2-5	62 800	103 600	64 800	109 400	-1.6	+5.6	8	O.K.		
6F	21.9	20.0	12.5	7-2-5	—	122 000	Didn't fit Template						O.K.	
6G	19.0	17.5	12.5	7-2-5-8	64 300	121 200	69 000	113 200	+7.3	-5.0	2	O.K.		
6H	30.6	20.0	12.5	7-2-5	69 400	111 300	67 000	63 700	-3.3	-36.8	2	O.K.		
6I	17.8	20.0	13.7	7-2-5	64 400	121 300	67 600	114 200	+5.0	-9.9	2	O.K.		
6J	3.4	2.5	2.5	7-2-5	69 500	94 400	66 100	83 200	-4.9	-11.9	4	O.K.		
Ave.							61 800	90 000	-6.4	-13.5				
Max.							69 000	115 200	-6.9	-72.2				
Min.							25 700	32 400	+0.1	+1.3				
Stdev.							14.6	22.5	21.7	130				
$\frac{3}{8}$ " ROUND BARS														
							V BEND TENSION TEST							
7L	25.6	27.5	16.2	5-4	67 800	116 000	66 400	117 800	+0.9	-0.2	6	O.K.		
7M	25.1	23.5	15.3	5-4	66 800	117 000	67 000	110 500	+0.3	-5.6	1	O.K.		
7N	6.1	5.3	7.6	5-4	73 000	114 200	75 100	121 900	+2.9	+6.7	6	O.K.		
7Q	11.4	9.0	8.8	5-4	73 400	107 300	73 600	87 900	+0.3	-16.6	2	O.K.		
7P	19.3	23.0	13.0	5-4	75 000	124 400	104 500	125 800	+30.3	+1.1	6	O.K.		
7Q	9.9	11.5	9.3	5-4	61 400	106 400	73 600	85 100	+20.2	-30.0	2	O.K.		
7R	22.2	28.0	14.1	6-4	61 300	106 000	70 400	107 200	+14.9	+1.1	6	O.K.		
7S	21.8	16.5	14.0	5-4	62 100	108 800	Broke in Bending						4	O.K.
7T	9.1	15.0	7.8	5-4	62 500	106 200	Broke in Bending						4	O.K.
7U	19.7	25.0	13.8	5-4	72 800	122 200	73 400	119 800	+0.6	-2.0	2 & 6	O.K.		
Ave.	16.4	16.7	12.6		66 700	112 000	75 800	109 400	+9.9	-4.7				
Max.	30.6	28.0	17.5		75 000	124 400	104 500	125 800	+30.3	-30.0				
Min.	3.4	2.5	2.5		61 300	94 400	67 000	85 100	+0.3	-0.2				
Stdev.	25.6	28.0	22.0		5.6	6.1	9.5	11.1	113	160				

TABLE NO. 9
TESTS OF $\frac{1}{2}$ " SQUARE
REROLLED DEFORMED BARS

Spec. No.	TENSION TEST					U BEND TENSION TEST		COLD BEND TEST		2 ND COLD BEND TEST	
	% Red of Area	% Elongation 2 in.	% Elongation 8 in.	Change of Fracture	Unit Stress—Lbs. per Sq. In. Yield Point Ultimate	Ultimate Lbs. per Sq. In.	% Failed Ompg. At	Bent 180°	Angle of Failure	Bent 180°	Angle of Failure
5A	7.7	7.5	5.0	2-4-5-6	90500	125000	Broke in Bending		15°		70°
5B	9.8	12.5	10.6	2-4-5	77600	129100	Broke in Bending		95°		160°
5C	30.8	15.0	12.0	1-2-7-8	70200	119000	+2.5 8	O.K.			
5D	23.1	15.0	10.6	1-2-7-8	69200	117000	+2.6 2		75°		
5E	12.9	12.5	9.4	2-4-5	76300	126700	Broke in Bending		55°		90°
5F	28.4	13.5	12.0	1-2-7-8	70400	123000	89500	+21.2 2	O.K.		
5G	29.2	17.5	12.5	1-2-7-8	68900	117300	119400	+1.6 2	O.K.		
5H	5.7	5.0	4.3	2-4-5	69900	96300	Broke in Bending		O.K.		30°
5I	37.7	20.0	12.0	1-3-6	73800	113600	+7.1 1	O.K.			
5J	40.7	20.0	11.2	1-3-6	65800	116000	Broke in Bending		O.K.		O.K.
5K	26.0	22.5	16.2	2-4-5-7	61100	93000	21200	-77.2 8	O.K.		
5L	29.2	16.5	12.5	1-2-7-8	63500	109700	11800	-98.2 4-6		80°	
5M	42.5	16.5	12.5	1-3-6	66800	117900	106500	-8.0 4-6	O.K.		
5N	2.0	1.5	1.2	2-4-5	65800	122000	Broke in Bending		50°		70°
5O	39.4	20.0	13.1	1-2-3-8	61500	107000	Broke in Bending		75°	O.K.	
5P	6.0	7.5	8.7	2-4-5	70400	113200	Broke in Bending		115°	O.K.	
5Q	6.7	5.0	5.0	2-4-5	—	137300	Broke in Bending		35°		15°
5R	40.5	20.0	12.5	1-2-6	80100	106500	113900	+3.0 6	O.K.		
5S	7.4	5.0	5.0	2-4-5	65500	99600	Broke in Bending		O.K.		O.K.
5T	47.5	25.0	17.5	1-2-3	85500	103000	24700	-76.0 6	O.K.		
55A	41.6	20.0	10.0	1-2-6-8	74000	127000	127700	+5.5 6	O.K.		
55B	4.7	0.0	0.0	2-4-5	80800	128200	Broke in Bending		30°		30°
55C	2.7	2.5	1.2	2-4-5	78200	100300	Broke in Bending		75°		5°
55D	22.3	17.5	10.6	2-5-7-8	69200	113000	109700	-2.9 2	O.K.		
55E	21.2	12.5	10.0	2-5-7-8	71900	126000	100000	-20.6 2	O.K.		
55F	33.0	20.0	12.0	1-2-6-8	67000	116900	42700	-64.0 2	O.K.		
55G	36.7	17.5	12.5	1-2-6-8	66700	119000	Broke in Bending		55°	O.K.	
55H	40.0	17.5	12.0	1-2-6-8	70100	125000	125000	0 2	O.K.		
55I	49.2	25.0	16.2	1-2-6-9	61400	104600	73700	-29.5 2	O.K.		
55J	36.2	17.5	12.5	1-2-6	75300	116700	77700	-33.4 2	O.K.		
55K	17.5	12.5	9.4	2-4-5	76400	127500	Broke in Bending		40°		50°
55L	22.9	17.5	10.6	1-2-7-8	73200	124700	125100	+0.3 6	O.K.		
55M	13.9	12.5	10.0	2-4-5	77400	124500	8400	-93.4 6		45°	
55N	33.1	17.5	23.7	1-2-7-8	69600	122200	116300	-3.2 6		65°	
55O	10.0	10.0	9.4	2-4-5	78500	119700	84500	-29.4 2	O.K.		
55P	4.0	2.5	1.3	2-4-5	69300	103000	Broke in Bending		20°		45°
55Q	6.0	5.0	5.0	2-4-5	100000	140500	Broke in Bending		15°		55°
55R	20.2	17.5	10.6	2-4-5-7	76000	123500	Broke in Bending		45°		55°
55S	1.2	1.5	0.4	2-4-5-7	—	81600	Broke in Bending		50°	O.K.	
55T	49.5	25.0	15.0	1-2-3	61800	98000	22200	-77.5 2	O.K.		
Ave	23.5	13.8	9.9		73000	116000	85000	-27.6			
Max.	49.5	25.0	23.7		100000	140500	127700	-93.4			
Min	1.2	0.0	0.0		61100	81600	6400	0			
%Var	56.2	43.5	37.1		8.6	6.2	42.4	106			

TABLE NO. 10
TESTS OF $\frac{3}{4}$ " SQUARE
REROLLED DEFORMED BARS

Spec. No.	TENSION TEST						U BEND TENSION TEST				COLD BEND TEST	
	% Red of Area	% Elongation		Character of Fracture	Unit Stress—Lbs. per Sq. In.		Ultimate Lbs. per Sq. In.	% Change	Failed At	Bent 90°	Angle of Failure	
		2 In.	8 In.		Yield Point	Ultimate						
5A	37.3	17.5	17.4	1-2-3	57 500	97 600	15 400	-84.3	2	O.K.		
5B	31.8	25.5	15.7	1-2-6	57 900	104 000	4 300	-96.0	8	O.K.		
5C	21.3	13.5	12.1	2-4-5	71 600	123 300	Broke in Bending				60°	
5D	31.1	19.5	15.5	1-2-6	64 900	114 900	12 500	-88.2	2		65°	
5E	35.9	25.0	17.7	1-2-6	64 000	107 400	22 400	-79.1	8	O.K.		
5F	37.5	30.5	20.9	1-2-6	59 000	90 500	Broke in Bending			O.K.		
5G	29.5	21.0	12.4	2-4-5-6	70 700	119 000	Broke in Bending			O.K.		
5H	31.7	22.0	13.1	1-2-4	72 000	112 000	Broke in Bending			O.K.		
5I	35.4	31.0	17.0	1-2-6-8	60 300	99 000	20 200	-78.6	2	O.K.		
5J	33.0	22.0	14.7	1-2-6	66 800	112 000	16 300	-85.5	8	O.K.		
5K	24.7	19.0	14.2	2-4-5	63 900	106 500	Broke in Bending				65°	
5L	6.0	7.5	5.2	2-4-5	62 500	96 300	Broke in Bending			O.K.		
5M	23.2	19.5	14.5	2-4-5-8	61 400	106 100	11 800	-88.9	8		50°	
5N	7.0	7.0	8.5	2-4-5	67 000	107 500	Broke in Bending				55°	
5O	19.0	19.0	14.5	2-4-5	64 000	104 000	"	"	"		85°	
5P	42.5	23.0	17.7	1-2-6	66 000	105 000	"	"	"	O.K.		
5Q	20.8	11.0	11.5	2-4-5	—	120 000	"	"	"	O.K.		
5R	42.1	32.0	17.5	1-2-6	67 800	107 000	86 300	-19.3		O.K.		
5S	12.6	17.5	12.0	2-4-5	64 600	107 000	Broke in Bending			O.K.		
5T	38.6	27.5	15.2	1-2-6	62 800	104 000	"	"	"		50°	
55A	23.4	20.0	12.5	2-4-5-8	73 300	117 500	"	"	"		70°	
55B	10.8	10.0	7.5	2-4-5	64 000	105 000	"	"	"		40°	
55C	38.5	27.5	15.0	1-2-4-6	64 000	105 500	19 400	-81.5	8	O.K.		
55D	12.0	10.0	12.0	2-4-5	63 300	109 200	Broke in Bending				45°	
55E	20.4	20.0	15.0	2-4-5-8	66 700	110 500	16 500	-85.0	2	O.K.		
55F	13.8	15.0	12.5	2-4-5	66 300	106 500	Broke in Bending				55°	
55G	30.5	22.5	15.6	1-2-8	66 000	109 000	"	"	"		50°	
55H	15.6	20.0	13.7	2-4-5	67 200	110 000	"	"	"		55°	
55I	25.2	22.5	13.7	2-4-5	64 900	109 500	"	"	"		50°	
55J	6.3	7.5	7.0	2-4-5	67 000	105 000	"	"	"	O.K.		
55K	40.0	27.5	15.0	1-2-3	70 000	112 000	"	"	"	O.K.		
55L	25.6	22.5	14.4	1-2-7	72 000	111 000	"	"	"		60°	
55M	18.0	17.5	17.5	1-2-4-5	67 300	110 500	"	"	"		40°	
55N	25.6	17.5	15.7	2-4-5-7	62 600	104 500	38 100	-63.5	6		55°	
55O	34.6	20.0	16.2	1-2-3	74 000	119 000	18 400	-84.5	2	O.K.		
55P	7.8	7.5	6.2	2-4-5	62 700	104 000	Broke in Bending			O.K.		
55Q	46.6	25.0	16.2	1-2-3	66 400	99 900	"	"	"	O.K.		
55R	17.7	15.0	2.5	2-4-5-8	72 000	127 000	"	"	"		45°	
55S	34.4	22.5	15.1	1-2-3	62 000	111 000	"	"	"	O.K.		
55T	35.5	25.0	17.0	1-2-7	60 700	100 500	16 700	-83.4	4	O.K.		
Ave.	26.0	19.6	15.9		65 500	106 300	22 900	-78.4				
Max.	46.8	32.0	20.9		74 000	127 000	86 300	-96.0				
Min.	6.0	7.0	5.2		57 500	90 500	4 300	-19.3				
Stdev.	36.4	26.1	18.0		4.9	4.9	52.6	14.5				

Note: In the U-bend test specimen No. 5R was tested up 50 000 lbs. The test could not afterward be completed because the specimen was mislaid.

In Tables No. 7 and 8 there are 9 different bars out of 40 which failed to meet the ductility requirements as represented by the percentage of elongation in 8 inches and the cold bend test. All the bars met the strength requirements called for in respect to yield point and ultimate strength.

The results of the U-bend and the V-bend tests may be summarized as follows: In the U-bend test, 2 bars out of 20 broke while being bent into the U shape. Of the remaining bars for which results were obtained there was an average loss in yield point of 6.4%, and in ultimate strength of 14.2%. In the V-bend test 3 bars out of 20 broke in bending them to the V shape. Of the remaining bars for which results were obtained, there was an average increase in yield point of 8.9%, and an average loss in ultimate strength of 6.5%.

The uniformity of these bars may be judged by the fact that the mean variation from the average for all the bars in the items of percentage of elongation in 8 inches, yield point, and ultimate strength was 20.5%, 5.3%, and 5.5%, respectively.

Tables No. 9 and 10 give the results of the rerolled bars from the second company. Of these, 42 different bars out of 80 failed to meet the ductility requirements as represented by the percentage of elongation in 8 inches and the cold bend test. All the bars met the strength requirements called for in respect to yield point and ultimate strength.

The results of the U-bend test may be summarized as follows: Out of 80 bars, 45 broke while being bent into the U shape. Of the remaining bars for which results were obtained there was an average loss in ultimate strength of 46.5%.

The uniformity of these bars may be judged by the fact that the mean variation from the average for all bars in the items of percentage of elongation in 8 inches, yield point, and ultimate strength was 27.6%, 6.7% and 6.6%, respectively.

Comparison of New Billet and Rerolled Bars.—The intermediate grade and hard grade new billet bars will be discussed as one group, and compared with the rerolled bars.

As to strength requirements represented by yield point and ultimate strength, all the rerolled bars satisfied the requirements. Of the new billet bars, 38 out of 120, or 31.7%, failed to meet the yield point requirement of the American Society for Testing Materials, falling 7.7% below that requirement; and 66 out of 120, or 55%, failed to meet the ultimate strength requirement, falling 8% below that requirement.

As to ductility requirements, represented by percentage of elongation in 8 inches and the cold bend test, only 2 out of 120 new billet bars, or 1.7%, failed to meet the requirements. The rerolled bars, on the other hand, especially those in Tables No. 9 and 10, showed serious lack of ductility. Of all the rerolled bars tested, 51 out of 120, or 42.5%, failed to meet the ductility requirements.

The U-bend test again showed this lack of ductility of the rerolled bars, 47 out of 100, or 47%, failing while being bent into the U shape. For the remaining bars which were tested in the U-bend test there was an average loss of 6.4% in yield point, and 36.8% in ultimate strength. None of the new billet bars broke in bending to the U shape, and the test showed no essential loss of strength due to cold bending.

The V-bend test showed results similar to those of the U-bend test. The new billet bars showed no essential loss of strength due to cold bending. Of the rerolled bars shown in Tables No. 7 and 8, three bars out of 20, or 15%, failed while being bent into the V shape. For the remaining bars which were tested in the V-bend test there was an average increase of 8.9% in the yield point, but an average loss of 6.5% in ultimate strength. The results of the V-bend test for the rerolled bars No. 5 and 55 are given in Table No. 11, and will be discussed separately later. Of these bars, 31 out of 80, or 38.8%, failed while being bent into the V shape. The remaining bars showed an average loss in yield point of 3%, and in ultimate strength of 7.4%.

It is seen, therefore, that a large percentage of the new billet bars fell somewhat below the yield point requirement, a still larger percentage fell somewhat below the ultimate strength requirement, but practically all showed satisfactory ductility. The rerolled bars, on the other hand, showed satisfactory tensile strength, but a large percentage showed a serious lack of ductility. The new billet bars showed no essential loss in strength after cold bending, while the rerolled bars showed a serious loss in ultimate tensile strength.

The uniformity of the material may be judged by the mean variation from the average in the items of percentage of elongation in 8 inches, yield point, and ultimate strength, in the standard tension test. This mean variation from the average was 9.7%, 8.1%, and 9.4% for the new billet, and 25.2%, 6.2%, and 6.2% for the rerolled bars, respectively. These results do not show much advantage of one steel over the other, except in ductility, in which the new billet has the advantage.

The difference in uniformity of the material was especially significant in connection with the U-bend and the V-bend tests. In the U-bend test the new billet intermediate grade bars showed a mean variation from the average, in the item of ultimate strength, of 1.6%, while the rerolled bars showed 38.6%. Here it must be remembered that the result for the rerolled bars was based only on the 53% of the bars which did not break in bending. The remaining 47% of the bars broke in bending to the U shape.

In the V-bend test the new billet bars of both grades showed a mean variation from the average, in the item of ultimate strength, of 2.2%, while the rerolled bars showed 12.3%. This includes the results given in Table No. 11 in which some of the rerolled bars were tested with the lugs ground off. The result for the rerolled bars was

again based only on the 66% of the bars which did not fail in bending. The remaining 34% broke in bending to the V shape.

It is seen, therefore, that as far as uniformity of material is concerned, the new billet bars were satisfactory, but the rerolled bars were not. The rerolled bars are evidently very unreliable when they are used under conditions which subject them to cold bending.

It should be stated here that the results on the rerolled bars of the U-bend test, the V-bend test, and the tests for ductility were very much better in the case of one company than they were in the case of the other company.

Further Tests on Rerolled Bars.—A representative of one of the companies which furnished the rerolled bars suggested that the projections on the deformed bars caused serious local deformation when they were bent cold, which accounted for the large proportion failing in cold bending.

In order to investigate this matter a specimen was cut from each of the 18 bars shown in Table No. 9 which had broken in bending to the U shape. The projections on these bars were milled off, and a second cold bend test was made with the plane surfaces next to the pins. The results are given in Table No. 9 under "Second Cold Bend Test," and show that 6 bars out of 18 passed the test, while 12, or 66.6%, failed to pass the test. In the first cold bend test 3 of these bars passed the test and 15 did not. The results indicate, therefore, that the cold bend test with the lugs on is slightly more severe than with the lugs off, but that in general the failure in cold bending was due to brittleness of material rather than to the presence of projections on the bars. A fact which should be particularly noted is that some bars failed which had passed the first cold bend test, while others passed which had failed in the first test. This indicates that some of the rerolled bars were not uniform along the individual bar. Since, in practice, these bars are bent cold with the projections on the bars as rolled, it is evident that they should have sufficient ductility to meet such service conditions.

Table No. 11 shows the results of some additional tests which were made on the rerolled bars No. 5 and 55 to determine whether the projections on the bars had much influence in determining the unsatisfactory results which had been obtained in the U-bend test. Of the bars reported in Tables No. 9 and 10 there was enough of the original material remaining so that a V-bend test could be made on each bar. Of the $\frac{1}{2}$ inch bars, 20 bars were bent into the V shape and tested in the usual way, the other bars first had the projections or lugs ground off, and were then bent into the V shape and tested. In each group of 20 bars there were 9 which had previously failed in bending to the U shape.

The results in Table No. 11 show that 7 bars out of 20 with the lugs on broke in bending to the V shape, while 4 bars out of 20 with the lugs ground off broke in bending. Of the bars with the

TABLE NO. 11
TESTS OF REROLLED DEFORMED BARS
½" SQUARE BARS ¾" SQUARE BARS
LUGS LEFT ON LUGS LEFT ON

V BEND TENSION TEST						V BEND TENSION TEST						
Spec. No.	Unit Stress—Lbs. per Sq. In.		% Change		Failed At	Spec. No.	Unit Stress—Lbs. per Sq. In.		% Change		Failed At	
	Yield Point	Ultimate	Y.P.	Ult.			Yield Point	Ultimate	Y.P.	Ult.		
5N	Broke in Bending						5L	60 700	94 200	-2.4	-4.2	2
55Q	.	.				5K	61 000	89 800	-4.5	-15.7	2	
55P	.	.				5J	63 800	69 200	-4.5	-38.2	4	
5A	.	.				5I	61 700	94 500	+2.3	-4.5	4-6	
5S	.	.				5H	69 900	103 200	-2.9	-7.9	2-5	
5Q	.	.				5F	60 300	78 300	+2.2	-13.5	2	
55B	.	.				5E	60 400	102 300	-5.6	-4.7	3-6	
5R	59 800	102 800	-25.2	-5.3	6	5D	—	31 500	—	-72.5	5	
5I	68 900	122 700	-6.6	+8.0	2	5B	62 600	102 800	+8.1	-1.2	6	
5B	66 500	79 000	-14.3	-38.8	4	5A	55 200	96 000	-4.0	-1.6	2	
5T	63 000	106 200	-24.0	+3.1	7	Ave.	61 700	86 200	-1.3	-16.4		
55H	70 000	127 000	-0.1	+1.6	1	Max.	69 900	103 200	+8.1	-72.5		
5C	67 000	122 300	-4.6	+2.8	1	Min.	55 200	31 500	+2.2	-1.2		
55A	68 900	129 500	-6.9	+2.0	6	%Var.	3.8	18.5	278	94.9		
55T	61 600	100 500	-0.3	+2.6	7	Lugs Ground Off						
55I	65 000	108 100	+5.9	+3.3	7	55T	58 300	94 800	-4.0	-5.7	2-5	
55J	63 900	116 000	-15.1	-0.6	1	55E	64 500	97 200	-3.3	-12.0	2-5	
5F	66 200	98 000	-6.0	-20.3	2	5M	68 600	103 400	+11.7	-2.5	6	
5O	63 400	86 700	+6.3	-19.0	6	55G	67 200	105 000	+1.8	-3.7	2	
55F	68 100	120 000	+1.6	+0.9	7	55N	62 300	104 200	-0.5	-0.3	6	
Ave.	65 900	109 100	-6.9	-4.6		55Q	60 700	99 100	-7.2	-0.4	2	
Max.	70 000	129 500	-25.2	-38.8		55S	57 300	96 200	-7.6	-13.3	2	
Min.	59 800	79 000	-0.1	-0.6		5R	63 400	106 800	-6.2	-0.2	2	
%Var.	3.4	11.7	110	218		5P	63 300	82 700	-4.1	-21.2	2-5	
5P	Lugs Ground Off						55K	—	48 200	—	-57.0	2
55K	Broke in Bending						Ave.	62 800	93 800	-2.2	-11.6	
55S	.	.				Max.	68 600	106 800	+11.7	-57.0		
55C	.	.				Min.	57 300	48 200	-0.5	-0.2		
5K	67 300	93 000	+10.1	0.0	6	%Var.	4.5	12.0	196	98.2		
5J	66 400	117 200	+0.9	+1.0	2							
55M	72 600	72 600	-6.2	-41.7	2							
55F	70 500	124 500	+5.2	+4.7	1							
55O	66 400	119 500	-13.2	-0.2	1							
5E	67 800	123 300	-3.7	+0.2	1							
55N	68 800	123 200	-1.2	+0.8	6							
5M	69 100	124 400	+0.4	+5.5	1							
5D	68 600	122 000	-0.9	+4.3	2							
5L	70 200	110 200	+1.1	+0.5	2							
55G	68 500	120 400	-0.3	+1.2	2							
5G	68 700	122 200	-0.3	+4.2	2							
55D	66 700	115 400	-3.6	+2.1	6							
55R	68 700	107 800	-9.6	-12.7	4							
5H	69 200	105 900	-1.0	+10.0	6							
55H	71 000	121 000	+1.3	-3.2	2							
Ave.	68 800	113 900	-1.3	-1.5								
Max.	72 600	124 500	-13.2	-41.7								
Min.	66 400	72 600	-0.3	0.0								
%Var.	1.8	8.6	285	446								

lugs on which broke, all had also previously broken in making the U bend; while of those which did not break, 2 had previously broken in making the U bend. Of the bars with the lugs ground off which broke, all had previously failed in making the U bend; while of the bars which did not break, 5 had previously failed in making the U bend. These results all indicate that in general, bending with the lugs ground off was not as severe as with the lugs on. The loss in yield point and ultimate strength indicate the same thing, being somewhat greater for the bars with the lugs on. Since 4 out of 20 bars which had the lugs ground off broke in bending, the failure of 20% of these bars must be attributed to brittleness of material rather than to the presence of projections on the bars.

The results on the $\frac{1}{2}$ inch bars with the lugs on in Table No. 11, as compared with the results in Table No. 9, would tend to show that the U-bend test was more severe than the V-bend test. This is seen by comparing the average loss in ultimate strength, which is 27.6% for the U-bend test, and 4.6% for the V-bend test.

The tests in Table No. 11 on the $\frac{3}{4}$ inch bars were carried out somewhat differently than were those on the $\frac{1}{2}$ inch bars. The bars were first bent into the V shape. Twenty bars out of 40 broke during this process, and Table No. 11 records the results only of the bars which did not fail. Before testing in the V-bend test, 10 of the bars had the lugs ground off while the remaining 10 were tested with the lugs on.

Table No. 11 indicates that the average loss of the $\frac{3}{4}$ inch bars in yield point and ultimate strength due to the cold bending was about the same whether the bars had the lugs ground off or not. This result would lead to the conclusion that when these bars had enough ductility so that they did not fail in bending, then there was no difference in their action due to the presence of the lugs.

Comparing the results for the $\frac{3}{4}$ inch bars with the lugs on in Table No. 11 and in Table No. 10 it is seen that for the U-bend test the average loss in ultimate strength was 78.4%, while in the V-bend test it was 16.4%. These results indicate again that for these rerolled bars the U-bend test was more severe than the V-bend test.

Micrographs and Chemical Analyses.—It was not the purpose of this investigation to make an exhaustive study of the reasons why the bars gave the results which are recorded in this bulletin. However, it seemed desirable to point out the probable reasons for some of the results.

Figures 5 and 6 show micrographs obtained from some typical samples of the various steels. In each case the steels of the larger diameter were used. In Figure 5, A and B are new billet, intermediate grade, and C and D new billet, hard grade bars. These micrographs show a large proportion of the soft ferrite, which explains the high ductility which these steels exhibited. This was further verified by the chemical analyses of carbon content for these steels which

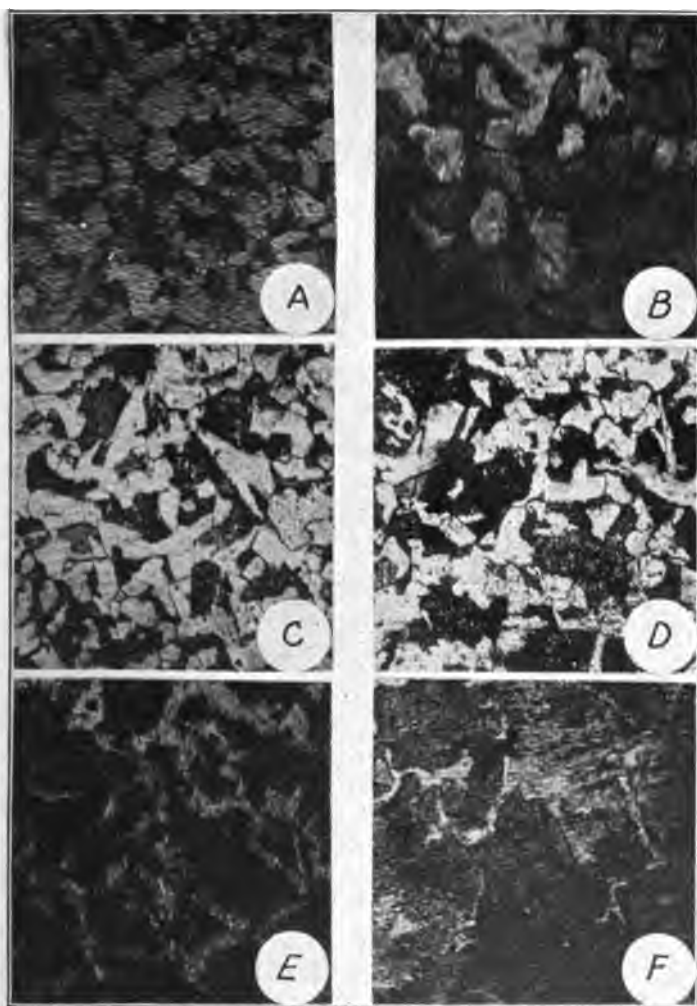


Figure 5.—Micrographs of Specimens

Magnification is 100 diameters

- A.—Specimen No. 9L, new billet, intermediate grade
- B.—Specimen No. 8T, new billet, intermediate grade
- C.—Specimen No. 11L, new billet, hard grade
- D.—Specimen No. 8F, new billet, hard grade
- E.—Specimen No. 7R, rerolled
- F.—Specimen No. 5B, rerolled

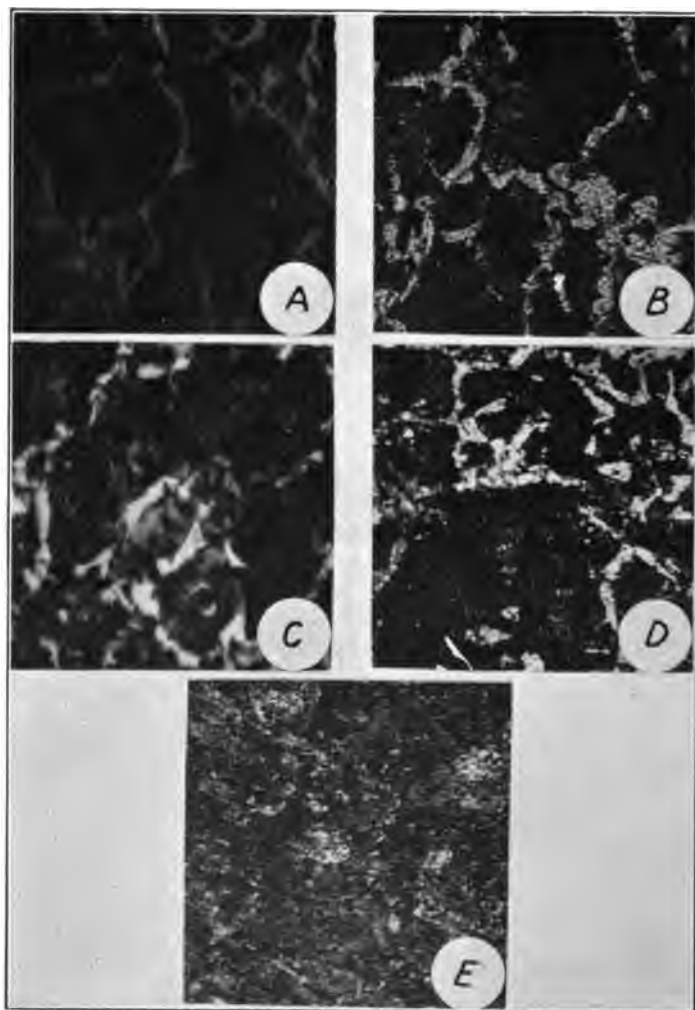


Figure 6.—Micrographs of Specimens

Magnification is 100 diameters

- A.—Specimen No. 5R, rerolled
- B.—Specimen No. 55B, rerolled
- C.—Specimen No. 55J, rerolled
- D.—Specimen No. 55N, rerolled
- E.—Specimen No. 55R, rerolled

are given in Table No. 12. These analyses show a carbon content of 0.35%, 0.19%, 0.34%, and 0.38% for A, B, C, and D, respectively.

TABLE NO. 12
CHEMICAL ANALYSES AND RESULTS
OF PHYSICAL TESTS FOR SOME TYPICAL SPECIMENS

Spec. No.	Size of Bar	Carbon Content in %	Silicon Content in %	Manganese Content in %	Phosphorus Content in %	Elongation in 8 in. %	Yield Point Lbs per Sq. in.	Ultimate Lbs per Sq. in.	Cold Bend Test	U Bend Test
9L	1/8"	0.35				24.5	44 000	78 800	O.K.	
8T	3/8"	0.19				35.0	34 300	56 500	O.K.	
11L	1/8"	0.34				27.2	45 500	78 800	O.K.	
10F	3/8"	0.38				21.0	43 600	78 800	O.K.	
7R	7/8"	0.30	0.09	0.69	0.087	14.1	61 300	106 000	O.K.	
5B	3/4"	0.32	0.21	0.65	0.016	15.7	57 900	104 000	O.K.	
5R	3/4"	0.41	0.12	1.12	0.081	17.5	67 600	107 000	O.K.	
55B	3/4"	0.48	0.08	0.66	0.099	7.5	64 000	105 000	Failed	Broke in bending
55J	3/4"	0.44	0.10	0.76	0.099	7.0	67 000	105 000	O.K.	
55N	3/4"	0.48	0.12	0.77	0.072	13.7	62 600	104 500	Failed	
55R	3/4"	0.61				12.5	72 000	127 000	Failed	Broke in bending

Table No. 12 also records the results obtained in the tension test, cold bend test, and U-bend test, of the individual bars for which the micrographs and chemical analyses were made. A comparison of these results with the carbon content is of interest. It will be noted that Specimen No. 9L, new billet intermediate grade steel, with a carbon content of 0.35%, met the requirements of the American Society for Testing Materials in yield point and in ultimate strength.

Specimens No. 11L and 10F, new billet hard grade steel, had practically the same carbon content as intermediate grade Specimen No. 9L, and these two specimens failed to meet the specifications for yield point and ultimate strength for hard grade deformed bars.

The remaining specimens in Table No. 12 are rerolled steel bars. Here the carbon content alone did not explain the results obtained in the physical tests. For instance, Specimens No. 7R and 5B, with 0.50% and 0.52% carbon, respectively, passed all the tests; but Specimens No. 55B, 55J and 55N, with 0.48%, 0.44%, and 0.48% carbon, respectively, failed in ductility. Of the latter, No. 55B failed in percentage of elongation, in the cold bend test, and in the U-bend test; No. 55J failed in percentage of elongation and in the V-bend test; while No. 55N failed in the cold bend test. Specimen No. 55R, with a high carbon content of 0.61%, failed in ductility. It will be noted that the 4 bars which failed in ductility had either a high carbon content or a medium carbon content in combination with a high phosphorus content.

In making the tension test of Specimen No. 55B there was observed a phenomenon which helps to explain further its lack of ductility. It showed a flaw 2 inches long, the full width of the bar being peeled off, and at one point near the side, the thickness of the bar was reduced from 0.720 inches to 0.645 inches. When this bar had been broken, a second test was made on it and developed another flaw, a wide flat sliver 1.5 inches long peeling off.

It may be mentioned here that Specimens No. 6D and 6G, $\frac{3}{8}$ inch round rerolled bars, also developed seams or longitudinal cracks over their entire length, thin layers peeling from the surface. These specimens, however, passed all the tests satisfactorily. Some of the other rerolled bars numbered 5 and 55 were examined under the microscope and showed fissures in some cases and ferrite streaks in others.

While these individual cases of flaws just mentioned help to account for some of the failures of these bars, yet it does not seem likely that they explain why so large a proportion of the rerolled bars were especially lacking in ductility.

In order to make a further study of the relation of the results of physical tests to the chemical composition of the steels, Table No. 13 was prepared. This table gives the carbon and phosphorus content and also the results from the physical tests for certain bars.

The first 6 bars listed in Table No. 13 are intermediate grade new billet bars. Bars No. 9R, 3A and 3B met the strength requirements for yield point and ultimate strength, while bars No. 8L, 4J and 4D failed either in the yield point or ultimate strength requirements. All the weaker bars except No. 4D had a lower carbon content than the bars which passed the tests.

Bars No. 11M to 10F in Table No. 13 are hard grade new billet bars. Bars No. 11M, 10D and 11S met the strength requirements for yield point and ultimate strength, while bars No. 11 O, 10C, and 10F did not meet either the yield point or ultimate strength requirements. It will be noted that the weaker bars were all lower in carbon content.

Bars No. 6B to 6 I are rerolled bars from one company. Bars No. 6B, 6D and 7L met the requirements for ductility, while bars No. 7D, 7H and 6 I failed in ductility in the cold bend test, in percentage of elongation, or in both. It will be noted again that the bars which lacked ductility were the ones that had a high carbon content.

Bars No. 5M to 5H are rerolled bars from a second company. Bars No. 5M to 55F met all requirements for ductility as represented by the per cent of elongation and the cold bend test, and none of them failed in bending for the U-bend test. Bars No. 5A to 5H all failed in bending for the U-bend test, all but 2 failed in the cold bend test, and all but 4 failed in per cent of elongation. It will be noted that in every case failure in ductility was accompanied either by a high carbon content, or a medium carbon content in combination with a high phosphorus content. For instance, bars No. 55H and 55B both had a high carbon content of 0.59%; but 55H had a low phosphorus content of 0.023% with satisfactory ductility, while 55B had a high phosphorus content of 0.135% with unsatisfactory ductility. On the other hand, bar No. 5K, with a high phosphorus content of 0.147% was ductile enough because it has a low carbon content of 0.30%.

Tables No. 12 and 13 indicate, therefore, that the low strength of many of the intermediate grade and hard grade new billet bars was largely due to a low carbon content. These tables also indicate that

TABLE NO. 13
CHEMICAL ANALYSES AND
RESULTS OF PHYSICAL TESTS

Spec No	Size of Bar	Carbon Content in %	Phosphorus Content in %	Elongation in 8 in %	Yield Point Lbs. per Sq. in.	Ultimate Lbs. per Sq. in.	Cold Bend Test	U Bend Test
9R	$\frac{7}{8}$ "	0.40	0.026	24.5	47100	81400	O.K.	
3A	$\frac{7}{8}$ "	0.30	0.010	25.0	55400	90600	O.K.	
3B	$\frac{7}{8}$ "	0.41	0.010	18.7	56100	96600	O.K.	
8L	$\frac{3}{4}$ "	0.17	0.013	32.0	35000	55000	O.K.	
4J	$\frac{7}{8}$ "	0.27	0.007	26.9	42300	67600	O.K.	
4D	$\frac{7}{8}$ "	0.31	0.008	26.2	44000	67500	O.K.	
11M	$\frac{7}{8}$ "	0.51	0.035	21.2	50400	86700	O.K.	
10D	$\frac{3}{4}$ "	0.45	0.017	18.0	52700	92400	O.K.	
11S	$\frac{7}{8}$ "	0.46	0.048	21.2	50600	91900	O.K.	
11O	$\frac{7}{8}$ "	0.36	0.022	24.0	47600	78000	O.K.	
10C	$\frac{3}{4}$ "	0.32	0.039	21.7	43700	78100	O.K.	
10F	$\frac{3}{4}$ "	0.38	0.043	21.0	43600	78800	O.K.	
6B	$\frac{7}{8}$ "	0.43	0.097	16.2	67400	112300	O.K.	
6D	$\frac{7}{8}$ "	0.40	0.086	17.5	62700	102300	O.K.	
7L	$\frac{7}{8}$ "	0.45	0.097	16.2	67800	118000	O.K.	
7D	$\frac{5}{8}$ "	0.63	0.047	4.4	75700	123100	Failed	
7H	$\frac{7}{8}$ "	0.62	0.036	9.0	76000	137700	Failed	
6I	$\frac{7}{8}$ "	0.62	0.022	12.5	71700	128900	Failed	
5M	$\frac{1}{2}$ "	0.57	0.022	12.5	68800	117900	O.K.	
55T	$\frac{1}{2}$ "	0.29	0.116	15.0	61800	96000	O.K.	
5K	$\frac{1}{2}$ "	0.30	0.147	16.2	61100	93000	O.K.	
55H	$\frac{1}{2}$ "	0.59	0.023	12.0	70100	125000	O.K.	
5T	$\frac{1}{2}$ "	0.24	0.104	17.5	69500	103000	O.K.	
55I	$\frac{1}{2}$ "	0.29	0.117	16.2	61400	104600	O.K.	
5C	$\frac{1}{2}$ "	0.32	0.026	12.0	70200	119000	O.K.	
5R	$\frac{1}{2}$ "	0.29	0.102	12.5	80100	108500	O.K.	
5G	$\frac{1}{2}$ "	0.53	0.024	12.5	68900	117300	O.K.	
55J	$\frac{1}{2}$ "	0.59	0.021	12.5	75300	116300	O.K.	
55F	$\frac{1}{2}$ "	0.56	0.030	12.0	67000	118900	O.K.	
5A	$\frac{1}{2}$ "	0.72	0.031	5.0	90500	125000	Failed	Broke in bending
5O	$\frac{1}{2}$ "	0.61	0.020	13.1	61500	107000	"	"
5P	$\frac{1}{2}$ "	0.48	0.165	6.7	70400	113200	"	"
5B	$\frac{1}{2}$ "	0.42	0.114	10.6	77600	129100	"	"
55S	$\frac{1}{2}$ "	0.68	0.042	0.4	—	81600	"	"
55B	$\frac{1}{2}$ "	0.59	0.135	0	80800	128200	"	"
55P	$\frac{1}{2}$ "	0.63	0.161	1.3	69300	103000	"	"
5Q	$\frac{1}{2}$ "	0.83	0.047	5.0	—	137500	"	"
55G	$\frac{1}{2}$ "	0.59	0.023	12.5	68700	119000	"	"
5S	$\frac{1}{2}$ "	0.55	0.132	5.0	65500	99600	O.K.	"
5H	$\frac{1}{2}$ "	0.56	0.128	4.3	69900	96300	O.K.	"

the poor ductility of many of the rerolled bars was largely due either to a high carbon content or a combination of medium carbon with a high phosphorus content.

The American Society for Testing Materials, in the case of Bessemer steel rails, calls for carbon ranging from 0.37% to 0.55%, and phosphorus with a maximum of 0.10%; while for open hearth steel rails the carbon ranges from 0.50% to 0.75%, with a maximum phosphorus of 0.04%. It may be expected, therefore, that many of the rerolled bars will be too hard to have proper ductility, either due to

high carbon or, in Bessemer rails, of medium carbon in combination with high phosphorus.

Conclusions.—The conclusions which may be drawn from the tests recorded in this bulletin are as follows:

1. Deformed bars of new billet, intermediate grade steel showed satisfactory ductility. A large percentage of the bars fell below the strength requirements, especially for ultimate strength, 50% of the bars falling 11.4% below the ultimate strength requirement. This lack of strength seemed to be due to low carbon content.

2. Deformed bars of new billet, hard grade steel showed a small percentage which failed to meet the ductility requirements. A large percentage of the bars failed to meet the strength requirements, 65% of them falling 4.2% below the ultimate strength requirement. This lack of strength seemed to be due to low carbon content.

3. Deformed bars of rerolled steel all showed satisfactory strength. A large percentage of the bars, 42.5%, failed to meet the ductility requirements. However, the results for the bars from one company were considerably better than those from the other company.

4. Cold bending of new billet bars, both of intermediate grade and hard grade, did not decrease the tensile strength of these bars. None of these bars failed in bending to the U or V shape.

5. Cold bending of rerolled bars seriously decreased the tensile strength of these bars. Furthermore, 47% of these bars failed in bending to the U shape. However, the results for the bars from one company were very much better than those from the other company.

6. The standard cold bend test and also the U-bend and V-bend tests, showed that the rerolled bars were likely to be very unreliable in the matter of uniformity of material. The micrographs and the chemical analyses helped to explain this non-uniformity.

7. Projections on the rerolled bars made it more difficult for these bars to withstand cold bending, but when the bars had proper ductility the presence of projections was of little importance.

8. The poor ductility of the rerolled bars seemed to be due either to a high carbon content, or a medium carbon content in combination with a high phosphorus content.

9. For the new billet bars the results in the U-bend and the V-bend tests were practically the same. For the rerolled bars, on the other hand, the U-bend test was considerably more severe than the V-bend test.

10. In the opinion of the writer, the new billet bars, while not meeting rigidly all the ordinary requirements for such steel, yet may be considered as reasonably satisfactory. The rerolled bars, on the other hand, while of satisfactory strength, must be considered as very unreliable for work under conditions subjecting them to cold bending.

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**EXPERIMENTAL STUDY OF AIR LIFT PUMPS AND
APPLICATION OF RESULTS TO DESIGN**

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EXPERIMENTAL STUDY OF AIR LIFT PUMPS AND APPLICATION OF RESULTS TO DESIGN

INTRODUCTION

The experimental work on the air lift pump presented in this bulletin was started in 1919 when the problem arose of selecting foot pieces for air-lift pumps which were to be installed in certain wells belonging to the State of Wisconsin. Mr. John C. White, Chief Engineer State Power Plants, and Mr. W. G. Kirchoffer, Consulting Engineer, who were in direct charge of the work for the State, invited Professor C. I. Corp of the University of Wisconsin, and the writer to assist in certain tests which they proposed to make on particular foot pieces that were being considered for the new installations. The apparatus for these tests was erected at the Capitol Heating Plant where the first part of the work was completed, particularly that portion relating to the comparison of foot pieces. This preliminary work confirmed the deductions made from data taken in previous air lift experimental work, which was done at the University of Wisconsin. That is, that if certain fundamental and simple principles were observed in the design of a foot piece, it appeared to give essentially the same performance as any other one; in other words, the foot piece is relatively the minor consideration in successful air lift performance. The conditions in the suction pipe, where the energy of the air is expended in producing motion of the water, are the factors which really determine the success or failure of design for economical operation of the air-lift pump. To investigate this question it seemed desirable to conduct further experiments along lines suggested by the preliminary tests upon apparatus which was somewhat larger—with suction pipes of greater diameter and of greater length—than those used in the previous University tests. To facilitate

the work, the apparatus was set up at the Hydraulic Laboratory of the University where the remainder of the work was completed.

The problem was divided into several parts which were assigned to the following students who performed the experimental work under the writer's direction, and presented their results as theses for the degree of Bachelor of Science in Engineering: Messrs. H. A. Lange, B. H. Puermer, K. P. Barth, F. W. Krez, L. H. Kessler, O. N. Rove, and W. W. Pidcoe. Messrs. R. O. Ruble, H. H. Gumprecht, and W. B. Newing have assisted the writer in an able manner in the calibration of the air orifices.

The writer acknowledges the assistance given by Mr. John C. White, Mr. W. G. Kirchoffer and Professor C. I. Corp, each of whom took an active part in the preliminary work. Mr. White cooperated in this work not only by giving valuable suggestions but also by furnishing equipment for various parts of the work.

Mr. Lewis H. Kessler, a thesis student mentioned above, became an Instructor in the Hydraulic Engineering Department of the University of Wisconsin in the fall of 1922. He has given very valuable assistance in computing and compiling the summary data and in preparing illustrations for this bulletin.

SCOPE OF EXPERIMENTAL WORK

The laboratory investigation of the air lift pump, presented here, began with a series of tests of different foot pieces which are shown in figure 1. Four of these foot pieces—A, B, C and D—were tested in an eduction pipe $2\frac{1}{2}$ inches in diameter and about 61 feet long. The general arrangement of the apparatus in these tests is shown in figure 2 for runs made with foot-pieces A and B, while that for the other comparative foot-piece tests is shown in figure 3.

A real well was not used, but the entire pump was erected above ground as shown in figure 3. This arrangement made possible the control of the pressure on the foot-piece which would not be the case in a real well. The arrangement shown in figure 3 was devised to permit a closer duplication of the conditions that are found in a real well than was the cases in the use of the apparatus of figure 2, particularly with respect to entrance conditions at the bottom of the foot-piece. In both of the arrangements used for the above tests the same eduction pipe was used. The Venturi foot-piece, B, figure 1, was tested in an 8 inch well casing, first with the air supply line leading through the top of the well casing to the foot-piece and, second with the air admitted to the top of the casing which served as a receiver for the air. As the air pressure increased in the casing, the water level was lowered until opening *a*—threaded opening into which the air line was screwed in the previous test—was uncovered, at which time air began to flow into the foot-piece. It was thought that possibly the storage of air right at the foot-piece, which tended to cause more uniform conditions of flow in the eduction pipe, might have some influence on the efficiency of the pump. A three inch Harris foot-piece was tested in the same casing and connected to a 3 inch eduction pipe 61.26 feet long. Since this same pipe had been used in a test with the slotted foot-piece (3 inch size) of the same general design as that of C, figure 1, a comparison of a commercial foot-piece with the others could be made.

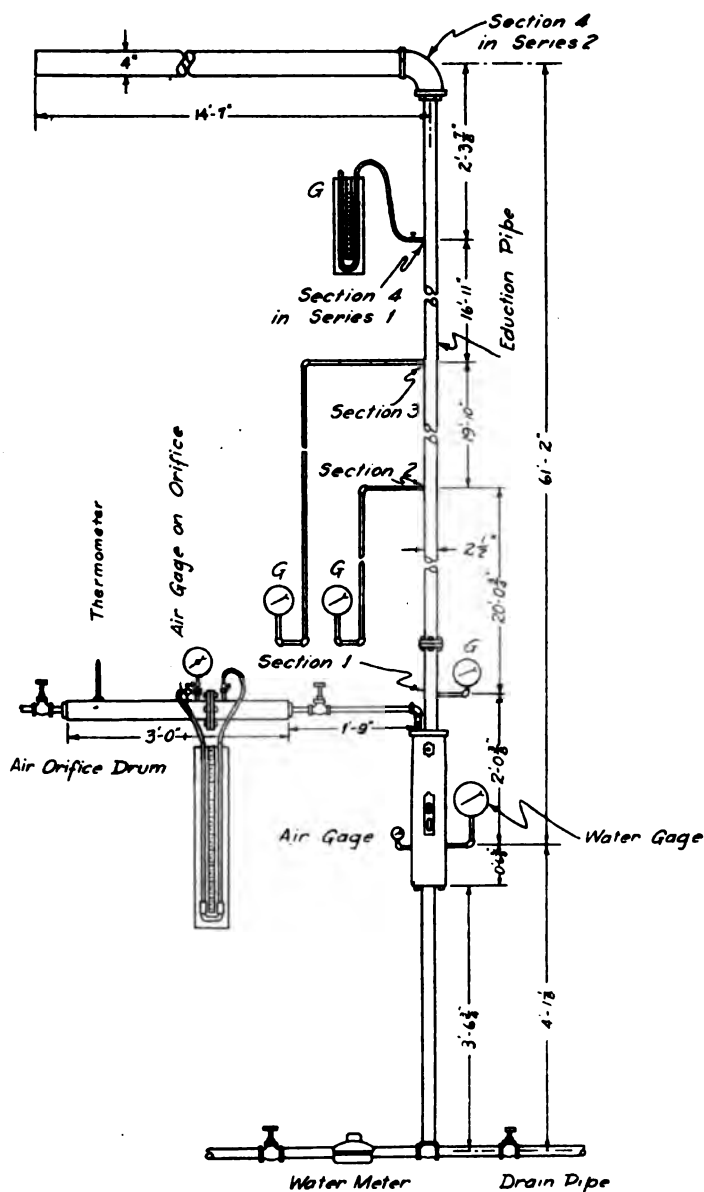


Fig. 2—Arrangement of apparatus in series 1 and 2

The above investigation showed that little advantage was to be gained by refinement of design in foot-pieces. The conclusions reached in regard to foot-pieces were similar to those of Messrs. Davis and Weidner in a previous University Bulletin.¹ It was concluded that the important consideration of air lift pump performance was that connected



Fig. 2a—Photograph of apparatus used in series 2.



Fig. 2b — Photograph of window in foot-piece A.

with the flow in the eduction pipe. Further information which might throw light on this phenomenon seemed desirable.

The experimental work which followed took up the investigation of flow in eduction pipes. Eduction pipes of different lengths of a given diameter, of different diameters all of the same length, and of eduction pipes made up of pipes of different diameter in a particular pump, were tested. In this study, the simple slotted foot-piece of the design shown in C, figure 1, was adopted because it could be easily produced in the various sizes that were required.

¹ *An Investigation of the Air Lift Pump* by G. J. DAVIS and C. R. WEIDNER, Bulletin of the University of Wisconsin, Engineering Series, Vol. 6, No. 7

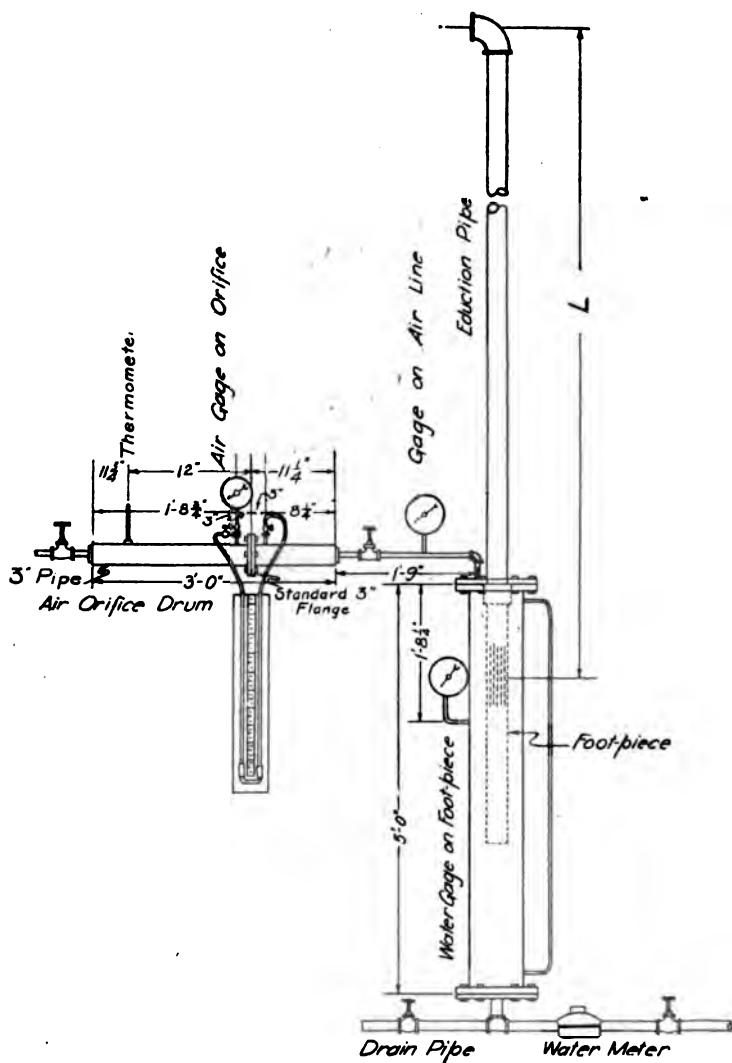


Fig. 3—Arrangement of apparatus in series 3 to series 13.

In certain series, pressure observations were taken at various points along the eduction pipe, and in one series, pressures and pressure variations were measured at gaging points at various levels of the eduction pipe by the use of an indicator with a special motor driven drum. The indicator as modified for the above purpose is shown in figure 4.



Fig. 4—Photograph of pressure indicator as arranged for making pressure-time measurements.

Each air lift pump installation, both in the series on foot-piece comparison and the series on the study of various eduction pipes, was tested at per cents of submergence ranging in some cases as low as thirty per cent and up to 80 or 85 per cent. At each of these submergences, runs were made with flows ranging from the minimum to the maximum attainable. Particular attention was given to the study of the relations which existed between velocity of flow in the eduction pipe and the accompanying losses of head with the view of determining how those relations were modified by such factors as size of eduction pipe, lift, and per cent submergence. It was thought that a knowledge of these relations was necessary in order that test data might be used in a rational way in designing air lift pumps. Equations of the form ($V = \frac{h_d}{C \log r}$), which are generally used for determining the quantity of air necessary for design purposes, are apt to be but rough guides. Such a form-

ula can be of very little value in design unless it is based on test data taken from an existing pump in which all of the conditions, such as depth of well, lift, and rate of discharge, are similar to those encountered in the design. In such a case no formula is needed. In other words, such an empirical formula is a very poor basis for extending experience since it does not include separate terms to care for the important individual factors which are involved.

DESCRIPTION AND GENERAL DISCUSSION OF THE AIR LIFT PUMP

An air lift pump is a device for raising liquids or a mixture of liquids and solids through a vertical pipe, partially submerged in the liquid, by means of compressed air introduced near the lower end of the pipe. The air admitted into the column of water in the eduction pipe causes the water column to rise and to overflow at the top of this pipe.

The general principles of air lift pumping have been understood for a long time. Carl E. Loëscher, a German mining engineer, invented it in 1797. About fifty years later it was put into practical use. Several patents have been issued by the United States patent office which cover certain features of this type of pump.

The particular merit of the air lift pump is the mechanical simplicity of the pump itself. When such a pump is installed in a well there is not a moving part below the surface of the ground. It is not affected by sand or grit which may be carried in suspension in the liquid pumped. Another advantage which it possesses is that of a large pumping capacity. The air lift pump can usually be designed so as to give a greater rate of discharge from a well of a given size than is practical with other types of pump which might be installed in that well. Where there are several wells in any particular system, each well may be operated and controlled from a central station where the air compressor is located. Repairs which are required on an air lift system usually can be made with a short shut-down of the pump. In order to

insure a reasonably constant service with other pumps, where days are required to make such repairs, an extra well and pumping unit must be held in reserve.

Its efficiency is lower than that which may be obtained with certain other pumps. Due to the difficulty in maintaining other deep well pumps, it is doubtful if the efficiency of a properly designed air lift pump is, in the course of continued operation, much less than that which is actually attained by other pumps.

An air lift pump cannot ordinarily be used economically in a well in which the depth of liquid is not greater than the height through which the liquid is to be raised. Its efficiency is very low when the conditions in the well prevent the installation of the proper air lift pump.

The fact that there is considerable discordance in data presented and in the conclusions reached by different observers on the performance of air lift pumps, is not surprising when one considers the relative complexity of the factors involved in it. A failure to understand and to properly take into account all of the elements affecting the flow of water in pipes—a problem relatively simple as compared to the flow of the combined air and water column in the air lift pumps—has brought forth results which are far from uniform and consistent. Experiments on a particular installation, where many of the factors involved are fixed, do not usually furnish a sufficient basis for the determination of any general law governing the problem studied. The complexity of a natural phenomenon does not necessarily increase directly as the number of factors increase, but, rather as the control of the individual factor becomes more difficult. Where a change in one condition affects all others, or where it is impossible or impractical to separate the effects of various causes, greater difficulty is experienced in analyzing the problem; and the deductions drawn by different observers are likely to be in greater discord. The air lift pump involves several of such interrelated factors.

The relative importance of the influence of the foot-piece on the operation of the air lift pump has been uncertain because of different deductions reached by various experimenters and writers on this subject.

DEFINITION OF TERMS USED

Two air lift pumps are illustrated in the sectional drawings of figure 5. The essential features of the pump itself are the eduction pipe and the air supply line. These parts are clearly indicated in this figure. The part A, of the pump

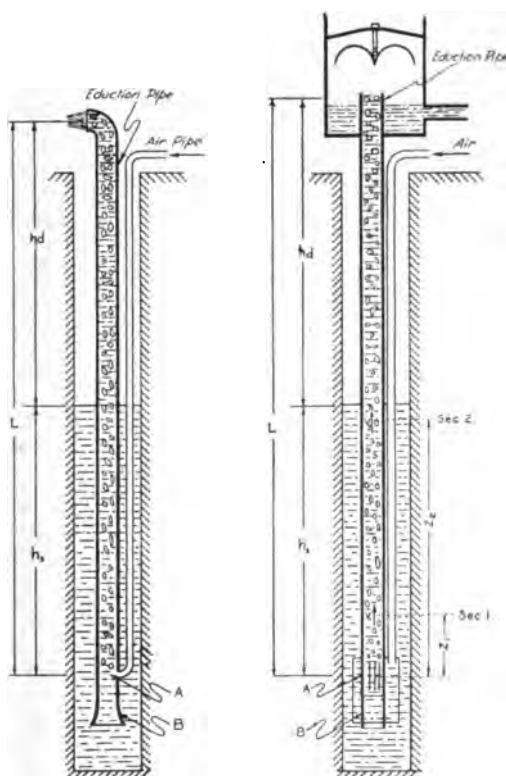


Fig. 5—Sections through wells to illustrate two typical air lift pump installations.

through which air is delivered to the eduction pipe and in which the air mixes with the liquid, is called the foot-piece. The lower end of the eduction pipe B, which serves as an inlet for the fluid, is called the tail-piece.

The explanation of other special terms used in this bulletin is given in the following table of nomenclature.

NOMENCLATURE

See Fig. 5

- A = internal cross sectional area of eduction pipe, square feet.
 C = coefficient of discharge of air orifice.
 d = diameter of air orifices, inches.
 E = efficiency of air lift pump, per cent.
 h_a = atmospheric pressure head, expressed in feet of water¹.
 h_d = delivery head, feet.
 h_s = depth of submergence of foot-piece, feet.
 G = gallons of water pumped per minute.
 H_1 = total head at any section (1) of eduction pipe, feet of water.
 H' = loss of head due to friction in eduction pipe, feet of water.
 H'_{1-2} = loss of head between any two sections (1) and (2) in eduction pipe, feet of water.
 H'' = head equivalent of energy added or given to the liquid due to expansion of the air, feet of water.
 H''_{1-2} = head equivalent of energy added by expansion of the air between any two sections (1) and (2), feet of water.
 H''' = energy in the air used per gallon of water pumped, foot pounds.
 i = drop in pressure through air orifice, inches of water.
 L = length of eduction pipe, feet.
 P_a = atmospheric pressure, pounds per square inch.

¹ The word "water" used in this nomenclature refers to the liquid which was used in the experiments discussed here. In general, the word "water" would be replaced by the name of the liquid which is pumped.

- p = pressure of air in front of air orifice, pounds per square inch, absolute.
 P_1 = pressure at any section (1), pounds per square foot absolute.
 Q = quantity of free air used per gallon of water per foot of lift, cubic feet.
 R = 53.37 for air in the relation $PV = WRT$.
 r = ratio of expansion of air.
 S = per cent submergence, $= \frac{100h_s}{L}$ per cent.
 v = velocity of mixture in eduction pipe, feet per second.
 v_{1-2} = average velocity of mixture between sections (1) and (2).
 V_a = volume of free air at 14.7 pounds per square inch pressure and at 60 degrees Fahrenheit, cubic feet per second.
 V_1 = volume of air passing section (1), cubic feet per second.
 V_{av} = average volume of the air in the eduction pipe, cubic feet per second.
 t = temperature of air or water, degrees Fahrenheit.
 T = temperature in degrees Fahrenheit, absolute.
 w = weight of liquid pumped, pounds per cubic foot. In these tests, w equals 62.4 pounds.
 w_a = weight of one cubic foot of air at atmospheric pressure, pounds.
 W = pounds of air per second used by the air lift pump.

DESCRIPTION OF APPARATUS

Water Supply.—The supply of water was taken from a high pressure main through a $1\frac{1}{2}$ " pipe in which was connected a $1\frac{1}{2}$ " Hersey water meter and a $1\frac{1}{2}$ " control valve. Beyond the valve, the water was delivered to the bottom of the foot-piece as shown in figure 2, or to the bottom of the 8 inch casing as shown in figure 3. The water meter was calibrated by passing water through it at different rates and by comparing the meter registration with the actual discharge at each rate of flow as determined by weighing the water discharged. The results of this calibration were

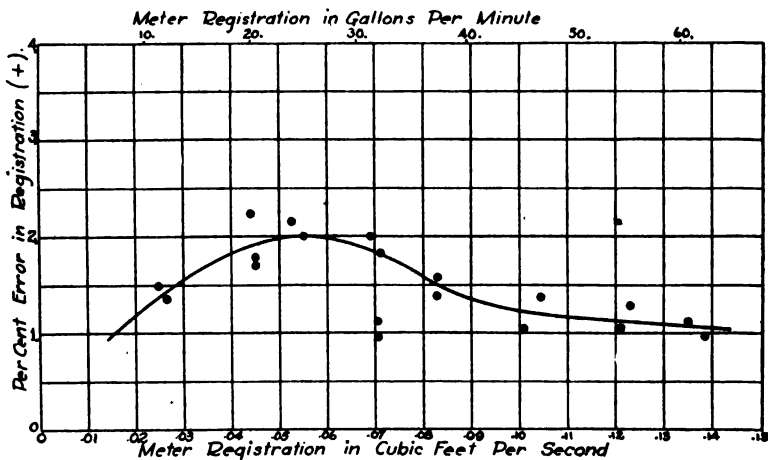


Fig. 6—Water meter calibration curve.

put in the form of a curve, shown in figure 6, which was used in correcting the meter registration of flow in the air lift experiments. In the last series of runs with the one inch eduction pipe, the $1\frac{1}{2}$ " meter was replaced by a $\frac{5}{8}$ " Hersey water meter for which a calibration was also made.

In calibrating the meters, care was taken to avoid errors due to eccentricity and imperfect dial graduation by passing such a volume of water through the meter in each run as to cause the dial pointer to make an integral number of revolutions.

Air Supply.—In the part of the work done at the Capitol Heating Plant, the air supply was furnished by a compressor which discharged into a small receiver. A $1\frac{1}{2}$ " pipe carried the supply about 200 feet to the air lift pump. In the University experiments the supply of air was taken from a tank 8 feet in diameter and 67 feet long. This tank is a part of the University pneumatic water supply system. It carries air under a pressure of about 125 pounds per square inch. Before entering the pump, the air passed through an orifice, the details of which are shown in figure 3. An angle valve immediately in front of the 3 inch orifice drum controlled the rate of flow of air. A handle 8 inches long attached to the valve stem made possible fine adjustments of the flow of air. As shown in figure 3, the orifice plates were attached between two flanges of the standard 3 inch pipe, which formed the orifice drum. Rubber gaskets were used to make up the joints between the orifice plates and the flanges. Orifices were made in iron plates about .065 inches thick. The opening was beveled on the downstream side so as to leave the metal around the orifice about .02 inches thick. Dowels fastened to the flanges of the drum fitted in holes made in the different orifice plates and served to center the orifices in the drum. The drop in pressure through the orifice was measured by a water manometer consisting of a simple water U tube, one leg being connected to the air drum above the orifice, and the other below the orifice. The hose connections from the U tube were attached to $\frac{1}{4}$ inch nipples fitted into the 3 inch drum. These $\frac{1}{4}$ inch piezometers were inserted so that the ends of the pipes were just flush with the inside of the drum. The pressure in front of the orifice was obtained from a Bourdon gage which was connected to the drum as shown in figure 3. A thermometer well extending nearly through the 3 inch pipe carried the thermometer which measured the temperature of the air in front of the orifice. It was found in the first series of runs that the pressure pulsations set up in the eduction pipe were reflected back to the gages on the orifice drum so as to make readings of the gages difficult under certain conditions of flow in the pump. To avoid the necessity of dampening gages by throttling valves in the

gage connections, a small tank 18 inches in diameter and 2 feet long was connected in the air line between the orifice drum and the foot-piece. The pressure variations on the orifice gage were reduced considerably by this means. Later, however, it was found that a valve connected in the air line beyond the orifice, as shown in figure 3, was more effective in ironing out pressure fluctuations than was the air tank. This valve served not only the purpose of reducing pressure fluctuations in the air orifice drum, but also made possible the control of the air pressure there. This increased the range of the rate of flow of air which could be measured with a particular orifice. Attached to the air line immediately in front of the point of connection to the foot-piece, there was a gage for measuring the air pressure. The orifice and orifice drum followed the general arrangement used by Professor Elmo Harris in measuring air by orifices discharging against pressure other than atmospheric. The 3 inch drum, which is smaller than the one used by Professor Harris, was selected for these tests because of the desirability of having a small compact apparatus which could be easily carried about for field measurements.

The air orifice coefficients obtained by Professor Harris were used in a preliminary analysis of the air lift pump tests. Since Professor Harris' coefficients were obtained with an orifice arrangement which was different than that used in the Wisconsin experiments, the preliminary results were only comparative among themselves. The air orifices were calibrated after the completion of the air lift pump experiments, as explained later. All air flow computations presented in this bulletin are based upon the correct coefficients of the orifices.

Air Orifice Calibration.—The air orifice drum was connected to a cylindrical steel tank 5 feet in diameter and about 12 feet long. A compressor was used to charge the tank with air at a pressure of about 110 pounds per square inch. After filling the tank, it was allowed to stand until the temperature of the air within became uniform throughout the tank and approximately equal to the atmospheric temperature outside. Water which condensed in the tank was blown out through valves in the bottom of the tank.

During a test, air was allowed to flow from the tank to the orifice drum. Because of the decrease of pressure in the tank it was necessary to regulate a valve in the air line in front of the orifice to obtain a uniform rate of flow during a test. Observations of pressure and temperature of the air in the tank were taken every two minutes during a run. The pressure was read from two standard Bourdon test gages. The temperature was obtained from thermometers located in long thermometer wells which extended to the center of the tank and which were located near the ends of the tank. The actual rate of flow, as determined from measurements at the tank, divided by the theoretical discharge of the orifice gave the coefficient of the orifice.

Air Orifice Formula.—The following form of the Fliegner¹ formula, which is recommended by Professor Harris² for orifices of the type used in these experiments, was adopted.

$$(1) \quad W = .1639 d^2 C \sqrt{\frac{i}{T-p}} \quad .$$

where

- W = weight of air discharged per second, pounds.
- p = absolute pressure in front of orifice, pounds per square inch.
- d = diameter of orifice, inches.
- i = pressure drop through orifice, inches of water.
- T = absolute temperature of air, degrees Fahrenheit.
- C = experimental coefficient.

The above formula is applicable to either an orifice discharging into the atmosphere or into a reservoir in which the pressure is different than atmospheric, providing the pressure drop through the orifice is low. If the pressure drop is too great, the change of temperature and change of density of the air passing the orifice become appreciable factors.

The diameters of orifices used in the air lift experiments

¹ A. S. M. E., Transactions, vol. 27, page 197

² *Compressed Air* by Elmo G. Harris, page 36.

were as follows: .2874, .5015, .7085, and 1.0033 inches. In referring to the orifices they will be named by the following nominal diameters: $9/32$, $1/2$, .707, and 1, although the actual measured diameters were used in all computations. It is important to note that considerable refinement in measuring the diameter of a small orifice is necessary in order to determine the area of such an orifice with any degree of accuracy. An error of .01 inch in the diameter of a $9/32$ inch orifice would introduce an error of almost 7 per cent in the area of that orifice.

Results of Air Orifice Calibration Tests.—It was found for those orifices for which the calibration equipment was adequate, that the coefficients given by Professor Harris for orifices discharging into the atmosphere could be applied to our small orifices. The coefficients obtained for the $1/2$ inch orifice checked very closely with the values given by Mr. Harris for an orifice of that size. Since there was a leakage in the tank which was more or less variable, as indicated by leakage tests at different pressures taken on different days, more uncertainty existed in the data for the $9/32$ inch orifice than in the data for the $1/2$ inch orifice. The magnitude of the leakage becomes an appreciable percentage of the quantity actually passing the small orifice. This is particularly true for the small rates of flow through such an orifice. Leakage of the tank was cared for in the computations of actual discharge, but since leakage is more or less erratic, and since average values must be used in leakage corrections, some uncertainty is carried into the orifice coefficient. The effect of discrepancies in leakage became apparent in the $9/32$ inch orifice in runs with a head drop through the orifice less than 2 inches. Coefficients for these runs failed to show as great consistency as those for runs at higher discharges. In the case of large orifices, particularly in the one inch orifice, and to a slight extent in the .707 inch orifice, the capacity of the tank was too small for the higher discharging capacities of these orifices. The range of drop of pressure that could be used and measured in the tank took place in a very short period of time, and the length of a run had to be shortened. Correct average temperatures in the tank could not be obtained when the pressure changed

so quickly, because of the wider range of temperature change which would occur under this condition.

Comparison of Orifices.—Because of the uncertainties in the calibration that existed at the lower rates of discharge of the small orifices and the upper rates of discharge of the large orifices, these orifices were checked against each other. The coefficients obtained for the one-half inch orifice were

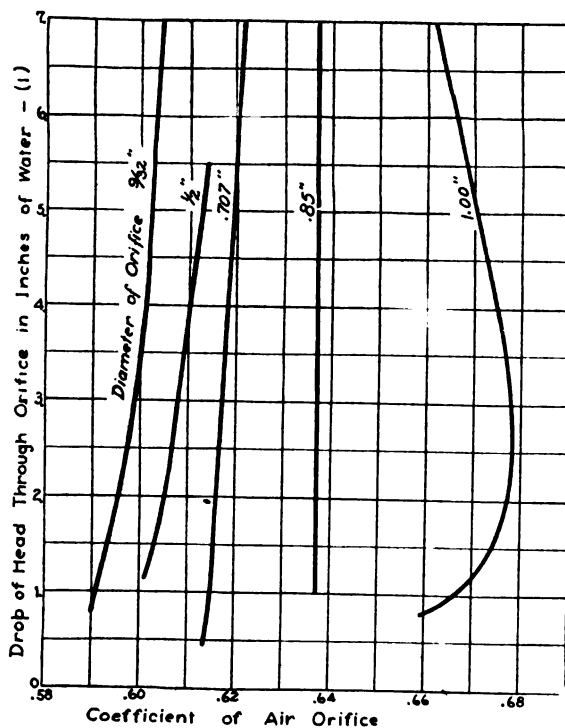


Fig. 7.—Air orifice coefficient curves.

consistent in themselves and agreed well with Harris' experiments so that these coefficients were accepted as correct. By passing air through the 9/32 inch orifice, which was located in a second drum similar to the one used in the previous experiments, and then through the one-half inch orifice—the two orifice drums being connected in series—the coefficients of the 9/32 inch orifice could be obtained. A valve located between the two orifice drums gave a pressure control which allowed a wider range of comparative tests

than one might anticipate. With the small orifice upstream, a high pressure could be maintained upon it by the use of the above valve. With a given (i), or head drop through the small orifice, and with a high pressure on this orifice, such a quantity of air would be discharged as to cause a considerable head drop through the larger downstream orifice which was discharging under lower pressure. The same comparative study was made then between the .707 inch orifice and the $\frac{1}{2}$ inch orifice and then between the .707 inch orifice and the 1 inch orifice. The coefficients determined from these comparative tests are shown in figure 7. In this figure (C) is plotted against (i), a separate curve being drawn for each orifice. These coefficients checked quite closely with the Harris' coefficients for the small orifices. Runs on the air lift pump using orifices greater in diameter than .707 inches, which had been computed previous to the orifice calibration test, were recomputed using the new coefficients which were obtained for these orifices. Since the coefficients obtained for the $\frac{9}{32}$ inch orifice and the .707 inch orifice checked so closely with those which had been adopted, these runs were not recomputed.

A $1\frac{1}{4}$ inch and a $1\frac{1}{2}$ inch orifice were each calibrated by the above method. It was found that the coefficients for the $1\frac{1}{4}$ inch orifice were still higher than those for the one inch orifice, the average value being about .69. The $1\frac{1}{2}$ inch orifice had coefficients averaging about .76. The results of the tests on this $1\frac{1}{2}$ inch orifice were quite erratic; coefficients were obtained which varied from .72 to .78. In the case of the larger orifices it appeared that influences other than those measured or observed affected the coefficient. The curves for these last two orifices are not shown in figure 7.

The charts in appendix, Figures 58, 59 and 60, page 104, have been prepared to facilitate the solution of the Harris formula for the $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 and $1\frac{1}{4}$ inch orifices. These charts are based on values of (C) taken from the curves of figure 7.

Foot-Pieces.—As stated before, this work began with the investigation of the performance of different foot-pieces. Figure 1 illustrates the various types of foot-pieces tested.

Four different types were tested with the 2½ inch eduction pipe at the Capitol Heating Plant. The eduction pipe was not altered for these tests except for minor changes which were necessary for the connection of the different foot-pieces.

Foot-piece A, figure 1, was tested first with the brass rings above *aa* removed. The foot-piece in this test consisted of a cylindrical casing with a central open end air nozzle. Water passed upward in the casing to the end of the nozzle and there mixed with air. From this point the mixture moved upward through a gradual contracting section to the bottom of the eduction pipe. The entrance conditions of the water in this foot-piece were different than those which would be encountered in a real well. An investigation of the velocity heads of the water just below the nozzle in the water supply pipe led the writer to think that this condition of entrance had little effect upon the performance of the pump.

The next series of runs was taken with the same foot-piece except that the brass rings were attached to the nozzle above *aa* as shown in figure 1, foot-piece A. Metal projections on these rings were provided to separate them so as to leave a slot between successive rings. On the top ring a conical shaped cover was attached which had openings through it for the passage of air. In figure 2a is shown a photograph of the set-up used in the first two series. Figure 2b illustrates the window which was located in the wall of this foot-piece. It was thought that an observation of the conditions at the point of delivery of air to the foot-piece might be instructive. This window was of little value, however, for at ordinary rates of pumping little could be seen because of the turbulent conditions of flow.

Foot-piece B, shown in figure 1, was the second type tested. The general set-up was changed so as to more nearly duplicate the conditions in a real well. Figure 3 illustrates the condition under which this foot-piece was tested. An 8 inch pipe served for the well casing. Instead of running the casing up the entire length of the eduction pipe, however, it was extended only a short distance above the foot-piece and was stopped off there by a blank flange.

In all subsequent tests this arrangement was used. Two series of tests were taken with foot-piece B. In the first series, air was taken through the air supply pipe which passed through the flange of the well casing, and down to a side opening in the foot-piece. In this foot-piece there was an annular space for the passage of air around the central axial water passageway. The walls between these two spaces contained vertical axial slots which allowed the passage of air inward from the outer air chamber. The essential difference between foot-piece A and foot-piece B is that in foot-piece B there is no central obstructing projection to disturb the flow of water. The second series taken with this foot-piece was made with the same set-up as above, excepting that the air supply pipe was removed between the well casing flange and the foot-piece. When air was admitted to the casing, a pressure was built up there which depressed the water surface in the well until opening *a*, air supply connection, was uncovered, thus allowing air to enter the foot-piece. The effect of this air storage chamber, in the upper part of the well casing, at the foot-piece seemed to be a factor well worth studying. If local storage at the foot-piece was desirable it could be obtained in a real well very easily as illustrated by the pump shown in the right hand side of figure 5. Comparatively few runs were taken with this second arrangement of foot-piece B. Since the data are not sufficiently complete to accurately indicate the performance of the pump, they are not presented in this bulletin. The results obtained, however, seem to be essentially the same as were obtained with this foot-piece as tested with the air line leading to the foot-piece.

A preliminary study of the data obtained in these first series of runs indicated that refinement in design of foot-pieces was not essential. Any device that allowed the air to pass into the water freely, and which allowed free passage of the water, was all that appeared to be essential. Since tests with different diameters of eduction pipes were to be made, new foot-pieces were required, so that it was essential to adopt some type of foot-piece for the new runs. In an attempt to find a simple type of foot-piece, which could be readily duplicated in the different sizes required, a $2\frac{1}{2}$

inch nipple was connected to the eduction pipe as in figure 1 C. There were no slots in the pipe, however. Air was delivered to the top of the well casing. As the air pressure increased in the casing, the water level lowered until the end of the $2\frac{1}{2}$ inch pipe was uncovered or partially uncovered. Air then entered the eduction pipe with the water. This arrangement caused intermittent action of the pump under all conditions of pumping. When the end of the pipe was uncovered, there was a sudden rush of air into the eduction pipe which resulted in a dropping of the pressure in the air supply line and air reservoir. This allowed the water level in the well to rise. The air pressure in the casing increased so as to prevent further rise of the water level. A greater increase of pressure caused the water surface to lower again. Thus, a cycle of conditions was produced which caused anything but smooth and continuous operation. Gages connected to the pump and to the air measuring drum varied over such a wide range that they could not be read with much accuracy, so that this foot-piece had to be modified. It may be said, that the tests made with this foot-piece indicate, at least roughly, that there was no apparent difference in the air consumption for a given rate of flow of water and for a given lift, as compared with the air consumption with the other foot-pieces. For the next trial the short $2\frac{1}{2}$ inch pipe was removed and eight axial slots were cut in its walls. These slots were .115 inches wide and about $9\frac{1}{2}$ inches long. With the above modification of foot-piece C, the undesirable pressure fluctuations were avoided to a great extent and the air fed into the water almost continuously. The general performance of the pump was not altered by the use of this type of foot-piece, so that it was adopted as a standard type for the tests to be made with other eduction pipes.

A 3 inch Harris foot-piece, figure 1D, was tested with an eduction pipe 3 inches in diameter and 60 feet long in order that at least one commercial foot-piece might be compared with the other types.

Eduction Pipes.—All of the air lift pumps used in these experiments were constructed of commercial black iron pipes. Special attention was given to the making of a smooth joint in all of the connections of the various educ-

tion pipes tested. Where ordinary screw couplings were utilized, pipes were threaded so that the ends of the pipes abutted against each other when the joint was made up. Precaution was also taken so as to avoid a change in alignment or area of cross-section at the joint in those cases where bolted flange couplings were used. Where different diameters of pipe were connected to each other, bolted flanged couplings were used. Cement mortar was placed in the end of the larger pipe and shaped so as to cause a gradual reduction in area of the cross-section in passing from the large pipe to the smaller pipe in order to avoid sudden change of cross-section at the joint. This was accomplished by clamping a thin piece of sheet metal to the end of the flange of the larger pipe. There was a circular opening in this metal sheet of the same size as the bore of the smaller pipe, the opening being concentric with the bore of the larger pipe. Cement paste was then filled in the large pipe behind this plate. A straight edge was used to press the cement against the pipe, and finally to strike off the cement so as to leave the surface of a frustum of a cone 6 diameters (of the larger pipe) in length. The bases of the frustum were equal to the sections of the pipes to be connected.

At the upper end of the $2\frac{1}{2}$ inch eduction pipe the conditions of exit were somewhat different than those conditions in other set-ups. A standard 4 inch elbow was connected to the top of the $2\frac{1}{2}$ inch eduction pipe by the use of bushings. A horizontal 4 inch pipe carried the water to a point where it could be discharged. In all other eduction pipes tested there was a standard short sweep elbow attached directly to the top of the eduction pipe. The opening of the elbow was horizontal in each case and discharged freely into the atmosphere.

Pressure Study in Eduction Pipe.—A special study of the hydraulic gradient was made in some of the $2\frac{1}{2}$ inch eduction pipe tests. Figures 2 and 2a illustrate the gages which were used in this study. Piezometers marked 1, 2 and 3 in the figure were connected to Bourdon gages located near the floor. Piezometer 4, near the top of the eduction pipe connected to a mercury U tube gage. In preliminary runs

it was observed that data of sufficient accuracy on the pressure drop between successive sections could not be obtained from the difference of readings of the Bourdon gages. Mercury differential gages were connected in the gage lines between sections 1 and 2, and between sections 2 and 3. Blow-off cocks were provided in the gage pipes for the purpose of discharging air which might be entrapped there. The mercury U tube gage at section 4 was abandoned after the preliminary trial because of the difficulty of reading this gage, an extra observer being required just to read this one gage. Section 4 was taken at the center of the 4 inch elbow and the pressure there was assumed to be atmospheric since the horizontal 4 inch pipe never flowed completely full.

In another study of pressure in eduction pipes a special device for measuring pressure variation was used. This apparatus, shown in the photograph of figure 4, consisted of an ordinary steam pressure indicator with a special cylinder for carrying the paper on which the pressure records were made. The drum was driven by a motor at practically a constant speed. Since there was some variation in the speed of the motor, a pendulum was connected in an electric circuit with an electromagnet recording device which marked one second intervals on the pressure diagram. The connection of the indicator to the piezometers was made by a very short hose. The pressure recording device was shifted up and down in order to connect it to the different piezometers. This device was used in the study of conditions in an eduction pipe made up of pipes of different diameters. Observations of pressure variation were taken at a section in the eduction pipe near the foot-piece, at sections on either side of joints in the pipe, and at a section as far removed from the foot-piece as could be reached. The indicator diagrams served the purpose of measuring pressure fluctuations at various sections and for comparing the range of pressure fluctuations at different sections. Average values of pressure taken from these diagrams are not likely to be sufficiently accurate for the study of the loss of head between different sections. A comparatively small pressure difference between two sections, which

must be obtained in computing the pipe friction loss between those sections, cannot be obtained with sufficient accuracy from these indicator diagrams. Inherent faults of the indicator cause small discrepancies which are a relatively large per cent of the difference in pressure between two sections.

METHOD OF CONDUCTING EXPERIMENTS

A series of experiments consists of runs made with a particular air lift pump. A complete series of runs includes the tests made at different rates of flow, or rate of pumpage of water, at different per cents of submergence of the foot-piece. In a particular series of runs the length of the eduction pipe was constant, and a change in the percentage of submergence involved a change in the delivery head. The percentage submergence might be held constant for consecutive runs while the rate of flow was varied by regulating both the water supply valve and the air control valve. Normally an increased rate of supply of water tended to build up a greater pressure head in the well, providing the air supply remained constant. In order to maintain a given submergence, which means also a constant lift (h_a) in these experiments, the supply of air had to be changed for different rates of pumping. The range over which a particular orifice could be used, usually was not sufficient to cover the range of air supply required for all of the different rates of pumpage of water at a given submergence. The tests of the pumps were taken in such a way as to avoid the necessity of changing air orifices too frequently. Runs were taken covering all percent submergences, and all rates of flow which could be accomplished with a particular size of air orifice, before changing the orifice. This accounts for the apparent lack of order in the tables of the summary data. A run consisted of a set of observations taken during a period of time varying from five to ten minutes with the rate of flow of water, supply of air, and the submergence constant. Two men performed the experiments. One man attended the apparatus and

read the gages and the water meter; the other man recorded data, acted as timekeeper, and made certain preliminary computations. In addition to the gages used on the air orifice drum and those connected to the eduction pipe which have already been described, there was a standard Bourdon test gage connected so as to record the pressure head in the well casing. This gage is called "water gage on foot-piece". Another gage called "air gage on foot-piece" was connected in the air line below the air orifice drum a short distance in front of the foot-piece. The gages and the water meter were arranged so that one observer could read all of them. From four to six readings of the pressure gages were taken in a given run. The water meter and the thermometer on the air drum were each read at the beginning and at the end of each run. In case the conditions of operation did not remain sufficiently constant during a run, the data taken were discarded and the run was started over again. All Bourdon gages were calibrated frequently by comparing them with an American Dead Weight Gage Tester. Care was taken to avoid errors in gage readings due to parallax by locating the gages in a convenient position for observation. The pressure at the foot-piece measured by the water gage on foot-piece varied considerably during certain ranges of operation in many of the pumps tested. It was necessary to throttle the gage connection in order to read this gage. Where it was necessary to throttle a gage, it was done only to such an extent as was necessary to reduce large and sudden vibrations of the gage arrow. Special attention was given to the observation of gages which measured pressures that varied.

THEORETICAL CONSIDERATIONS

Size and Shape of Air Bubbles in Water.—The eduction pipe contains both air and water, the air being more or less intimately mixed with the water. Observations made on small glass models of an air lift pump indicated that the mixture of air and water never very closely approached the condition of a homogeneous mixture. The size of the air bubbles and their distribution in the water changed greatly with the different conditions of operation. Regardless of how the air was admitted at the foot-piece, whether it was broken up in a fine spray by passing it through small openings or simply allowed to enter as large bubbles, there was a readjustment in the size of the bubbles a short distance above the foot-piece. For a given rate of discharge and a given lift there was a grouping of small air bubbles to form larger ones in case the air had been admitted in a fine spray. A division of large bubbles took place if the air was admitted in bubbles of great size. In the length of the eduction pipe immediately above the foot-piece, where this readjustment in the size of bubbles took place, there was considerable turbulence and eddying. The lower the rate of pumping the larger the bubbles of air became, until they finally were of such a size as nearly to fill the bore of the eduction pipe. At still lower rates of pumping the bubbles became elongated, their length being in some cases as great as two feet. Between the large bubbles of air there were layers of water in which there were many small bubbles of air. The individual bubbles and their distribution in the water could not be observed at fast rates of pumping. In this case, however, waves of light color flashed up the tube and indicated that there was not an even distribution of the air in the water. It is interesting to note that the action appeared to follow a definite cycle for a given rate of pumping.

Head Produced by Air in a Vertical Column of Water.—Certain theoretical analyses of the conditions which are associated with the motion of bubbles of air in water have been made, and laws governing this phenomenon have been

presented. These laws do not agree with practice, usually, because many important factors have been omitted or have been improperly considered. The motion of a bubble in water involves such factors as viscosity of the water, loss of head due to flow past an obstruction, skin friction on a surface moving through a liquid, and pipe friction.

The following discussion illustrates certain fundamental considerations of the motion of a bubble which is submerged in a liquid.

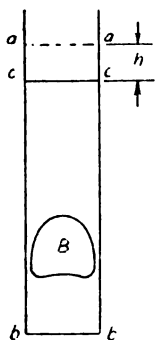


Fig. 8—An air bubble in a vertical cylinder of water; bottom of the cylinder closed.

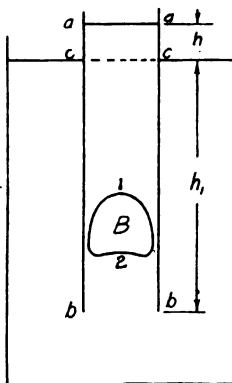


Fig. 9—An air bubble in a vertical cylinder of water; bottom of the cylinder open.

Case I.—Let aa , bb in figure 8 be a vertical tube closed at bb , and filled with water to the level cc . If a bubble of air B of volume (V) cubic feet be placed in the tube, (V) cubic feet of water will be displaced and the surface of the water will be raised to some level, aa . The volume of water between aa and cc will equal the volume of the bubble. Let (h) equal the height that the surface of the water was raised above cc and (A) equal the area of the pipe in square feet.

$$\text{Then, } Ah = V, \text{ and } h = \frac{V}{A}.$$

Case II.—In figure 9 there is a tube similar to that of case I, except that the lower end of it is open and the tube is submerged to the level cc in a reservoir of water. The problem under this condition is to determine the height to which the

water level will rise in the tube when a bubble of volume (V) is introduced into the tube. This arrangement is quite similar to that of an air lift pump. The length (h) will be the same as in case I, providing there is no flow of water through section bb after the bubble has been injected into the pipe. If the presence of the bubble tends to produce a pressure head at section bb different than (h_1), water will flow past section bb and h is unequal to $\frac{V}{A}$.

Suppose that a bubble of air in the tube should be restrained so that it could not move upward in the liquid. For example, imagine the air bubble to be enclosed in a thin membrane so that it may be held at rest in water. It is obvious that the level of the water in the tube would be the same as that outside of the tube and (h) would equal zero. It should be noted that the air must move with a velocity upward, relative to the water, in order that the water in the tube may be maintained above the level cc . In this discussion it is assumed that the bubble does not completely fill

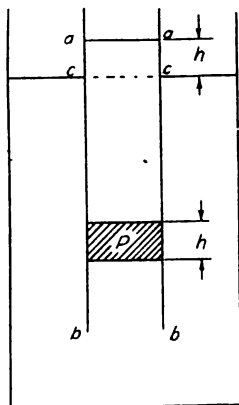


Fig. 10—A weightless piston in a vertical cylinder; bottom of cylinder open.

the bore of the tube since experience shows that this is always the case. No matter how large the bubble, there is always water surrounding it on all sides. If the bubble did fill the tube as in figure 10, so as to form a perfect piston, then of course (h) would have the same value as in Case

I and the bubble would remain at rest. Any reasoning based on this assumption leads to erroneous conclusions.

The movement of a bubble in a column of water is quite similar to that of a loosely fitting piston moving in a cylinder. In figure 11 the above similarity is illustrated. Consider the piston at the right to be forced upward by some force F pounds and with a velocity (u) feet per second equal to the velocity of the bubble or the "slip". The water level will rise to some height aa until the difference in the head on the two sides of the piston is such as to cause ($A \cdot u$) cubic feet of water per second to leak past the piston. Apply the energy equation between points 1 and 2.

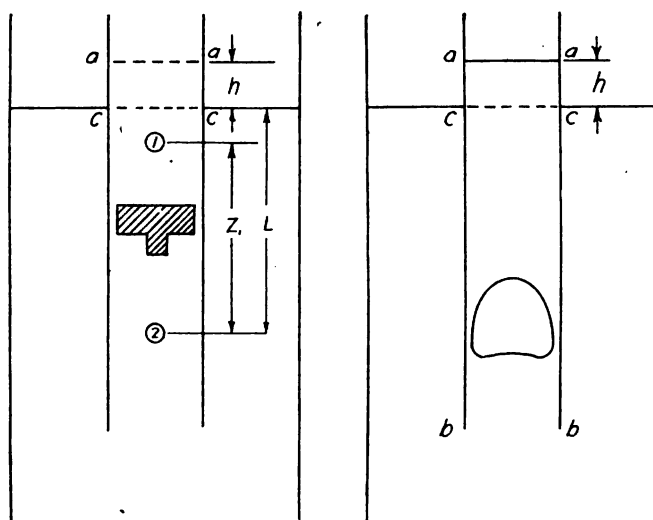


Fig. 11—Comparison of a loose fitting piston with an air bubble.

The total head at section

$$(1) \quad H_1 = z_1 + L + h - z_1 + \frac{v_1^2}{2g} .$$

$$H_2 = L + \frac{v_2^2}{2g} .$$

$H' =$ loss of head due to the flow around the piston.

$$(2) \quad H_1 = H_2 + H'; L + h + \frac{v_1^2}{2g} = L + \frac{v_2^2}{2g} + H' .$$

Since $\frac{v_1^2}{2g} = \frac{v_2^2}{2g} = 0, H' = h .$

The water rises to a height (h) above level cc equal to the head required to pass ($A \cdot u$) cubic feet of water per second past the piston. Since losses due to flow past an obstruction in a channel vary approximately with the velocity squared

$$(3) \quad H' = K \frac{v^2}{2g} = h.$$

In this equation (v) is the velocity of the water relative to the passageway between the piston and the walls of the cylinder. It is proportional to the velocity of the piston. The coefficient (K) is a constant for any particular size and shape of piston in a particular size of cylinder. This same reasoning applies to the bubble shown at the left of this figure. The coefficient (K) is probably not a constant since losses are likely to be modified by such factors as size of bubble, diameter of water column, and viscosity of the liquid in which the air bubble moves.

Velocity of Bubble Upward in Column of Water.—(Slip of Bubble). By assuming, as is commonly done, that the force moving a bubble upward is equal to the buoyant force—weight of the displaced fluid—and that the head produced by a volume (V) of air is $\frac{V}{A}$ feet, where (A) is a cross section of the water column, the following value of (u), the “slip” of the bubble, is obtained:

$$(4) \quad u = \sqrt{\frac{gV}{2A}} \text{ feet per second.}$$

The “slip” of the air is directly proportional to the square root of the volume of the bubble and inversely proportional to the square root of (A), according to the above assumption. The assumptions made, however, are not correct.

The law of hydrostatics which states that submerged bodies are buoyed up by a force equivalent to the weight of the fluid displaced holds under only one condition. The pressure intensity at different points in the fluid surrounding the body must be directly proportional to the depth of those points below the free surface of the fluid. The water surrounding a bubble is in motion with different velocities

at different points. Pressure does not increase directly with difference in elevation of these points. In figure 9, the pressure of the air at different points in the bubble is practically constant. Since the weight of the air in the bubble may be neglected, the pressures at points 1 and 2 in the surface of the bubble are equal. In fact, the pressure at all points in the water surrounding the bubble is practically constant. It will be seen that where the surface of contact of air and water is curved that surface tension modifies this above equality of pressure to a slight extent. The bubble moves with a velocity (u) and takes a shape such that changes of velocity of the water around the bubble, impact of water on the upper part of the bubble, and loss of head due to flow around the bubble causes practically a constant pressure at various depths in the water surrounding the bubble. Viscosity of the water influences the shape and also the velocity (u) of the bubble. There is little reason for thinking that the force moving the bubble equals the buoyant force.

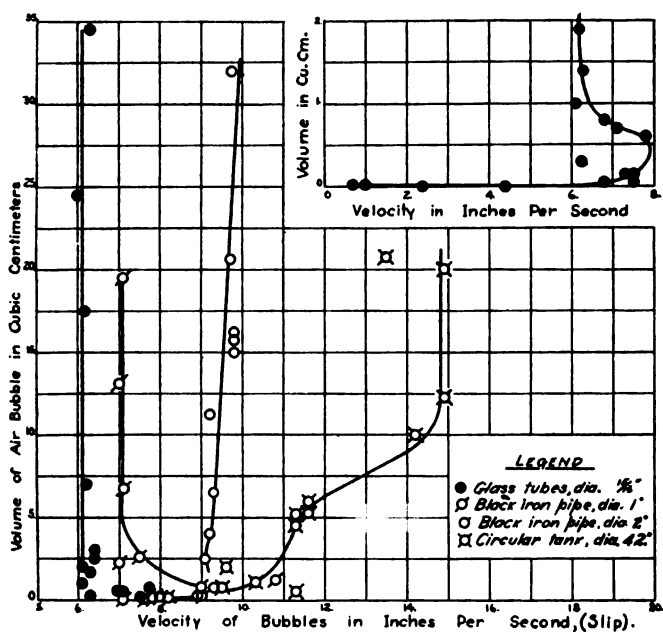


Fig. 12—Results of experimental study of the movement of bubbles of air of different sizes in various vertical columns of water. Water at rest.

Experimental Investigation of Slip.—The results of a laboratory study of the velocity of bubbles in vertical columns of water are shown in figures 12 and 13. Observations were made on the travel of bubbles in a glass tube, in iron pipes, and in a large tank. In each of the sizes of pipe used in the first series of tests there was no flow of water so that the absolute velocity of the bubble was the slip. Figure 12 gives the "volume of bubble-slip" relations when the water was at rest in the pipes.

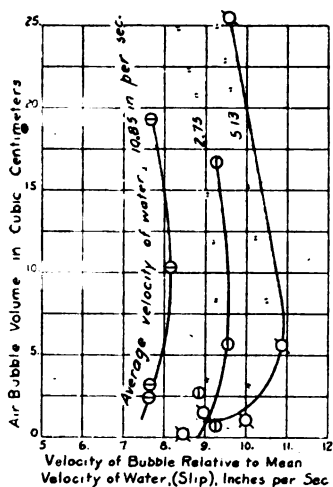


Fig. 13—Results of experimental study of the movement of bubbles of air of different sizes in a vertical column of water in a two inch black iron pipe. Water flowing upward.

A second series of tests was made with the water flowing upward in the 2 inch black pipe. The "slip-volume" relations for three different mean velocities of flow of water are plotted in figure 13. The distance of travel was about six feet in each case, except in the tank where it was four feet $8\frac{3}{4}$ inches. The apparatus and the method used in making observations are described in the appendix, page 100. While great accuracy could not be obtained in the short lengths of bubble travel used, the results indicate that there is no simple relation between size of bubble and velocity of the bubble relative to the water. There is a distinct indication,

at least in smaller pipes, that slip increases as the volume decreases over a certain range of size of bubble. A reduction in size of bubble might, under certain conditions, be actually detrimental. The effect of an increase in area (A) is to increase the slip for a given value of (V) rather than to decrease it as was indicated in equation (4), page 40. The relations obtained between volume of bubble and slip in the above experimental study probably do not represent those relations in an air lift pump. In an eduction pipe there are many bubbles close to each other which undoubtedly influence each other's motion. It is almost certain that, if equation (4) does not apply even approximately to the simple case which was assumed in its derivation, it will not apply to the conditions in an eduction pipe.

METHODS USED IN ANALYZING AIR LIFT PUMP DATA

Efficiency.—The efficiency of the pump is taken as the useful work done—the lifting of one pound of water through (h_a) feet—divided by the energy in the air delivered to the pump per pound of water delivered. Since the air is in contact with the water it expands isothermally. The pump is charged with the energy involved in the isothermal expansion of the above quantity of air from the pressure at the foot-piece to atmospheric pressure. The exit velocity, which in many practical cases is high, usually can not be recovered for any useful purpose so that it is charged as a loss in the pump. The loss of energy due to friction in the air pipe leading from the ground to the foot-piece is not charged against the pump. This loss can be determined separately and can be controlled by the proper consideration of the size of the air line. In these experiments the air was delivered to the foot-piece a short distance from the air orifice. Actual efficiencies for the air lift as a pumping system are less than those obtained by the above method of computing efficiency. The energy delivered to the system is greater than that involved in the expansion of the air from pressure at

the foot-piece to atmospheric pressure. Mechanical losses in the compressor, losses in transmitting air to the foot-piece, and the fact that air can not be compressed isothermally in the compressor are factors which make the total expenditure of energy in the compression of the air greater than that of expansion in the eduction pipe. However, for studying conditions in the eduction pipe and for comparative purposes, the method adopted is more satisfactory than that which takes into account losses outside of the eduction pipe.

$$(5) \text{ The efficiency } E = \frac{100 h_d}{\frac{P_a V_a}{8.33 G/60} \log_e \frac{h_a + h_a}{h_a}} .$$

Here $\frac{V_a}{8.33 G/60}$ is the cubic feet of air used per pound of water pumped.

$$(6) \quad \text{Since } P_a V_a = W R T, \text{ and } \frac{h_a + h_a}{h_a} = r,$$

$$E = \frac{100 h_d}{\frac{2.303 W R T}{8.33 G/60} \log_{10} r} .$$

R for air equals 53.37.

$$(7) \quad E = \frac{100 h_d}{887. \frac{W T}{G} \log_{10} r} .$$

The denominator of this equation is (H''), the foot-pounds of energy involved in the expansion of the air used per pound of water pumped. It equals the head equivalent of the energy of the air in feet of water.

$$(8) \quad E = \frac{100 h_d}{H''} .$$

The pressure head (h_s) on the foot-piece was obtained from the gage labeled "water gage" in figures 2 and 3, instead of from the gage on the air line. These two gages, however, read practically the same pressure in each run.

The cubic feet of free air used per second was determined

by reducing (W) to its equivalent volume at the standard conditions of pressure and temperature of 14.7 pounds per square inch and 70 degree Fahrenheit, respectively.

$$P_a V_a = W R T.$$

$$V_a = \frac{53.37}{(14.7)} \frac{(70 + 460) W}{(144)} \text{ , cubic feet per second.}$$

$$(9) \quad V_a = 13.38 W, \text{ cubic feet per second.}$$

Loss of Head Due to Friction in Eduction Pipe.—By applying Bernoulli's equation between any two sections 1 and 2 of an eduction pipe, see figure 5, the loss of head may be obtained between these two sections. The total heads are expressed in feet of water.

$$(10) \quad H_1 = H_2 + H'_{1-2} - H''_{1-2}.$$

$$H_1 = \frac{v_1^2}{2g} + \frac{P_1}{w} + z_1.$$

$$H_2 = \frac{v_2^2}{2g} + \frac{P_2}{w} + z_2.$$

If (V_1) is the volume of air per pound of water passing section (1) where the pressure is (P_1) (pounds per square foot, absolute),

$$(11) \quad H''_{1-2} = P_1 V_1 \log_e \frac{P_1}{P_2} \text{ feet of water.}$$

Substituting the above values for (H_1 , H_2 , and H''_{1-2}) in equation (10) the following relation is obtained:

$$(12) \quad \frac{v_1^2}{2g} + \frac{P_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{P_2}{w} + z_2 + H'_{1-2} \\ - P_1 V_1 \log_e \frac{P_1}{P_2}.$$

Solving for the loss of head (H'_{1-2}) there is obtained:

$$(13) \quad H'_{1-2} = P_1 V_1 \log_e r + \frac{v_1^2 - v_2^2}{2g} + \frac{P_1 - P_2}{w} + z_1 - z_2.$$

(14)

$$H'_{1-2} = \frac{2.303 W R T}{8.33 G/60} \log_{10} r + \frac{v_1^2 - v_2^2}{2g} + \frac{P_1 - P_2}{w} + z_1 - z_2.$$

In this bulletin equation (14) was used as the basis for determining the loss of head between any two sections of an eduction pipe.

General Equation Applying to the Air Lift Pump.—By applying the energy equation so as to include the entire eduction pipe, the relations between the various factors involved there may be obtained as follows: Let section (1) be at the surface of the water in the well and section (2) be at the upper end of the eduction pipe. Omitting losses at entrance to the foot-piece, and losses in the region where the air mixes with the water in the eduction pipe we have

$$(10) \quad H_1 = H_2 + H'_{1-2} - H''_{1-2}.$$

H'_{1-2} now represents losses in the entire eduction pipe.

H''_{1-2} is the head equivalent of the energy of expansion of the air, per pound of water pumped, from the pressure $w(h_s + h_a)$ pounds per square foot to $(w h_a)$ pounds per square foot. Letting (V_s) be the volume of the air at the foot-piece per pound of water pumped and expanding equation (10), the following relations are obtained:

(15)

$$\frac{v_1^2}{2g} + \frac{P_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{P_2}{w} + z_2 + H'_{1-2} - 62.4 (h_s + h_a) V_s \times \log_e \frac{h_s + h_a}{h_a}.$$

$$62.4 (h_s + h_a) V_s \log_e \frac{h_s + h_a}{h_a} = \frac{v_2^2 - v_1^2}{2g} + \frac{P_2 - P_1}{w} + z_2 - z_1 + H'_{1-2}.$$

The velocity in the well (v_1) is so small that $\frac{v_1^2}{2g}$ may be

dropped. Since usually the pressure at sections (1) and (2) are each atmospheric pressure,

$$\frac{P_2 - P_1}{w} = 0; z_2 - z_1 = h_d .$$

(16)

$$V_s = \frac{h_d + H'_{1-2} + \frac{v^2_2}{2g}}{62.4 \times 2.303 (h_s + h_a) \log_{10} r} \text{ cubic feet per second.}$$

Equations (13) and (16) furnish the basis for the design of air lift pumps. The determination of H'_{1-2} , the loss of head in the eduction pipe, and the use of the above equations in design is discussed later.

Analysis of Eduction Pipe Losses.—It was learned that there was no simple relation existing between loss of head as determined from equation (6) and the velocity of flow in the eduction pipe. When loss of head was plotted against the accompanying velocity, in many cases, the loss of head appeared to decrease as the velocity became greater. Frictional losses with a mixture of air and water flowing in a vertical pipe appeared to be much greater than would result from the flow of water alone at the same velocities. After a preliminary study it was learned that uniformity could be obtained in the "loss of head—velocity relations" when the ratio of the volume of air to the volume of water in the mixture was considered. For any particular eduction pipe and for any particular ratio of volume of air to volume of water, points plotted with loss of head and velocity as coordinates fell close to the regular curve.

Average Volume of Air in Eduction Pipe.—It must be remembered, in investigating the "loss of head and velocity" relations in any length of eduction pipe, that the velocity of the mixture varies throughout the eduction pipe because of the expansion of the air. The ratio of the volume of air to the volume of water changes always at different sections. Knowing the pressure at a section and the pounds of air used per second, the volume of the air in cubic feet passing that section in one second can be determined. This volume plus the rate of flow of water in cubic feet per second divided

by the cross-sectional area of the pipe gives the velocity at the section. If P_1 is the pressure at some section 1; W , the pounds of air used per second; G , the flow of water in gallons per minute, the velocity,

$$(17) \quad v_1 = \frac{G}{449} + \frac{WRT}{P_1 A} \text{ feet per second.}$$

The ratio of the volume of air to the volume of water at section (1) equals

$$(18) \quad \frac{WRT}{P_1} \bigg/ \frac{G}{449} = \frac{449 WRT}{G P_1}.$$

The ratio of volume of air to volume of water at the section at the other end of the length of eduction pipe may be found in a similar manner. The arithmetical mean of these ratios is nearly equal to the actual average ratio and it may be used when the length of the pipe investigated is short. For determining the ratios of air to water and average velocities in long eduction pipes—lengths greater than 40 feet—the above method is not sufficiently accurate. The average volume of the air in a long section of an eduction pipe was determined from the following equation which is discussed in the appendix, page 102.

Average volume of air equals

$$(19) \quad \frac{2.303 h_1 V_1}{h_1 - h_2} \log_{10} \frac{h_1}{h_2}, \text{ cubic feet per second.}$$

h_1 = pressure head at the lower section 1, feet of water, absolute.

h_2 = the pressure head at the upper section 1, feet of water, absolute.

V_1 = volume of air at section 1, cubic feet per second.

The average velocity between sections 1 and 2 equals the above average volume of air plus the flow of water in cubic feet per second divided by the area of the pipe. The average ratio of volume of air to volume of water equals the volume of air as obtained from equation (19) divided by the cubic feet of water per second.

Sample Computation.—Run number 147, series number 2, is analyzed here to illustrate the methods used in the computation of results. See figure 14, which is a sample of the form of notes used in recording data and which gives the

THE UNIVERSITY OF WISCONSIN, HYDRAULIC LABORATORY

Experiment on *Air Lift Pump. Series 2*

Date by *Ward and Kirchhoff* Computed by

Date of Experiment *April 12, 1919* Checked by

General Data *Diameter of education pipe, 2½ inches*
Foot-piece (A), brass riggs used.
Diameter of air orifice, ⅜ inch.
Temperature of water 78° C.

Sheet No.

Run No.	Time.	Water Meter Head's	Water Gage on Foot Piece	Air Gage	Foot Piece	Air Gage	Diff. Gage on Orifice		Bourdon Gages on Education Pipe Sections			Mercury Gages on Education Pipe		Temp air	
							L	R	Sections			Sec's			Temp air
									1	2	3	L	R		
146	34.36	22.0	465	20.0	21.5	1-530	1-430	46.0	50.5	53.0	0.90	1.20	0.82	1.23	24.0
			455	19.5	21.5	1-528	1-430	45.5	50.0	53.0	0.90	1.20	0.83	1.22	
			46.0	20.5	21.7	1-529	1-431	46.0	50.0	53.0	0.89	1.21	0.83	1.23	
			46.2	20.0	21.8	1-529	1-430	45.5	50.5	53.0	0.89	1.21	0.82	1.22	
	41.17	32.0	46.2	20.5	21.5	1-529	1-429	45.5	50.5	53.0	0.90	1.19	0.82	1.23	24.0
Average			46.05	20.1	21.6	1-528	1-430	45.7	50.3	53.0	0.896	1.202	0.824	1.226	24.0
147	46.02	39.0	420	19.5	20.8	1-571	1-390	42.0	47.5	51.0	0.85	1.25	0.76	1.30	23.9
			42.2	19.3	20.5	1-570	1-385	41.0	47.3	51.0	0.86	1.25	0.77	1.29	
			42.5	18.5	20.5	1-571	1-387	42.0	48.0	51.0	0.85	1.25	0.77	1.29	
	52.43	49.0	42.5	19.0	21.0	1-568	1-389	42.0	47.5	51.0	0.85	1.25	0.77	1.29	23.9
	Average			42.3	19.07	20.7	1-570	1-387	41.75	47.6	51.0	0.852	1.250	0.770	1.292

Fig. 14—Sample data sheet.

observed quantities taken in run number 147, series number 2. Quantity of water pumped equals $49.0 - 39.0 = 10.0$ cubic feet.

Time interval = 401. seconds.

$$\text{Rate of pumping} = \frac{10}{401} = .025 \text{ cubic feet per second.}$$

The correction for the meter registration at this rate of flow is (—1.4 per cent) as determined from figure 6.

The actual flow is $.025 - .014 \times .025 = .0247$ cubic feet per second.

$$.0247 \times 449 = 11.02 \text{ gallons per minute.}$$

$$\text{The equation for the air orifice is } W = .1639 \text{ Cd}^2 \sqrt{\frac{ip}{T}}.$$

$$C = .606.$$

$$d = .2874 \text{ inches.}$$

$$i = 17.700 - 15.878 = 1.822 \text{ inches.}$$

$t = 23.9$ degrees Centigrade.

$T = 534.4$ degrees Fahrenheit, absolute.

$p =$ gage reading plus or minus gage correction plus barometric pressure, equals $20.7 + .35 + 13.85 = 34.9$ pounds per square inch.

$$W = .01354 \times .600 \times \sqrt{\frac{1.822 \times 34.9}{584.4}} = .00283 \text{ pounds per second.}$$

The pressure head (h_s) on the foot-piece, from the water gage on foot-piece, $= 42.3 + 0.97 = 43.3$ feet. Here, 0.97 is the correction for the gage discrepancy and for elevation of the gage.

The delivery head (h_d) equals the length of eduction pipe less the pressure head on the foot-piece, equals $61.17 - 43.3$, equals 17.9 feet.

$$\text{Per cent submergence (S) equals } \frac{100 \times 43.3}{61.17} = 70.7 \text{ per cent.}$$

Free air used per second equals $13.38 W = 13.38 \times .00283 = .0379$ cubic feet per second. (See equation 9.)

The free air per gallon of water per foot of lift equals

$$Q = \frac{.0379}{(G/60) \times h_d} = \frac{.0379}{(11.02/60) \times 17.9} = .0115 \text{ cubic feet.}$$

$$\text{Ratio of expansion (r) } = (19.0 + 14.2)/14.2 = 2.338.$$

In the above expression 19.0 pounds per square inch is the pressure taken from the air gage on the foot-piece corrected for gage discrepancy and 14.2 is the atmospheric pressure in pounds per square inch on April 12, 1919.

$$\text{Log}_{10} r = .3688$$

$$H'' = 887. \frac{W T}{G} \log_{10} r \text{ (See denominator of equation 7.)}$$

Here T is the temperature of the water, degrees Fahrenheit, absolute.

$$H'' = \frac{887. \times .00283 \times 504.6 \times .3688}{11.02} = 42.4 \text{ feet of water.}$$

$$E = \frac{100 h_d}{H''} = \frac{100 \times 17.9}{42.4} = 42.2 \text{ per cent.}$$

The frictional losses in the eduction pipe were not computed for the entire eduction pipe in series 2, as was the case in most of the other series. However, the losses in sections 1-2, 2-3, and 3-4, see figure 2, were obtained. The results of this special study of losses in the different sections are given in Table No. IV. The general method of computing losses is illustrated by the following sample of computation involving section 1-2 of the eduction pipe in run 147 series 2.

The pressure at section 1 equals $41.75 - 1.25 = 40.5$, feet of water (gage). The gage correction is 1.25 both for gage elevation relative to section 1, and also for the discrepancy in the gage itself. Using the mercury differential gage between sections 1 and 2, the pressure at section 2 may be found as follows: $(1.250 - .852) 12.58 = 5.00$ feet of water. The pressure at section 2 equals the pressure at section 1 plus 5.0 minus the difference in level of sections 1 and 2. Pressure at section 2 equals 40.5 plus 5.00 minus $20.06 = 25.5$ feet of water (gage).

Volume of the air passing section 1 where the pressure head is 40.5 and temperature is 7 degrees Fahrenheit equals, from the relation $P V = W R T$,

$$V = \frac{W R T}{P} = \frac{.00283 \times 53.37 \times 505.}{(32.8 + 40.5) \times 62.4} = .0166 \text{ cubic}$$

feet per second. Velocity of mixture at Section 1 equals volume of water plus volume of air passing section 1 di-

$$\text{vided by the area of the pipe} = \frac{.0247 + .0166}{.0332} = 1.24 \text{ feet}$$

per second. The velocity of the mixture at section 2 was found by a similar method to be 1.37 feet per second; the volume of air passing section 2 being .0209 cubic feet per

second. Ratio, average volume of air over volume of water, between sections 1 and 2 equals

$$\frac{(.0166 + .0209)}{2} \div .0247 = .76$$

The average velocity between sections 1 and 2 equals $\frac{1.24 + 1.37}{2} = 1.30$ feet per second. The total loss of head between sections 1 and 2, from equation (14), equals (H'_{1-2}) equals

$$\frac{2.303 \times .00283 \times 53.37 \times 505}{(8.33 \times 11.02)/60} \times \log_{10} \frac{40.5 + 32.8}{25.5 + 32.8} +$$

$$(.02 - .03) + (40.5 - 25.5) + (0 - 20.06) = 11.41 - .01$$

$$+ 15.0 - 20.06 = 6.34 \text{ feet of water.}$$

The loss of head per foot of pipe equals

$$\frac{6.34}{20.06} = .316 \text{ feet of water.}$$

DATA FROM TESTS OF LARGE PUMPS

In order to determine "velocity-loss of head" relations in larger eduction pipes than those tested in the laboratory by the author, an investigation has been made of published test data on large air lift pumps. A study of such data usually reveals the fact that there is not sufficient information presented to serve as a basis for the full analysis. Since most of the air lift pumps for which data are available are made up of pipes of varying diameter, it is not possible to analyze and separate the losses which occur in the individual sections of the pipe. Usually pressure observations have not been taken at points where one size of pipe is joined with a pipe of a different size. Certain data had to be abandoned because of obvious inaccuracies. In some of the field tests the rate of discharge of water is obtained by methods which cannot be relied upon to give accurate results. A volumetric measurement in a large tank is likely not to be accurate when the surface rise of the water

during a test is very small. Measurements of air based on compressor displacement and upon an assumed volumetric efficiency of the compressor are also apt to be in error. The writer has seen compressors whose volumetric efficiency dropped practically to zero at certain periods of time with no apparent indication at the compressor that this condition existed.

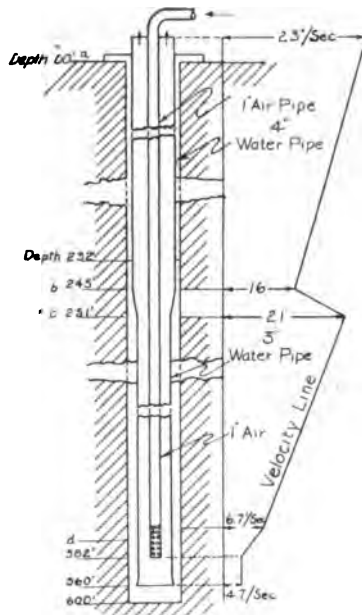


Fig. 15—Section through well at Rolla, Mo., and certain test data as given by Professor Harris in "Compressed Air", p. 85.

In *Compressed Air*, pages 85 and 86, Professor Harris gives test data on a pump whose eduction pipe consisted of pipes of different diameters. This pump is shown in figure 15. In this figure the velocities of flow which occurred at different sections of the pipe are shown. The velocities given by Professor Harris are presumably computed from pressure measurements taken at the different sections of the eduction pipe. By working back from the data given, with an assumed temperature of the water, the writer has computed the pressures at the various points of the eduction pipe, and also the loss of head in the different sections

of the eduction pipe. It should be noted that this pump has a central air line.

A similar analysis of friction loss has been made with test data published by Mr. Edmond M. Ivens in his book *Pumping By Compressed Air*, pages 139 and 140. This pump had an eduction pipe whose diameter was $3\frac{1}{2}$ in. throughout, and a central air pipe. Following are the data presented by Mr. Ivens:

Total depth of well	610 feet
Diameter of Casing	8 inches
Diameter of eduction pipe	$3\frac{1}{2}$ inches
Diameter of air pipe	$1\frac{1}{4}$ inches
Length of eduction pipe	355 feet
Length of air pipe	320 feet
Total pumping head	115 feet
Water pumped	120 gal./min.
Air consumed	74.5 cu.ft./min. free air
Submergence	65. %
Air Horsepower (by indicator)	12.1
“ “ (isothermal)	9.5
Pump efficiency	37 %
Over-all efficiency	29.7 %

Mr. S. F. Shaw gives the results of tests made upon some very large air lift pumps which were used in unwatering the Tiro General Mine at Charcas, S. L. P., Mexico. These data were published in *Mining and Metallurgy*, number 158, section 35, a paper published by the American Institute of Mining and Metallurgical Engineers. Two of the pumps which were tested had eduction pipes that were constant in diameter. The pump termed “air-lift No. 1”; in the above paper, had an eduction pipe of constant diameter of 6 in. and a length which ranged from 370 feet to 615 feet. A test of another pump made up of 5 in. spiral riveted pipe throughout called “San Fernando Air-Lift”, is presented in the same pamphlet.

The above pumps have been analyzed by the methods described in this bulletin in order to determine the relations existing between loss of head and average velocity of flow, and the results are given in table no. XVI.

RESULTS OF EXPERIMENTAL STUDY

The results of the Wisconsin Air Lift experiments are given in tabular form in tables II to XVI. Column headings in these tables were made quite complete so that the tables would be practically self-explanatory. Each table consists of the summary of data taken upon a particular air lift pump. The special pressure study made with the education pipe of series 2 is given in table IV. The curves which have been drawn to show more clearly the results given in the above tables will be discussed later.

Efficiency.—Curves shown in figures 16 to 30 represent the “efficiency-rate of pumping” relations for each pump tested. In each figure, lines connecting points of equal submergence have been drawn. The actual per cent submergence was recorded opposite the plotted point for the purpose of drawing the above lines. Due to lack of space it was necessary, in the final drawing, to adopt symbols to represent the range of per cent submergence in which each individual run came.

Effect of Foot-Pieces upon Efficiency.—Series 1, 2, 3 and 4 were taken in order to study the influence of foot-pieces. Figures 16 to 19 show that the maximum efficiency and the discharge at maximum efficiency are about the same for all $2\frac{1}{2}$ in. foot-pieces. At certain per cents of submergence, foot-piece A seems to give as much as $2\frac{1}{2}$ per cent greater efficiency than foot-piece D, the one of apparent lowest efficiency. It is believed that experimental errors, which were probably of greater magnitude in the test of foot-piece A since it was the first one tested, and failure to have a sufficient number of test runs at the efficiency point of operation, account for the apparent slight advantage which foot-piece A seems to possess. The efficiency-discharge curves for foot-piece C and for foot-piece D, the Harris foot-piece, each of which were tested with the same 3 in. pipe, are almost identical for all per cents of submergence.

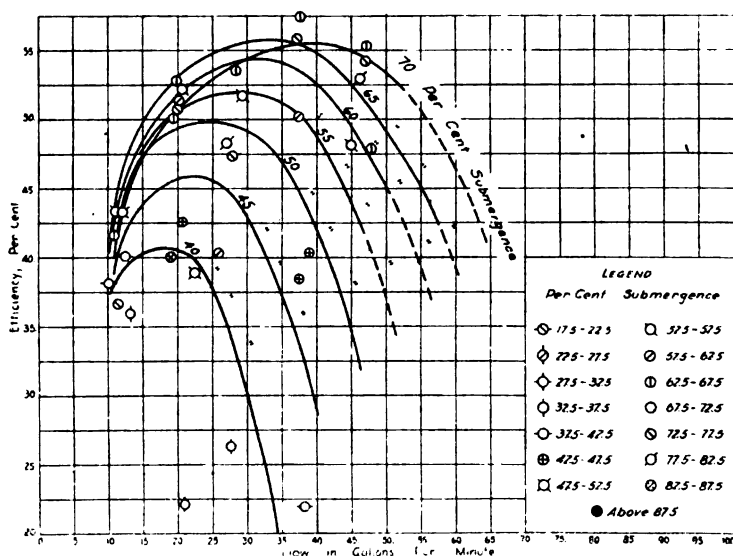


Fig. 16—Efficiency-flow relations in series 1, 2 $\frac{1}{4}$ inch eduction pipe 61.17 feet long, foot-piece A with brass rings removed. See figure 2.

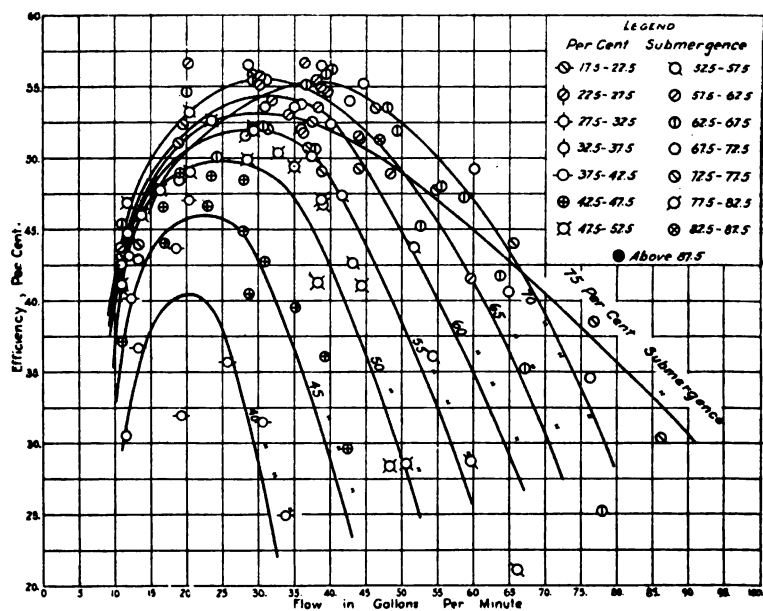


Fig. 17—Efficiency-flow relations in series 2, 2 $\frac{1}{4}$ inch eduction pipe 61.17 feet long, foot-piece A. See figure 2.

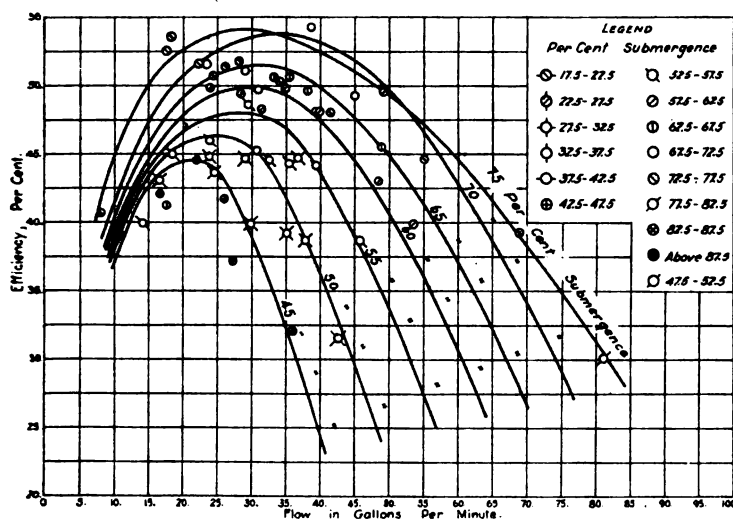


Fig. 18—Efficiency-flow relations in series 3, 2 1/2 inch eduction pipe 60.88 feet long, foot-piece B. See figure 3.

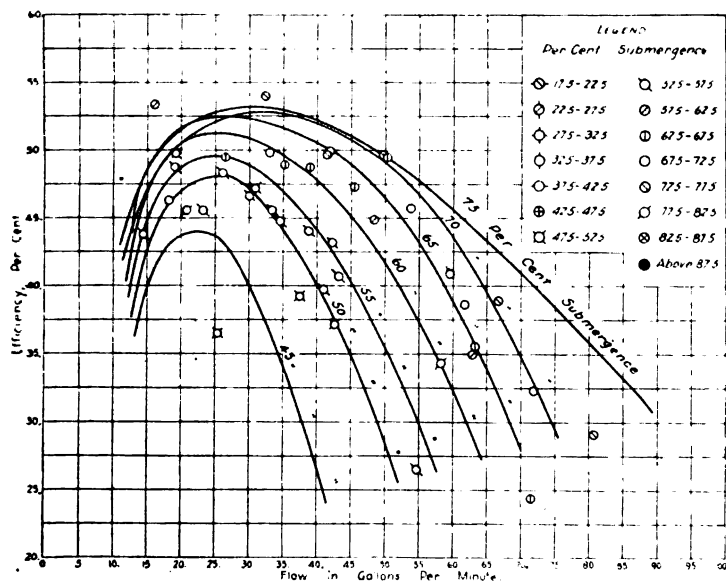


Fig. 19—Efficiency-flow relations in series 4, 2 1/2 inch eduction pipe 60.88 feet long, foot-piece C. See figure 3.

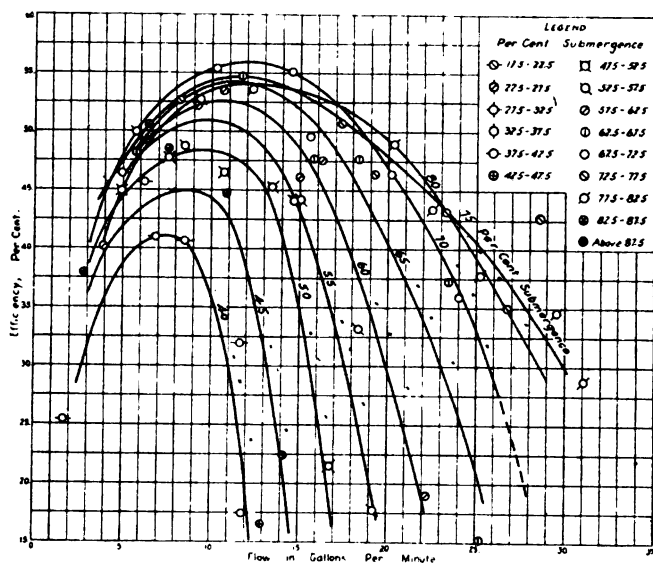


Fig. 20—Efficiency-flow relations in series 5, 1 1/2 inch education pipe 60.88 feet long, foot-piece C. See figure 3.

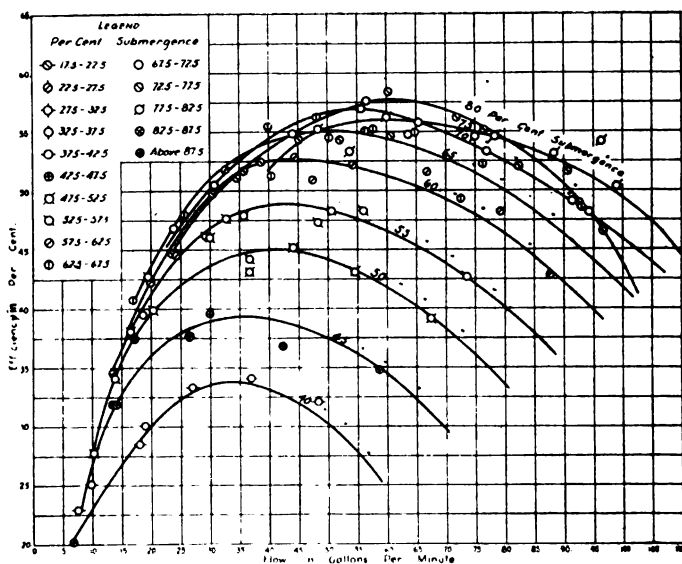


Fig. 21—Efficiency-flow relations in series 6, 3 inch education pipe 60.88 feet long, foot-piece C. See figure 3.

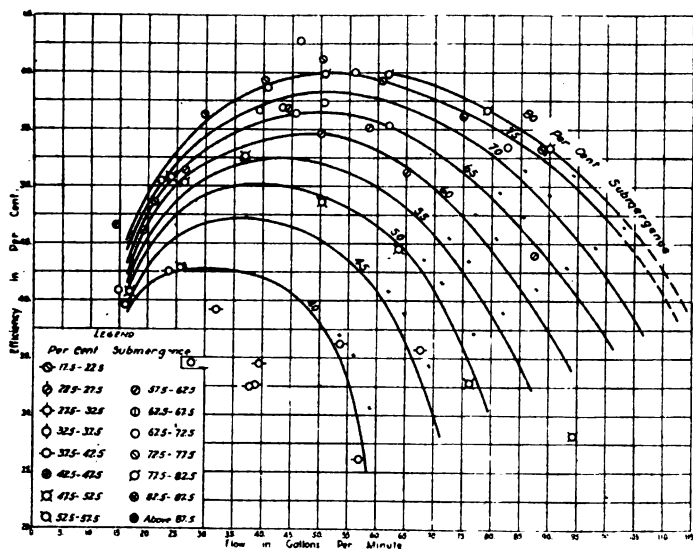


Fig. 22—Efficiency-flow relations in series 7, 3 inch eduction pipe 81.63 feet long, foot-piece C. See figure 3.

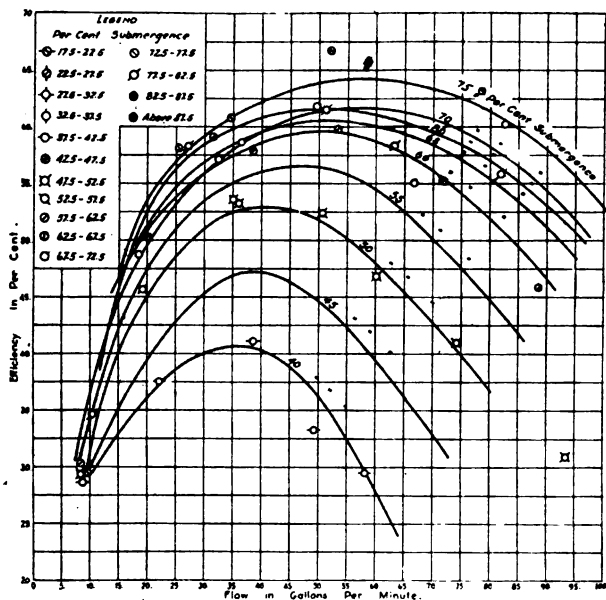


Fig. 23—Efficiency-flow relations in series 8, 3 inch eduction pipe 100.64 feet long, foot-piece C. See figure 3.

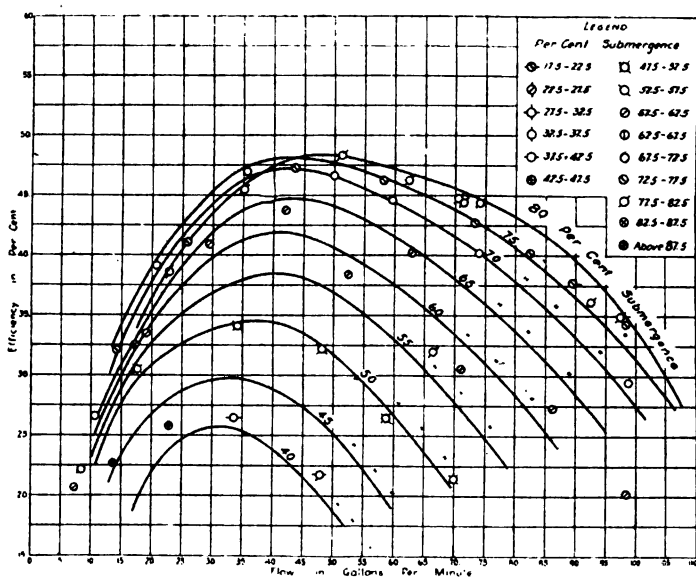


Fig. 24—Efficiency-flow relations in series 9, 3 inch education pipe 37.85 feet long, foot-piece C. See figure 3.

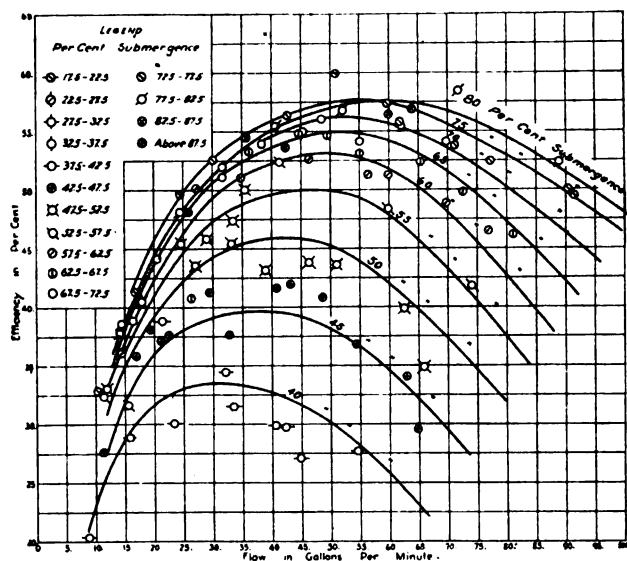


Fig. 25—Efficiency-flow relations in series 10, 3 inch education pipe 61.26 feet long, foot-piece D. See figure 3.

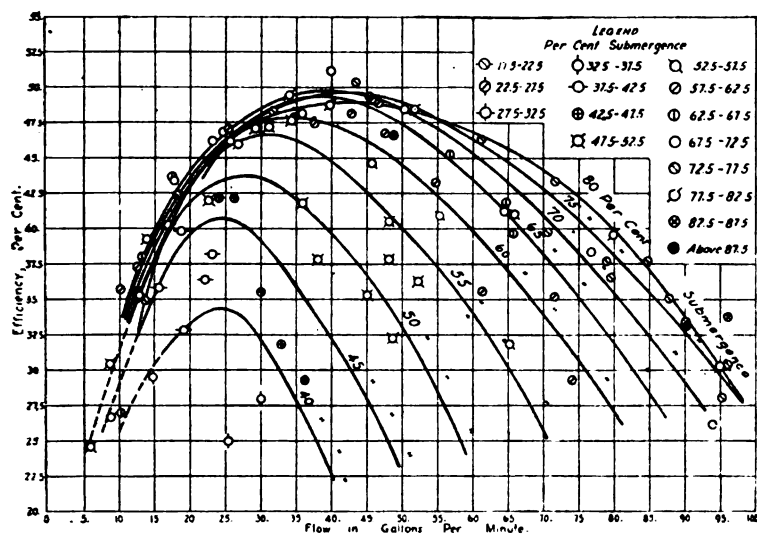


Fig. 26—Efficiency-flow relations in series 11; education pipe consists of 42.91 feet of 2½ inch pipe and 19.08 feet of 3 inch pipe; foot-piece C. See figure 3.

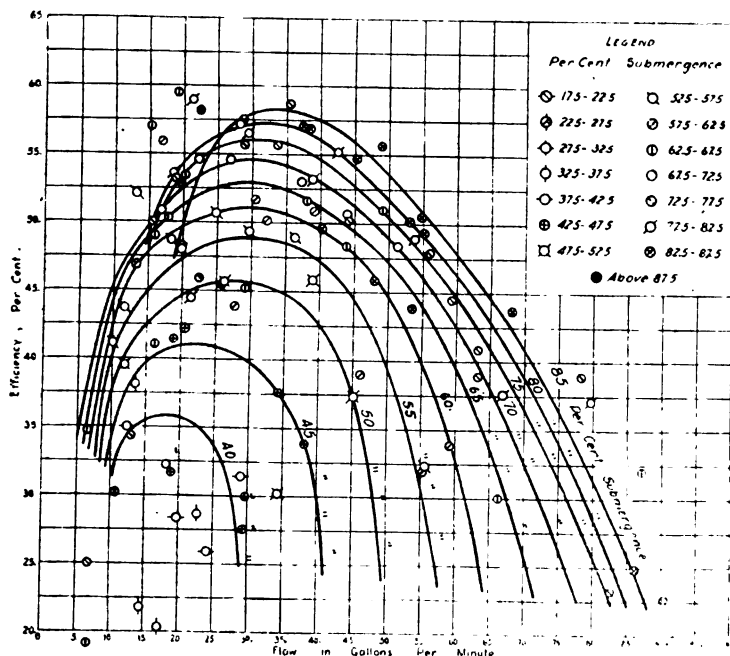


Fig. 27—Efficiency-flow relations in series 12; education pipe consists of 20.38 feet of 2 inch pipe, 21.62 feet of 2½ inch pipe, and 19.08 feet of 3 inch pipe; foot-piece C. See figure 3.

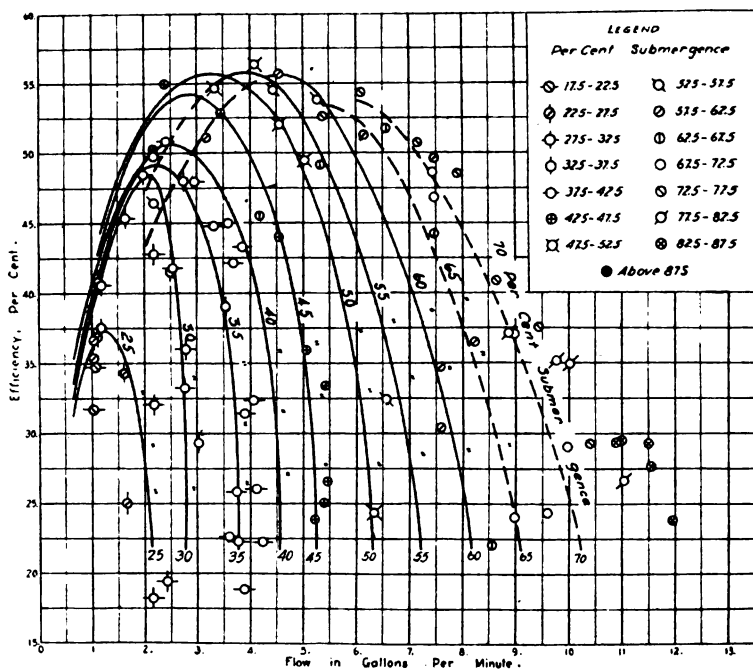


Fig. 23—Efficiency-flow relations in series 18, 1 inch eduction pipe 62.43 feet long, foot-piece C. See figure 3.

Diameter of Eduction Pipe Varied, Length Kept Constant.—See figs. 29 & 30.

With the same type of foot-piece and about the same length of eduction pipe, series 4, 5, 6 and 13 were taken with eduction pipes whose diameters were $2\frac{1}{2}$ in., $1\frac{1}{2}$ in., 3 in. and 1 in. respectively. The eduction pipes of greater diameter generally gave the highest efficiency. For each of the higher per cents of submergence the maximum efficiency obtainable was greater with the larger eduction pipes. For submergence of 40 per cent or less the small size of eduction pipe gave the highest maximum efficiency. The larger the eduction pipe, the greater the range of discharge over which a pump may be operated efficiently. Note that at 60 per cent submergence the one inch eduction pipe delivered water at a rate varying from one gallon per minute to eight gallons per minute, and it gave its maximum efficiency at $4\frac{1}{2}$ gallons per minute. A change in rate of pumping of only 2 gallons per minute from the rate which gave maximum efficiency reduced the efficiency more than 15%. The relation between discharge at maximum efficiency and the area of eduction pipe is practically a straight line relation.

Quantity of Air Used.—The curves of figures 31 to 43 give the relations between rates of pumping and the quantity of free air used. Instead of representing the amount of air consumed per gallon of water pumped, as is usually done, the amount of air per gallon per foot of lift has been computed. The curves for each of the pumps show the same general characteristics. At each per cent submergence there was a rate of pumping at which the volume of free air consumed per gallon pump per foot of lift was a minimum. The less the per cent submergence, the greater the relative quantity of air required. With the 1, $1\frac{1}{2}$, $2\frac{1}{2}$ and 3 inch pumps 60 ft. long, the range of volume of free air required per gallon per foot of lift for per cents of submergence between 45 and 70 per cent is .0087 to .0169 cu. ft. of free air. These values of the volume of the air required are those for minimum air consumption at each per cent submergence. In tests of the small eduction pipes the air supply could be increased so as to cause the pump to

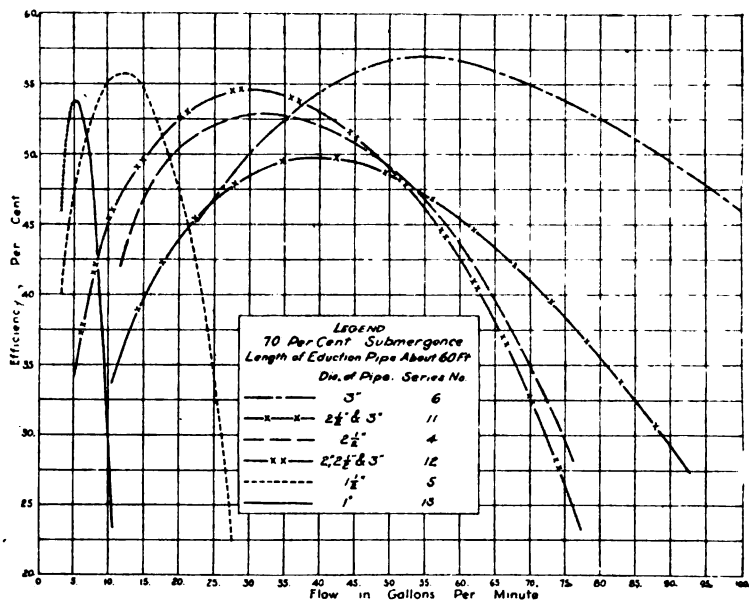


Fig. 29—Comparison of efficiency-flow relations in pumps with eduction pipes of different diameters and approximately of equal length. Submergence 70 per cent.

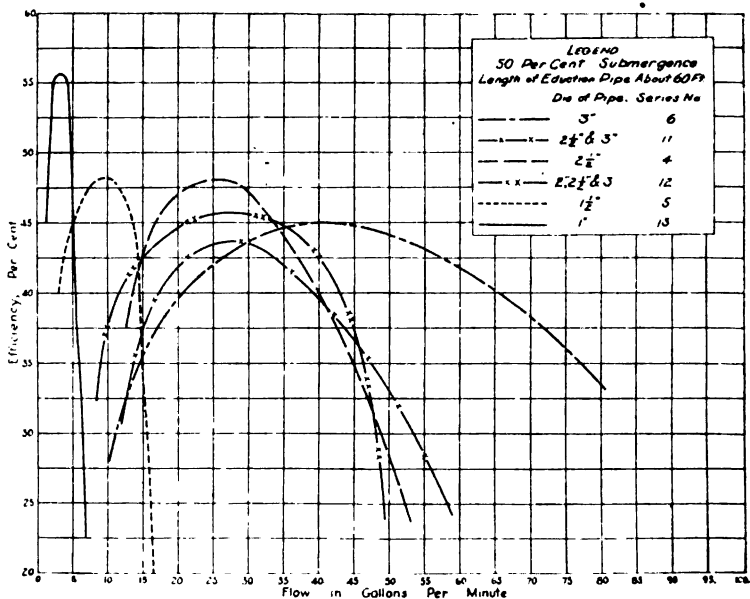


Fig. 30—Comparison of efficiency-flow relations in pumps with eduction pipes of different diameters and approximately of equal length. Submergence 50 per cent.

deliver water at rates much greater than the rate for maximum efficiency at each per cent submergence. This was particularly true in the case of the one inch pump. As the air supply was increased above that rate required for maximum efficiency the rate of water delivery increased, the increase in the rate of pumping being less than the rate of increase of the air supply. Finally, a point was reached where additional air did not increase the rate of pumping. Tests made on 1 inch eduction pipes subsequent to the preparation of the data for this bulletin indicated that a condition was reached where a further increase in the rate of air supply actually reduced the rate of pumping.

The volume of free air required per gallon per foot of lift increased as the total length of the pump became less, as shown in series 6, 7, 8, and 9. The range of air consumption at maximum efficiency for three inch pumps varying in length from 37 feet to 100 feet at 70 per cent submergence is from .0058 to .0159 cubic feet of free air per gallon per foot of lift. At 45 per cent submergence the range is .0105 to .0332 cubic feet of air per gallon per foot of lift.

The pumps made up of pipes of varying diameters, series 11 and 12, showed little advantage over pumps with eduction pipes of constant diameter. It appears that the air consumption was slightly higher than would be required in an eduction pipe of the same length made up of any one of the sizes of pipe which were used in these pumps.

Friction-Velocity Relations.—The “loss of head-velocity” relations have been brought together on one sheet for each of the sizes of eduction pipes tested. For example, all of the above relations in 3 inch pipe obtained from tests on three different pumps are plotted on one sheet. Figures 43 to 52 illustrate these relations. The exact ratio of air to water was originally recorded opposite each of the plotted points in the above figures. Lines then were drawn through points representing a particular ratio of air to water. In inking the above figures, symbols had to be adopted to represent the range of the ratio of air to water in which each of the plotted points fell.

These curves show that the “loss of head-velocity” rela-

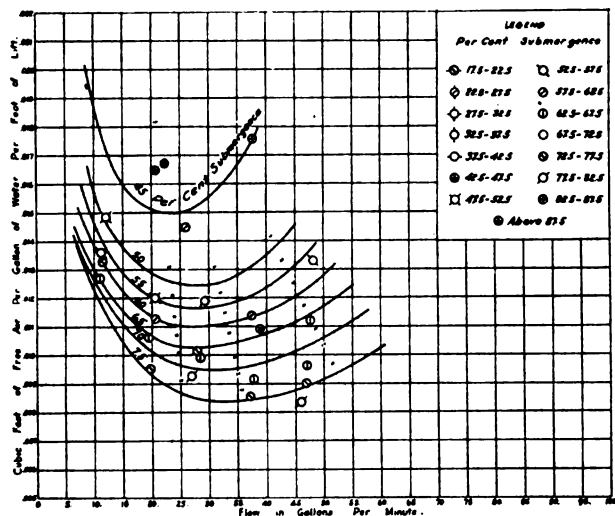


Fig. 31—Air consumption-flow relations in series 1, 2½ inch eduction pipe 61.17 feet long, foot-piece A with brass rings removed. See figure 2.

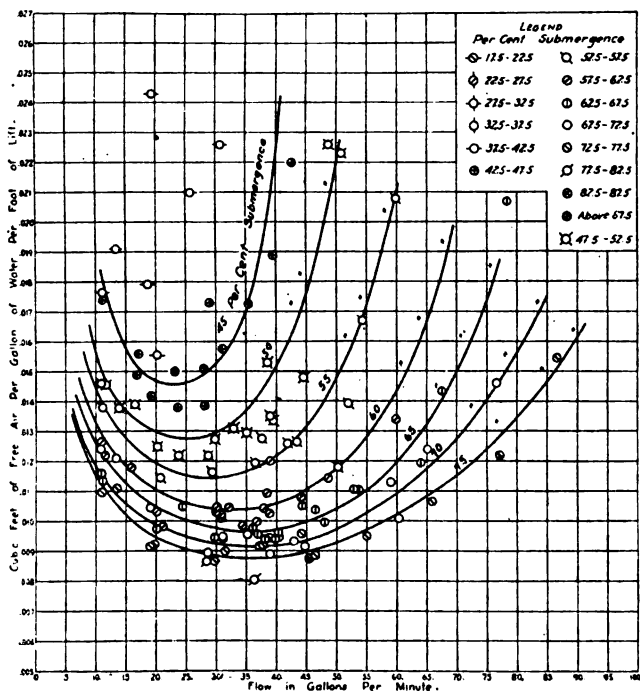


Fig. 32—Air consumption-flow relations in series 2, 2½ inch eduction pipe 61.17 feet long, foot-piece A. See figure 2.

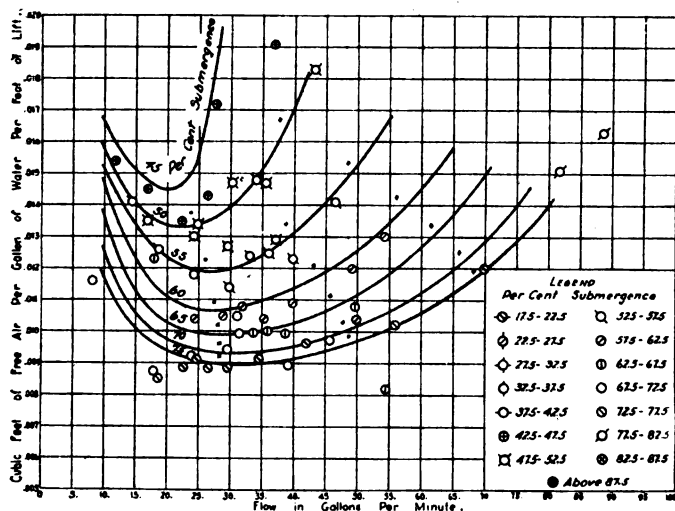


Fig. 33—Air consumption-flow relations in series 3, 2½ inch eduction pipe 60.88 feet long, foot-piece B. See figure 3.

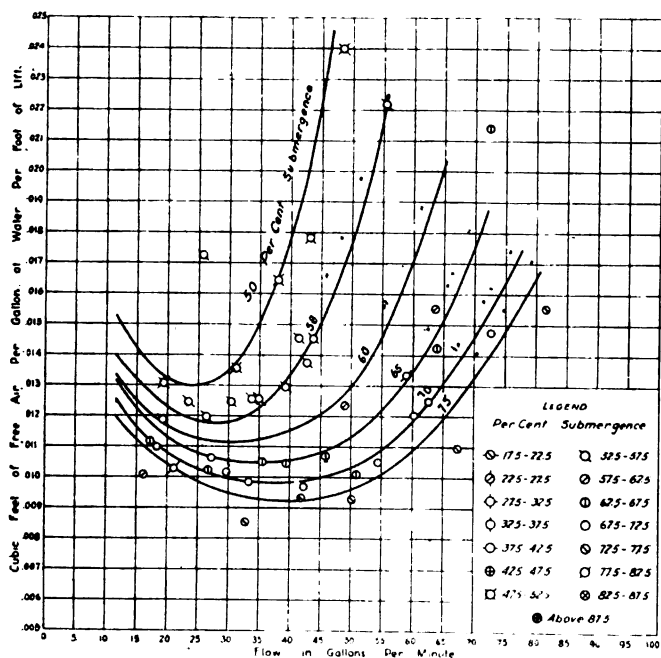


Fig. 34—Air consumption-flow relations in series 4, 2½ inch eduction pipe 60.88 feet long, foot-piece C. See figure 3.

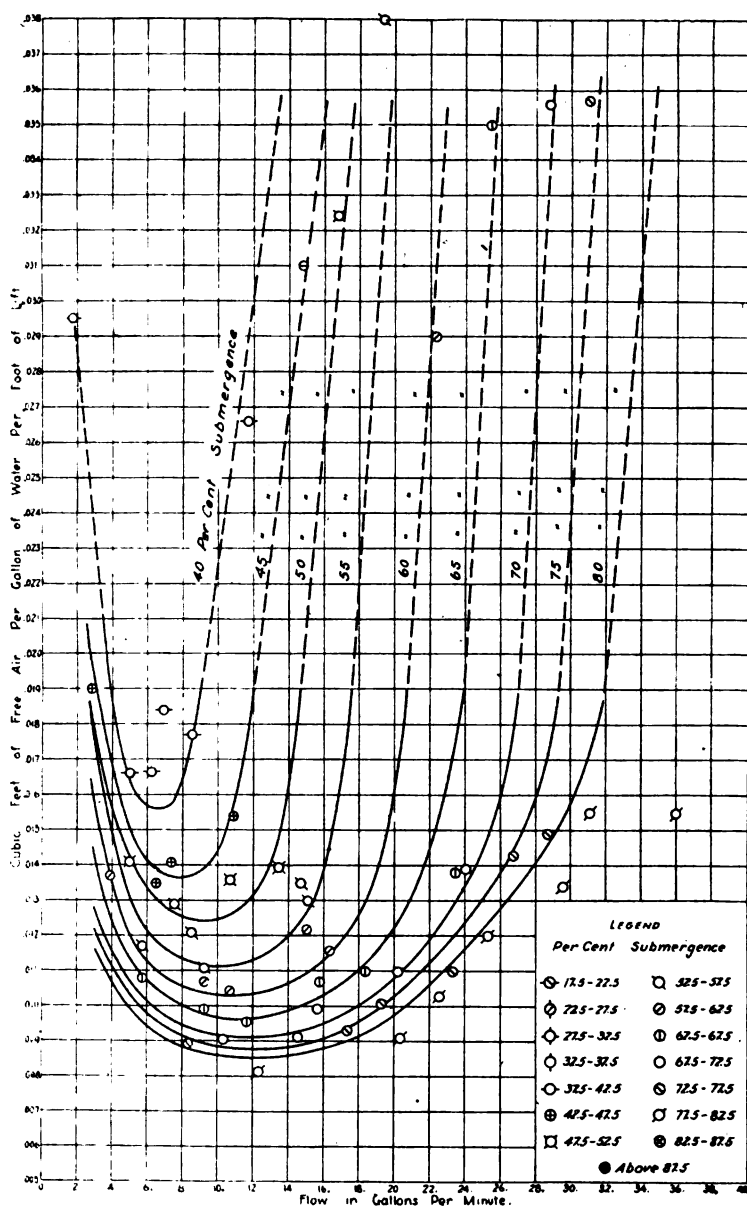


Fig. 35—Air consumption-flow relations in series 5, 1½ inch education pipe 60.88 feet long, foot-piece C. See figure 3.

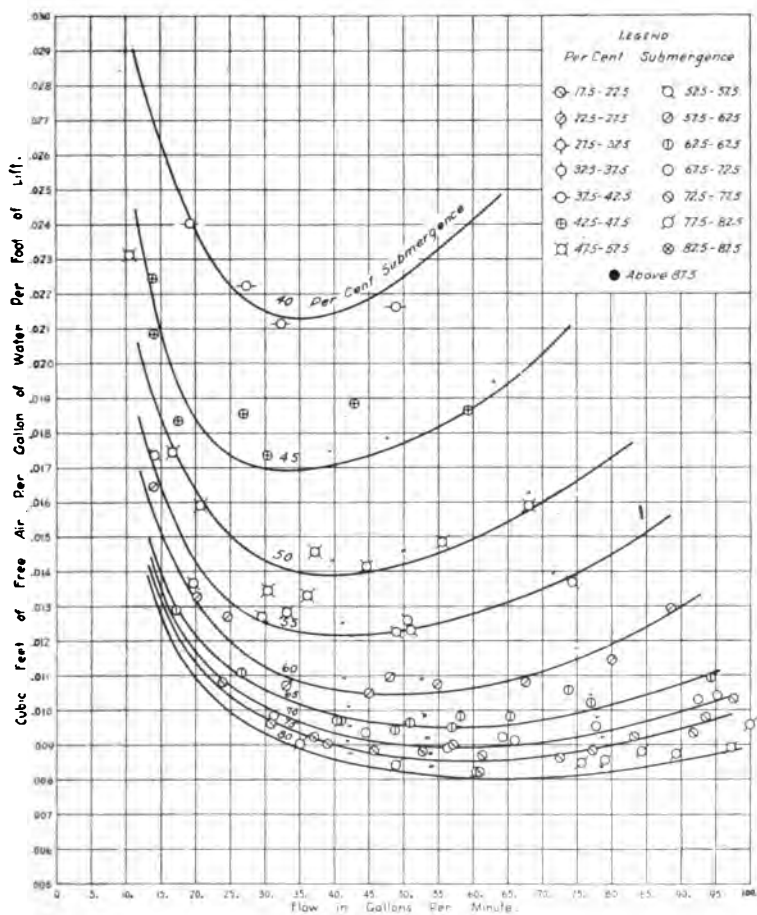


Fig. 36—Air consumption-flow relations in series 6, 3 inch education pipe 60.88 feet long, foot-piece C. See figure 3.

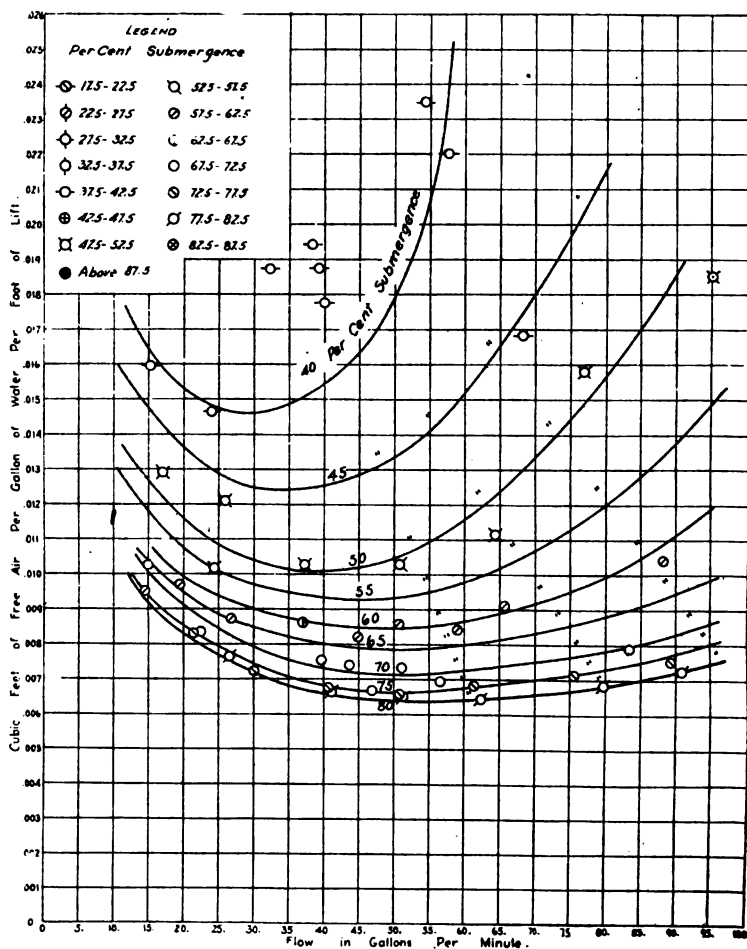


Fig. 37—Air consumption-flow relations in series 7, 3 inch education pipe 81.63 feet long, foot-piece C. See figure 3.

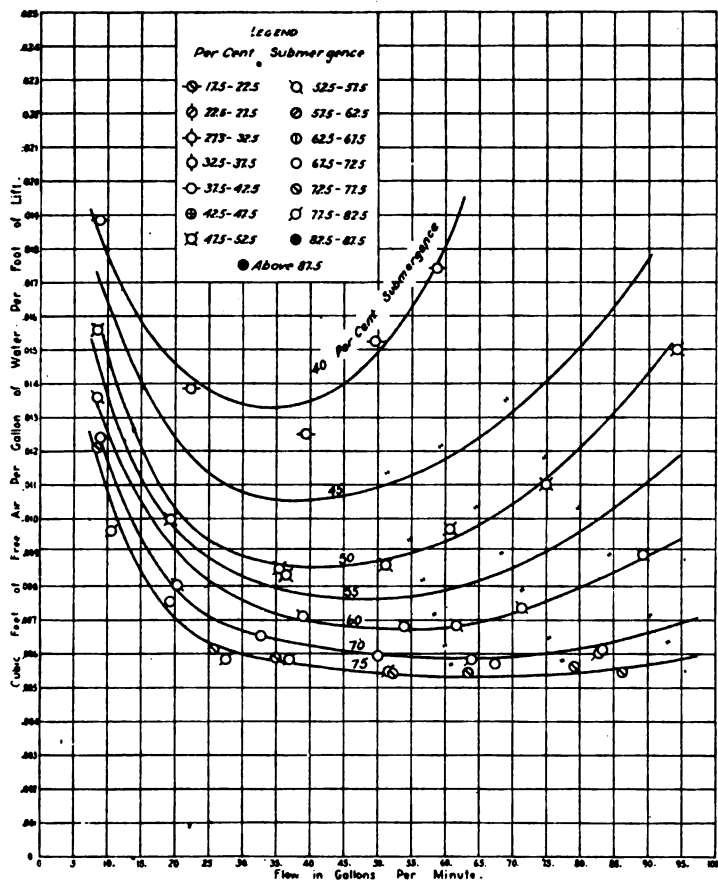


Fig. 38—Air consumption-flow relations in series 8, 3 inch education pipe 100.64 feet long, foot-piece C. See figure 8.

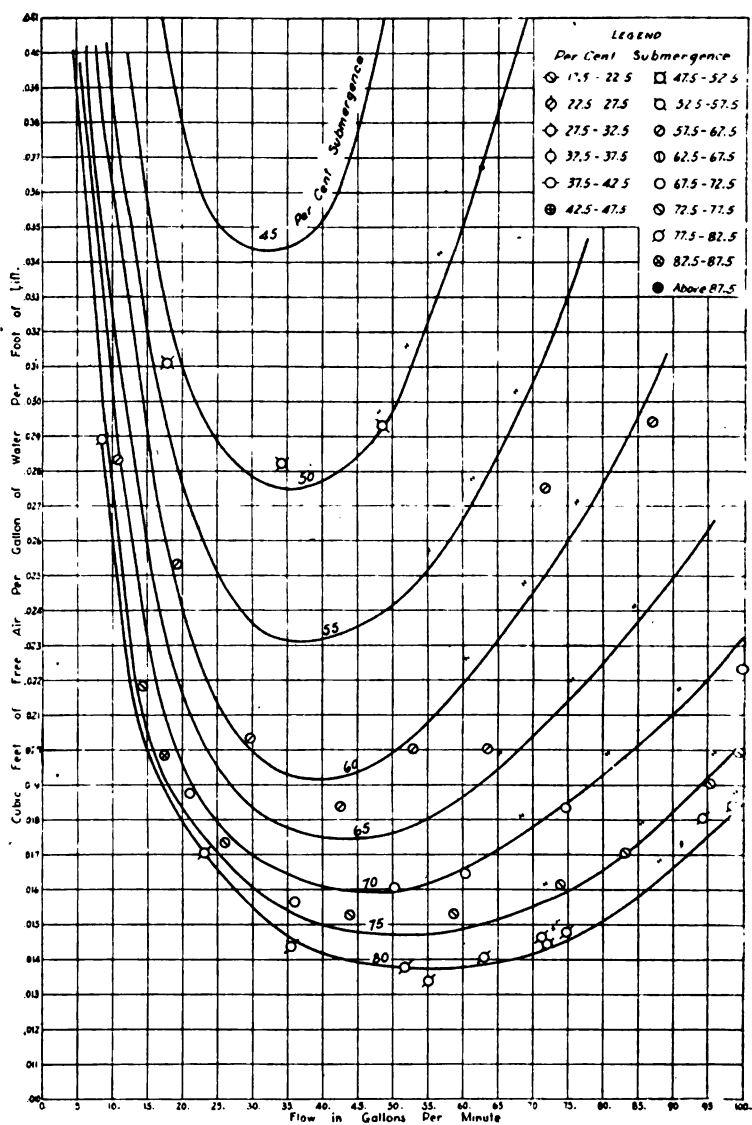


Fig. 33—Air consumption-flow relations in series 9, 3 inch eduction pipe 37.85 feet long, foot-piece C. See figure 3.

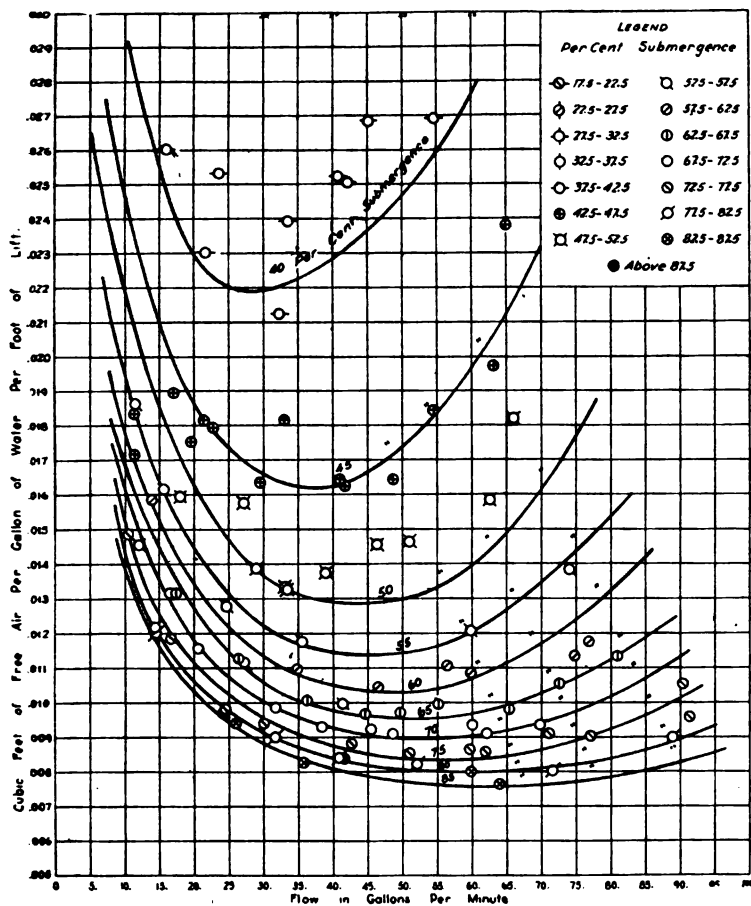


Fig. 40—Air consumption-flow relations in series 10, 3 inch eduction pipe, 61.26 feet long, foot-piece D. See figure 3.

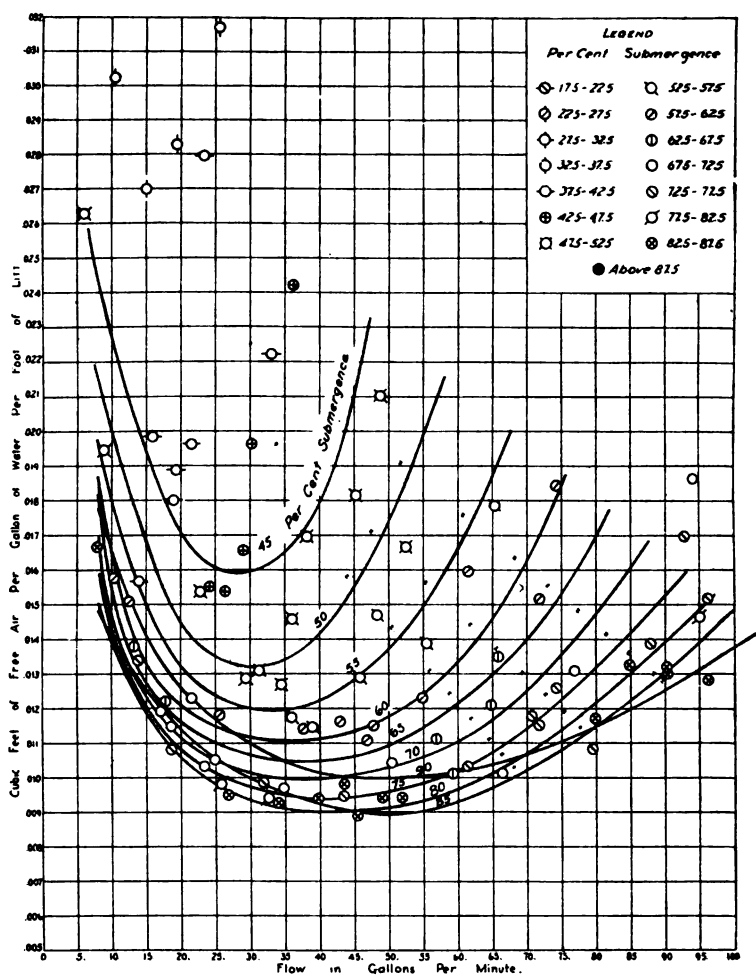


Fig. 41—Air consumption-flow relations in series 11, education pipe consists of 42.91 feet of 2½ inch pipe and 19.08 feet of 3 inch pipe, foot-piece C. See figure 3.

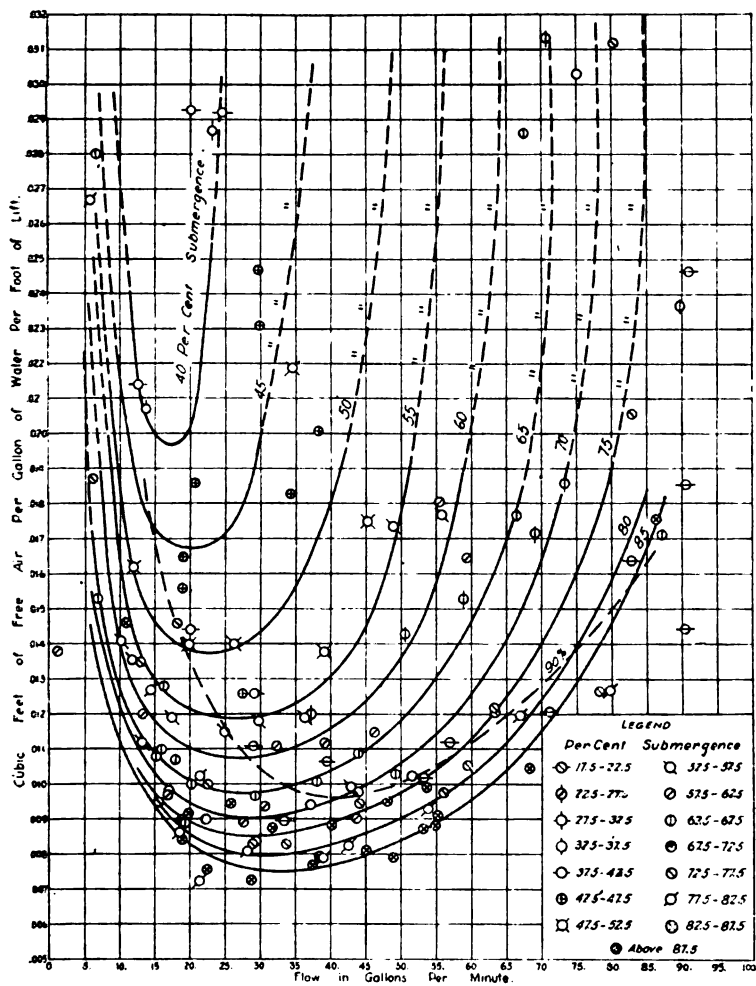


Fig. 42—Air consumption-flow relations in series 12, eduction pipe consists of 20.38 feet of 2 inch pipe, 21.62 feet of 2½ inch pipe, and 19.03 feet of 3 inch pipe. See figure 3.

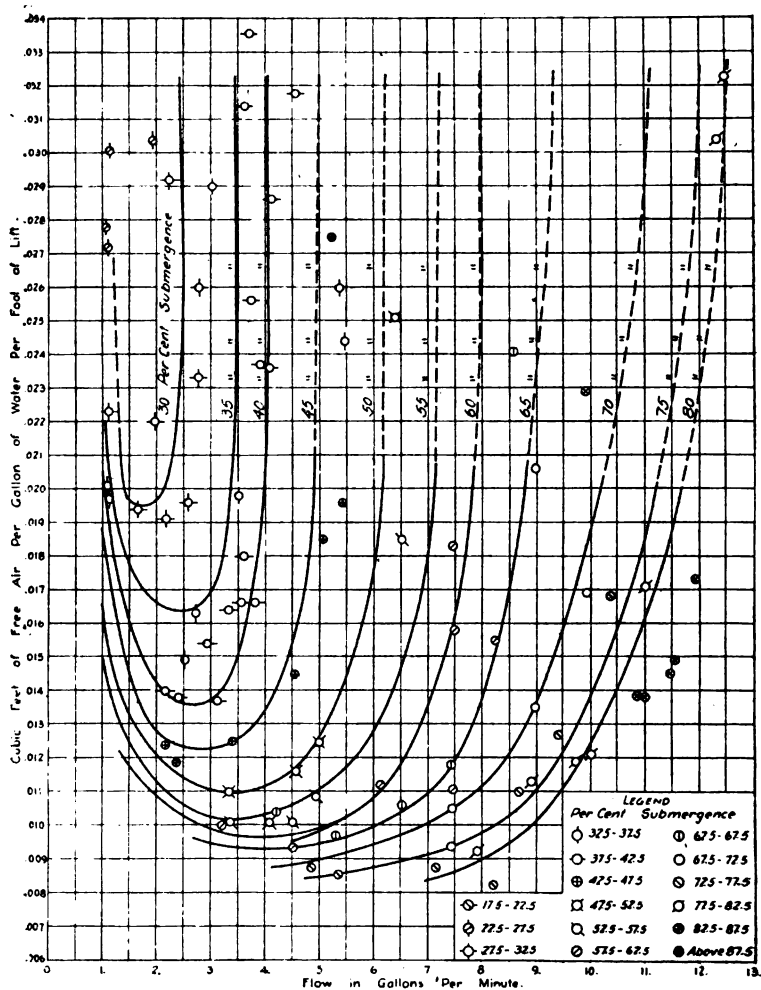


Fig. 43—Air consumption-flow relations in series 13, 1 inch eduction pipe 62.43 feet long, foot-piece C. See figure 3.

tions for air and water mixed are not similar to the relations which would be obtained in the case of either air or water flowing separately in pipes. For a given velocity the losses in any case are much higher than with water flowing alone. There is a velocity below which the losses increase as the velocities become less. It must be remembered that the loss of head in an eduction pipe consists not only of the ordinary friction losses which accompany the flow of liquid in pipes but also of another loss which is due to what has been termed "slip". The air in an eduction pipe does not mix with the water so as to form a perfect emulsion of air in water; the mixture is probably not homogeneous under any condition. The air mixes with the water in the form of bubbles which are small, depending upon the conditions of flow in the eduction pipe, the size of the eduction pipe, and possibly, to a slight extent, upon the conditions under which the air is admitted to the water at the foot-piece. It is not necessarily true that the bubbles will remain small throughout their passage upward through the eduction pipe simply because they were small when admitted at the foot-piece. In any case, the bubbles have a velocity upward relative to the water. This relative velocity is what is called "slip". Since there is no way of separating the losses due to friction which accompany the flow of the mixture of air and water, and the losses due to "slip", these losses must be treated together. A study of the "loss of head-velocity" curves and a comparison of these losses with those which accompany the flow of water alone, indicate that the effect of slip is relatively very great. At low velocities, the "loss of head-velocity" curves for all ratios of volume of air to water indicate that the losses increase very rapidly as the velocity of flow decreases. The supply of air may be decreased until finally the delivery of water becomes zero, and the entire energy of the air will then be used in maintaining a head in the eduction pipe which is less than the delivery head. The entire loss of head in this case is due to "slip". There is a particular average velocity in each eduction pipe for each average ratio of volume of air to volume of water at which the loss of head per foot of pipe is a minimum. The losses increase at a rate which depends upon the size of the

eduction pipe, for velocities greater than the above velocity. The loss of head increased more rapidly with an increase of velocity in small eduction pipes than in large eduction pipes. The velocity which gives the minimum loss of head for any particular ratio of volume of air to volume of water, increased as the size of eduction pipe was increased. The minimum loss of head in small eduction pipes was less than the minimum loss in larger eduction pipes, for the higher ratios of average volume of air to volume of water. These experiments indicate that the loss of head velocity curves would become less steep for eduction pipes greater in diameter than 3 inches, the maximum size tested in these experiments. The range of discharge at which a large eduction pipe may be operated with reasonable efficiency is relatively much greater than in the case of small eduction pipes.

The loss of head in various sections of the eduction pipe of a particular pump, the 2½ inch eduction pipe of series 2, are represented in figures 50 to 52. Figure 2 shows the location of sections 1, 2, 3 and 4. The shape of the curves are the same for each of the different sections of the pipe. At each ratio of volume of air to volume of water, however, the loss of head in the central part of the eduction pipe between sections 2 and 3 was somewhat less than in either of the two end sections. The loss in the gaging length between sections 1 and 2 should be relatively high. It is in the region immediately over the foot-piece where the adjustment of the mixture of the air and water takes place. It was observed in the glass model of an air lift pump that there was a region immediately above the foot-piece in which the flow was very turbulent. The effect of the elbow at the top probably accounts for the higher losses which were experienced between sections 3 and 4. At low rates of flow, when the discharge was intermittent, the elbow tended to retard the water and to cause a part of it to flow back down the eduction pipe. The effect of the foot-piece and the effect of appurtenances at the upper end of the eduction pipe have a relatively less influence on losses due to flow when the eduction pipe is long. Test data taken on a pump, whose length is much less than 40 feet, is apt to be misleading because of the relatively great influence of the above

factors upon the conditions of flow in the eduction pipe. This is illustrated in figures 44 and 48.

Frictional losses in 6 inch eduction pipes of lengths varying from 370 feet to 615 feet are given in table XVI. In this same table there are also represented the losses due to flow in 3, 3½ and 4 inch eduction pipes, each having a central air line.

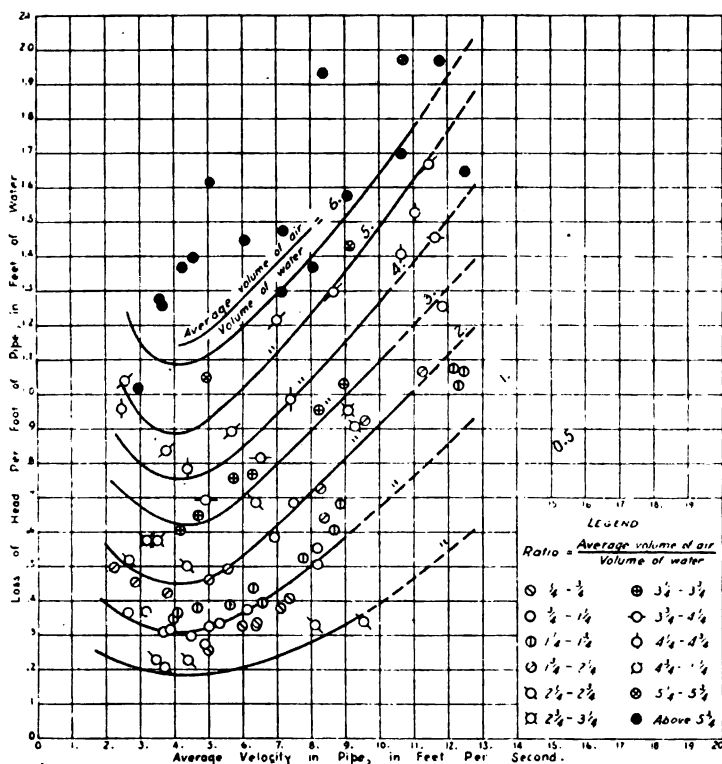


Fig. 44—Loss of head-velocity relations in 1 inch eduction pipe, series 13.

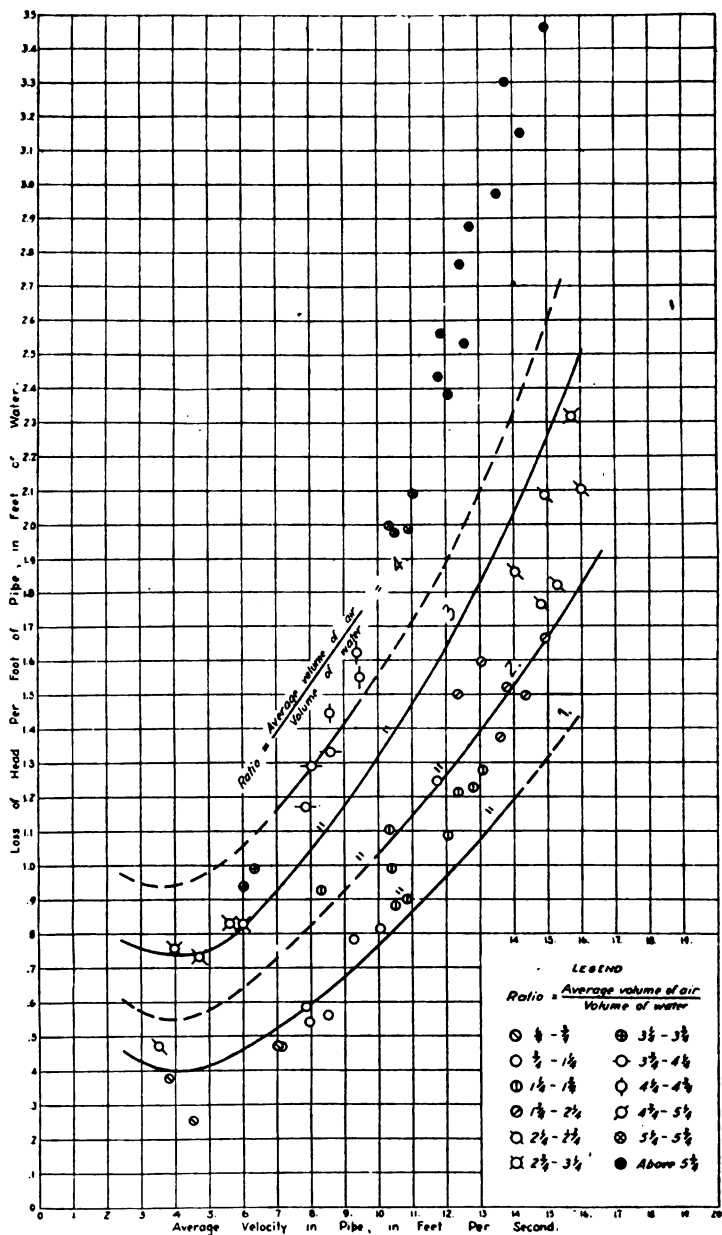


Fig. 45—Loss of head-velocity relations in 1 1/4 inch eduction pipe. Data taken from University of Wisconsin Bulletin, "An Investigation of the Air Lift Pump", by Davis and Weldner.

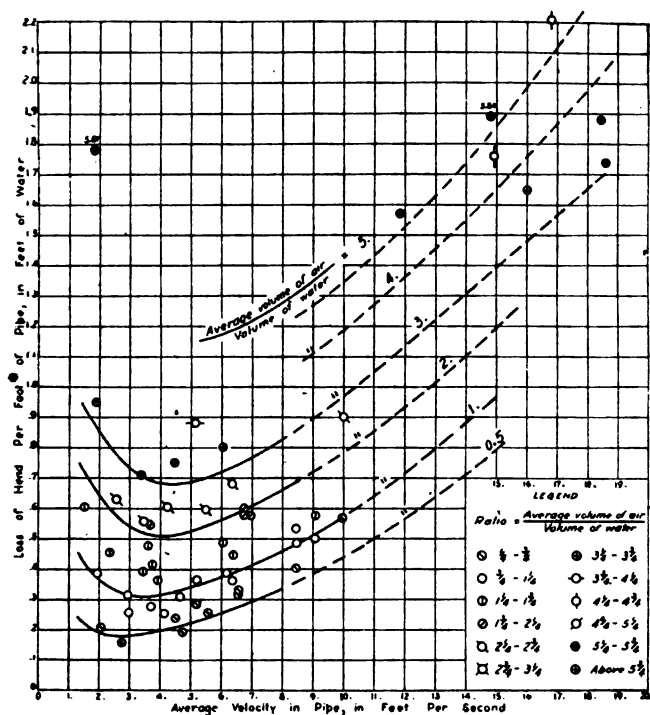


Fig. 46—Loss of head-velocity relations in $1\frac{1}{2}$ inch education pipe, series 5.

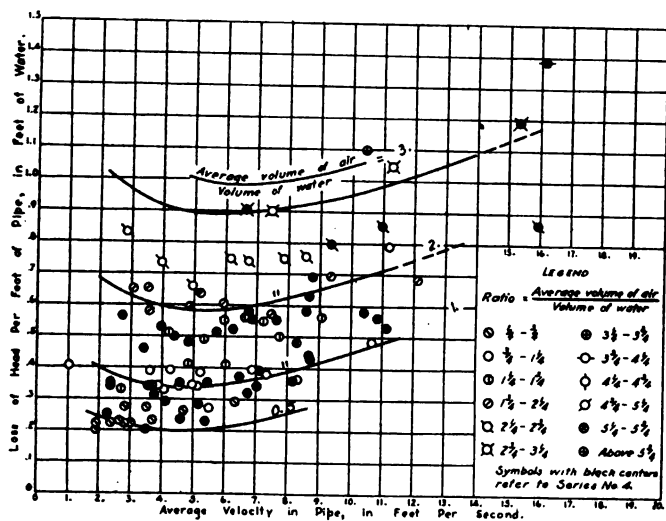


Fig. 47—Loss of head-velocity relations in $2\frac{1}{2}$ inch education pipes, series 3 and 4.

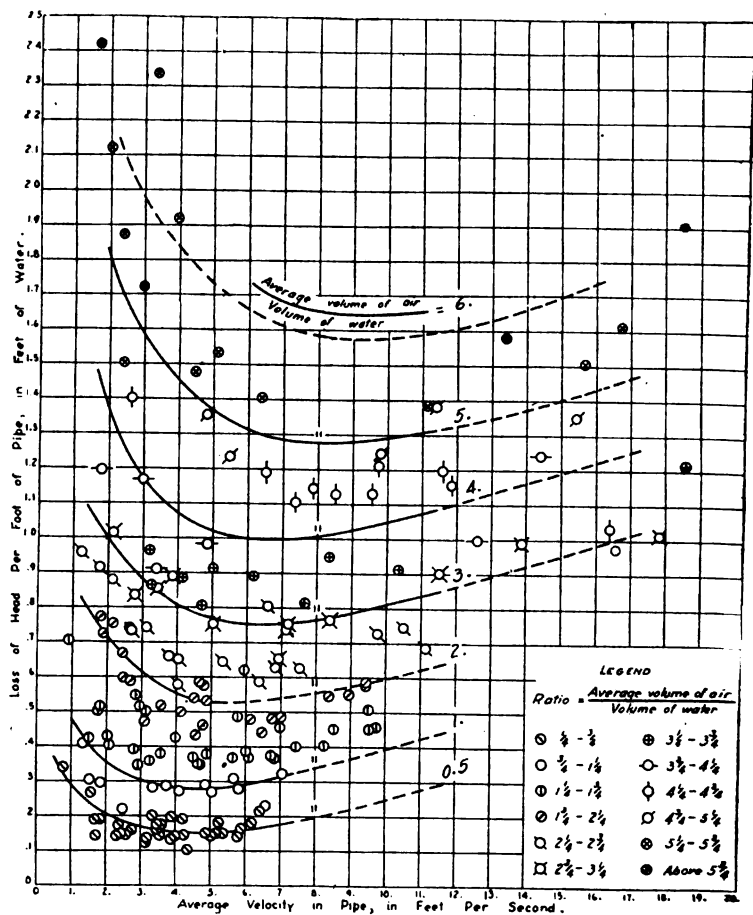


Fig. 48—Loss of head-velocity relations in 3 inch education pipes, series 6, 7, and 10.

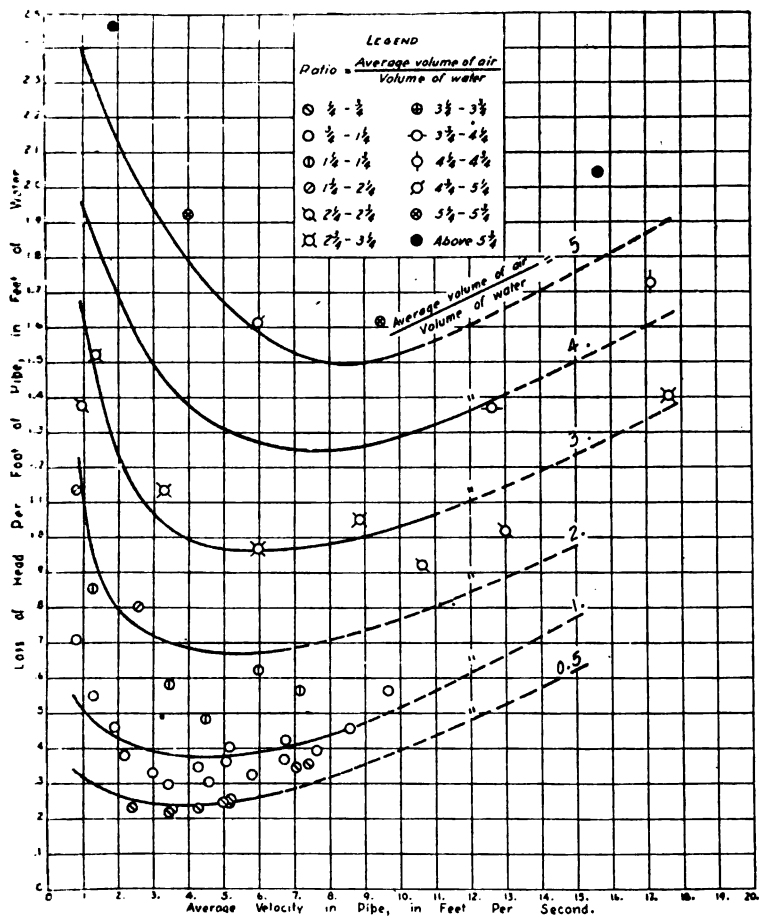


Fig. 49—Loss of head-velocity relations in 3 inch education pipe, series 9.

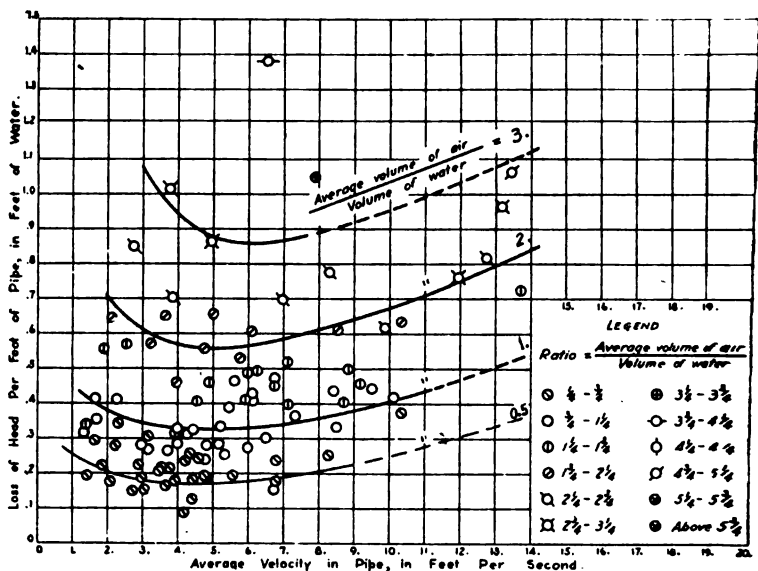


Fig. 50—Loss of head-velocity relations between sections 1 and 2 of the 2½ inch eduction pipe of series 2.

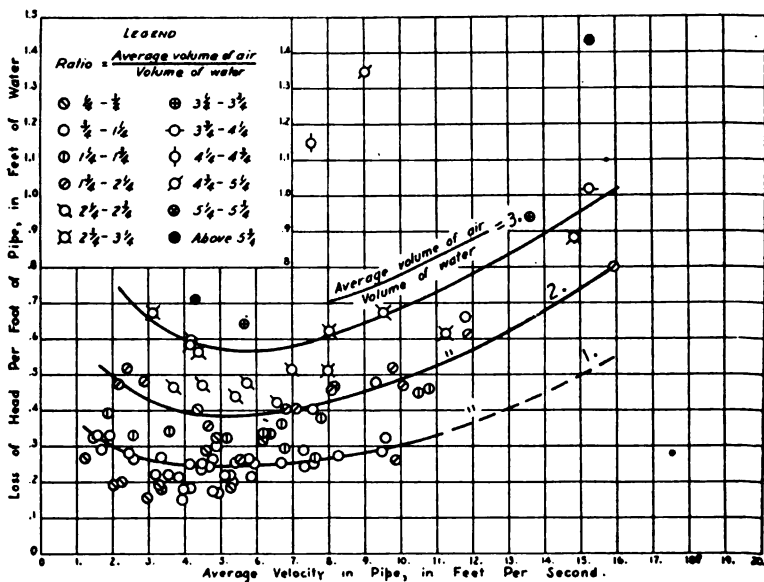


Fig. 51—Loss of head-velocity relations between sections 2 and 3 of the 2½ inch eduction pipe of series 2.

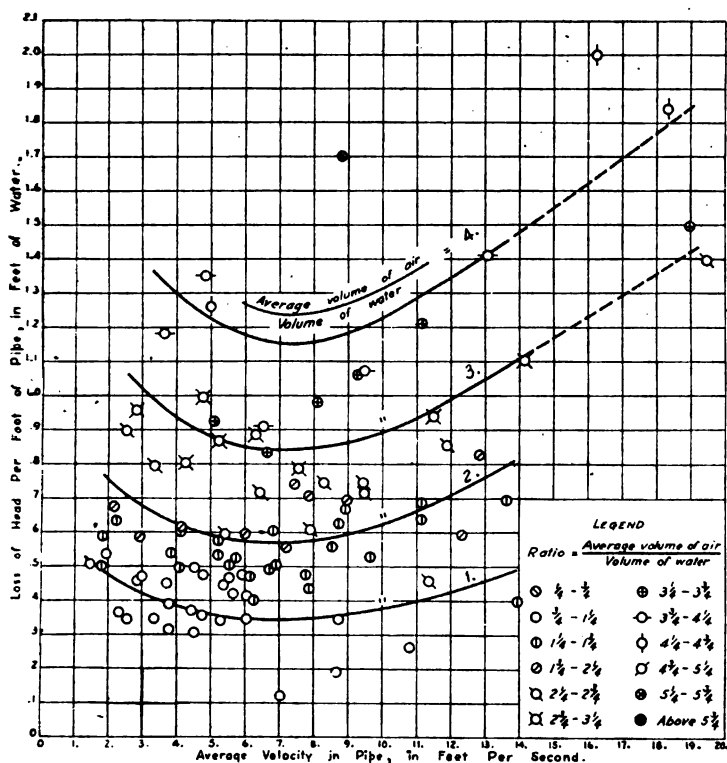


Fig. 52—Loss of head-velocity relations between sections 3 and 4 of the $2\frac{1}{2}$ inch eduction pipe of series 2.

Pressure-Time Relations.—Figures 53 and 54 are reproductions of “pressure-time” diagrams taken at different sections of the eduction pipe used in series 12. Section 1 was 2.51 feet above the foot-piece in the 2 inch pipe, section 2 was 20.45 feet above the foot-piece in the 2 inch pipe just below the point of connection with the $2\frac{1}{2}$ inch pipe, section 3 was 22.61 feet above the foot-piece in the $2\frac{1}{2}$ inch pipe, and section 4 was in the $2\frac{1}{2}$ inch pipe 30.43 feet above the foot-piece. These curves indicate that the pressure variation is greater at sections furthest from the foot-piece. It must be kept in mind, however, that in these figures the pressure scale is different for the lower sections than for the upper ones because of the necessity of using a heavier spring on the pressure indicator in the lower sections.

"Pressure-time" lines became more smooth as the ratio of volume of air to the volume of water decreased. This condition of flow was reached with high per cents of submergence, low delivery heads, and moderate rates of pumping.

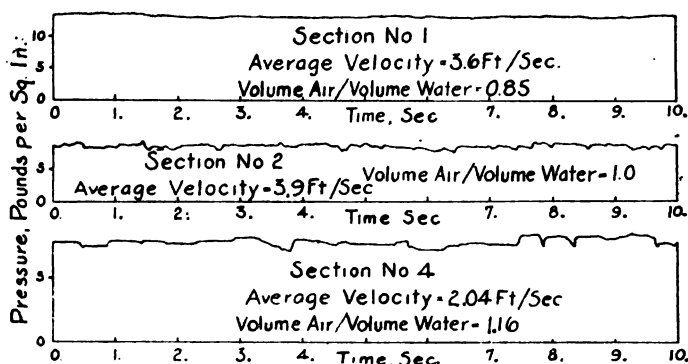


Fig. 53—Pressure-time diagrams taken along the eduction pipe of series 12. Rate of pumping 20.2 gallons per minute, submergence 62.5 per cent, and efficiency of pump 51 per cent.

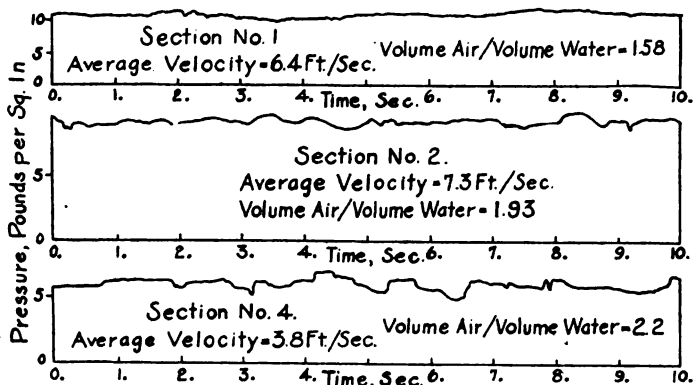


Fig. 54—Pressure-time diagrams taken along the eduction pipe of series 12. Rate of pumping 25.7 gallons per minute, submergence 51.6 per cent, efficiency of pump 47 per cent.

DESIGN OF AIR LIFT PUMPS

A method of using the "friction loss-velocity" relations, which have been developed in this bulletin, in the design of air-lift pumps, is illustrated below. Equation number (16),

$$V_s = \frac{h_d + H'_{1-2} + v^2/2g}{144 (h_s + h_a) \log_{10} \frac{h_s + h_a}{h_a}}$$

serves as a basis for determining the amount of air required. By representing the term

$$144 \left\{ (h_s + h_a) \log_{10} \frac{h_s + h_a}{h_a} \right\} \text{ by } K_1$$

the above equation may be written

$$V_s = \frac{h_d + H'_{1-2} + v^2/2g}{K_1}$$

(V_s) is the volume of air required at the foot-piece per pound of water pumped. In the above equation, (H'_{1-2}) varies with (V_s). The loss of head-velocity curves are used in a trial and error method of solving the equation. A direct solution could be developed if the relation between loss of head, velocity, ratio of volume of air to volume of water, and diameter of eduction pipe were expressed by empirical equations. These equations would not be simple and it is probable that the direct solution would be more difficult than the following indirect method.

The term K_1 in the equation above represents the foot pounds of energy in one cubic foot of air at the foot-piece. To facilitate the solution of the equation, values of K_1 were plotted against corresponding values of (h_s) in one of the curves of figure 55.

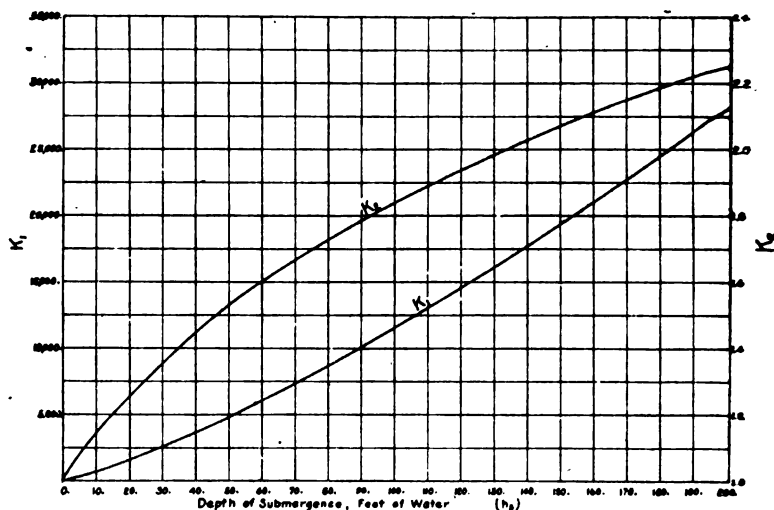


Fig. 55—Curves which may be used in the designing of air lift pumps.
See pages 87-92 inclusive.

The relation,

$$V_{av} = \left[\frac{2.303 (h_s + h_a)}{h_s} \log_{10} \frac{h_s + h_a}{h_a} \right] V_s .$$

gives the average volume of (V_s) during its passage up through the eduction pipe. Let

$$K_2 = \frac{2.303 (h_s + h_a)}{h_s} \log_{10} \frac{h_s + h_a}{h_a} .$$

Then $V_{av} = K_2 V_s$. Values of K_2 may be readily obtained for different values of (h_s) from figure 55. The curves of figure 55 are each based on a value of (h_a), the atmospheric pressure, of 34 feet of water.

The Design Problem

Let the conditions of the design be as follows:

Delivery head, $h_s = 50$ feet.

Rate of pumping = 50 gallons of water per minute.

The well is deep so that any desirable submergence of the foot-piece may be had.

Pumps with a submergence of 70 per cent will be tried first.

$$h_s = \frac{S}{1-S} h_d = \frac{.70}{.30} \times 50. = 117. \text{ feet.}$$

Length of eduction pipe, $L = 117. + 50 = 167. \text{ feet.}$

K_2 is found to equal 14,100. for h_s equal to 117. feet from figure 55.

Equation (16) then gives

$$V_s = \frac{50 + H'_{1-2}}{14,100}. \text{ The term } \frac{v_2^2}{2g}, \text{ which is relatively small,}$$

has been dropped in this first trial.

This equation is solved by taking an estimated value of H'_{1-2} and solving for V_s . H'_{1-2} may then be determined quite accurately with this approximate value of V_s and the "friction-velocity" curves. This process is continued until values of V_s and H'_{1-2} are obtained which satisfy the above equation and also the relations given in the "friction-velocity" curves.

To obtain an approximate value of H'_{1-2} assume an efficiency of the pump of 50%. Then $\frac{50}{H'_{1-2} + 50} = .5$ and $H'_{1-2} = 50 \text{ feet.}$

$V_s = \frac{50 + 50}{14,100} = .00710 \text{ cubic feet of air per pound of water pumped.}$

$$V_{a,v} = K_2 V_s, K_2 = 1.93$$

$$V_{a,v} = 1.93 \times .00710 = .0137 \text{ cu. feet per pound of water.}$$

The volume of 1. pound of water equals .01601 cubic feet

$$\text{The ratio of air to water, by volume } R_v, \text{ equals } \frac{.0137}{.0160} = .856$$

The average volume of mixture per pound of water equals $.0137 + .0160 = .0297 \text{ cubic feet.}$

50 gal. of water per minute = 6.95 pounds of water per second.

Average total volume of mixture passing the eduction pipe equals $6.95 \times .0297 = .2065 \text{ cubic feet per second.}$

A $2\frac{1}{2}$ inch pipe has an area of .0332 square feet. With this size of pipe the average velocity of flow equals $\frac{.2065}{.0332} = 6.2$ feet per second. Since this is a reasonable average velocity, the $2\frac{1}{2}$ inch pipe will be investigated.

Referring to the "loss of head-velocity" curves for $2\frac{1}{2}$ inch eduction pipes, figure 47, it is found that for a ratio of air to water of .86, and an average velocity of 6.2 feet per second that the loss of head per foot of pipe is about .31 feet of water.

The total loss $H'_{1-2} = 167. \times .31 = 51.7$ feet of water.

This happens to check unusually well with the assumed value of H'_{1-2} ; the check is closer than is usually experienced in the first trial.

Now the velocity head at point of discharge $v_2^2/2g$ can be considered. With $V_s = .0071$, the average volume passing section 2 is 3.30 cubic feet per second and $v_2^2/2g = 1.5$

$$\text{Now } V_s = \frac{50 + 51.7 + 1.5}{14,100} = .00732.$$

Since $H'_{1-2} = 51.7$ is based on a value of V_s so nearly equal to the new value of V_s , .00732 cubic feet per second may be taken as essentially the correct value of V_s .

The efficiency of the $2\frac{1}{2}$ inch pump, 167 feet long equals

$$\frac{50 \times 100}{50 + 51.7 + 1.5} = 48.5 \text{ per cent.}$$

The free air required equals

$$.00732 \times 50. \times 8.33 \times \frac{151}{34} = 13.5 \text{ cubic feet per minute,}$$

at the temperature of the water.

With a $2\frac{1}{2}$ inch eduction pipe, 150 feet long—60 per cent submerged—and a delivery of 50. gallons per minute, it was found by the above method that the efficiency would be 47.6 per cent. Volume of free air required would be 15.28 cubic feet per minute.

It would be necessary after investigating other $2\frac{1}{2}$ inch eduction pipes to try 3 inch eduction pipes in this particular problem. The pump giving the greatest efficiency then should be selected.

Determination of Characteristics of a Given Pump.—

Another problem which arises is that of determining what the rate of pumping would be with a given rate of air supply to some particular pump. Assume that 18 cubic feet of free air, at the temperature of the water, is delivered to the above $2\frac{1}{2}$ inch, 167. foot pump.

The equivalent volume at the foot-piece of 18 cubic feet of free air is $18 \times \frac{34}{151} = 4.06$ cubic feet per minute.

Av. volume = $4.06 \times 1.93 = 7.82$ cubic feet per minute = .1302 cu. ft. per second. Assume first that $H'_{1-2} = 50$. feet.

$$\text{Then } V_s = \frac{100}{14,100} = .00710, V_{av} = .0137.$$

$$\text{Ratio by volume of air to water} = \frac{.0137}{.0160} = .856$$

Since .1302 cubic feet of air per second—average volume—is available and .00710 cu. feet of air is required per pound of water,

$$\frac{0.1302}{.0137} = 9.5, \text{ the approximate number of pounds of water pumped per second.}$$

Average total volume of mixture =

$$9.5 (.0137 + .0160) = .282 \text{ cubic feet per second.}$$

$$\text{Average velocity} = \frac{.282}{.0332} = 8.49 \text{ feet per second.}$$

Check,—

With $v = 8.5$ and $R_v = .86$, the "loss of head-velocity" curves show that the loss of head per foot of pipe = .37 feet. Total loss equals $167 \times .37 = 61.8$ feet and $v_2^2/2g = 2.8$ feet. This does not agree with the assumed value of H'_{1-2} . Starting again with $H'_{1-2} = 61.8$ feet instead of 50.0 feet, we have

$$V_s = \frac{50 + 61.8 + 2.8}{14,100} = .00813 \text{ cubic feet.}$$

$$V_{av} = 1.93 \times .00813 = .0157 \text{ cubic feet.}$$

$$\text{Water pumped} = \frac{.1302}{.0157} = 8.29 \text{ pounds per second.}$$

$$\text{Av. velocity} = \frac{.1302 + 8.29 \times .0160}{.0332} = 7.9 \text{ feet per second.}$$

$$R_v = .982$$

With a ratio of air to water of .98, and an average velocity of mixture equal to 7.9, the loss per foot of pipe is found from the curves to be .375 feet. Total loss equals $167 \times .375 = 62.7$ feet, which agrees closely with the value of H'_{1-2} used in the last solution of V_s . The rate of delivery of water will be 8.29 pounds per second or 59.7 gallons per minute with 18 cubic feet of free air supplied to the pump per minute.

The "friction-velocity" relations furnish a basis for the design of air lift pumps made up of pipes of different diameters. The writer knows of no other rational method of design of this type of pump. Many pumps are designed on the assumption that the hydraulic gradient over such an eduction pipe is a continuous straight line. This is usually far from the truth. The drop of pressure per foot of pipe in the direction of flow is much greater in the smaller pipes than in the larger pipes which are used in the upper part of the eduction pipe. The velocities which actually exist in the pipes are much higher than the designer estimates them to be, and often much higher than they should be.

In some wells, the bore of the well is such as to prohibit a pipe of proper size to be installed. The writer is convinced that in many cases pumps are designed to operate at velocities which are too great. It is probably felt that high velocities, which give apparent smooth and uniform pumping, are essential for high efficiency. This is not the case. Some of the pumps made up of pipes of varying diameter give less efficiency than an air lift pump of uniform diameter would give. This is particularly true in wells with low lifts.

The design of eduction pipes of varying diameter by the use of the "friction head-velocity" curves is quite similar to that used above. The pressure at sections where pipes of different size join is first assumed and then computed by the

trial and error method. Finally, the total losses due to friction plus the velocity head at section (2) plus the delivery head must be equivalent to the energy supplied by the air.

SUMMARY OF CONCLUSIONS

The air lift pump is mechanically very simple. The principles of hydraulics, which should govern the design of these pumps, however, are complicated. Since the conditions to be encountered in the pumping of any two wells are not usually exactly similar, each pump installation requires special attention. As in the case of centrifugal pumps or hydraulic turbines, the air lift pump will not give reasonable efficiencies if improperly designed or if used under conditions for which it is not adapted. The limits of length and of diameter of suction pipes, which will give good efficiency in a particular well, are rather narrow.

Some of the other important conclusions reached in this study are given below:

1. The efficiency of an air lift pump depends primarily upon the conditions of flow in the suction pipe.
2. Great refinement in the design of foot-pieces is not necessary. Small air openings for dividing the air into fine bubbles, and special devices for mixing the air with the water are not necessary. There should be no central nozzle or projecting part to obstruct the flow of water in the foot-piece.
3. A particular pump in a given well gives maximum efficiency at some particular rate of pumping. The smaller the pump, the narrower the range of rate of pumping in which high efficiencies may be obtained.
4. The per cent submergence required for maximum possible efficiency ranges from 65. to 75. per cent in most cases. The lower range of submergence is approached in wells with a high delivery head.
5. Very small pumps give relatively high efficiencies with low submergences. A one inch pump shows good efficiency at a submergence as low as 45. per cent.

6. It is possible that the air lift pump may be satisfactorily adapted to the pumping of small wells such as are used for rural water supplies.

7. The combined friction and slip losses due to the flow in eduction pipes follow a different law than that which governs the flow of water, or of air, in a pipe.

8. There is a comparatively simple relation between frictional losses and velocity of flow in an eduction pipe for any particular mixture of air and water.

9. At a given velocity of flow in a given eduction pipe, the losses increase as the ratio of volume of air to volume of water increases.

10. There is a particular velocity of flow, for any ratio of volume of air to volume of water which is accompanied by a minimum loss of head. Losses increase very rapidly when the average velocity is reduced below the above velocity. The rate of increase of losses with increase of velocity depends upon the diameter of the eduction pipe. Relatively high velocities may be used in large eduction pipes. In the small eduction pipes, losses increase rapidly with increase of velocity above the velocity which gives maximum efficiency.

11. Special attention should be given to the making of smooth joints in eduction pipes. Changes from one size of pipe to another should be gradual. A sudden enlargement is very detrimental to efficient operation. Eduction pipes should be vertical. A horizontal travel of a mixture of air and water results in the separation of the air from the water.

12. In many cases, the writer believes that eduction pipes of uniform diameter could be designed to give better efficiencies than are now obtained with those of varying diameter. This is particularly true of pumps used in wells in which the delivery head is relatively low.

13. Many pumps of varying diameter are designed on the basis of a straight line hydraulic gradient from the foot-piece to the point of discharge. This is a faulty assumption which results in the using of velocities that are entirely too high in the lower sections of the eduction pipe. A very thorough analysis of the problem is required in order to design this type of eduction pipe.

14. The loss of head-velocity analysis is the most satisfactory method of correlating experience with air lift pumps. This method makes possible the extension of experience to the practical design of pumps to operate under conditions different than those encountered in the tests.

15. Many experimental tests of pumps do not include sufficient data for a complete analysis of the conditions of operation. The temperature of the liquid pumped should be recorded in field tests. Pressure measurements should be taken, when possible, at points in suction pipes where pipes of different size are joined. These pressures may be obtained at small cost, if one-quarter inch gage pipes are installed when the suction pipe is being assembled and lowered into the well. Pressure observations then can be made in the same way that the level of water in the well is usually obtained by the use of "tell-tale" pipes. There is particular need for further experimental tests on suction pipes of large diameter.

16. Test models of air lift pumps less than 40 feet in length are apt to give results which are different than those obtained with long pumps. Losses which are relatively insignificant in large pumps become important in short air lift pumps.

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APPENDIX—A

Apparatus and Methods Used in Determining the "Slip" of Bubbles.—The apparatus shown in figure 56 was used in the study of the velocity of bubbles of air in water. On the 6 inch tee, which served as a reservoir at the bottom of the

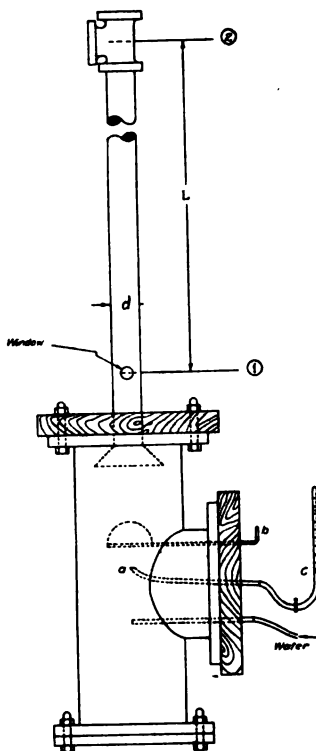


Fig. 56—Apparatus used in studying the motion of air bubbles in water.

vertical pipe, were three blank flanges made out of 2 inch planks. The various connections to the tee were made by boring holes through the wood flanges.

The method used in measuring the air and in delivering it to the water was as follows: The graduated glass tube *c* was removed from the rubber hose and was partly filled

with water. The tube was re-inserted into the hose and was held in a vertical position with the closed end of the tube upward. Air which was thus trapped moved up to the top of the tube where its volume could be read on the graduated scale. The pressure head on the air during this volume

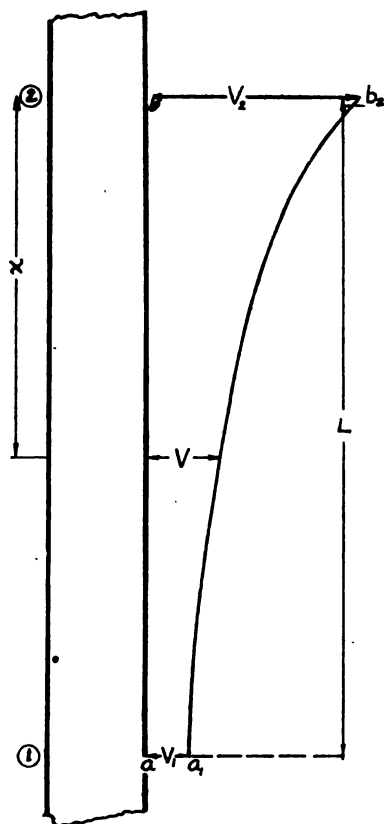


Fig. 57—Diagram illustrating the change of volume of air along education pipe.

measurement was equal to the difference in level of the water surface in the tube and section 2 in the pipe. This length was recorded opposite the volume of the bubble. After the above measurements were completed, the air was forced out through the pipe *a* and up to the inverted cup. By turning the spindle *b*, the cup was revolved so that the entire volume of the air could be started up into the pipe as a

single bubble. The air was broken up into many bubbles when the cup was not used.

The window at section 1 in the iron pipes was made by drilling a $\frac{1}{2}$ inch hole through the walls of the pipe. A thin sheet of celluloid and a rubber gasket were wrapped around the pipe at this section. Section 2 was at the surface of the water in the pipe.

In the tests made with the water in motion, water was delivered to the tee through the pipe shown in the figure. The average velocity of flow of the water was determined by weighing the water discharged in a known period of time, and by dividing the rate of discharge by the area of the pipe.

Determination of Average Velocities in Long Eduction Pipes.—In these experiments it was found that the hydraulic gradient in long eduction pipes of constant diameter is practically a straight line. For extremely high rates of pumping—rates of pumping greater than would be used in practice—the hydraulic gradient is slightly curved, being concave away from the eduction pipe.

Assume that pressures at sections in the eduction pipe vary directly with the depth of those sections. Referring to figure 57, let h_1 and h_2 be the pressure heads (feet of water, absolute) at the ends of eduction pipe ab . By plotting volume of air horizontally the isothermal "pressure-volume" curve a_1b_2 will be obtained. V_1 and V_2 are the volumes of air in cubic feet per second passing sections 1 and 2, respectively. The pressure p at any depth x below section 2 is

$$62.4 h_2 + \frac{x}{L} (p_1 - p_2), \text{ pounds per square foot.}$$

$$p = 62.4 h_2 + \frac{x w}{L} (h_1 - h_2), \text{ pounds per square foot.}$$

$$\text{since } p V = p_1 V_1$$

$$V = \frac{wh_1 V_1}{wh_2 + \frac{x w}{L} (p_1 - p_2)} .$$

The average volume of the air during its passage through this section ab of the pipe equals the area between the curve $a_1 b_2$, and the line ab , divided by the length L .

The average volume equals,

$$\begin{aligned} & \frac{\int_0^L V \, dx}{L} \\ &= \frac{1}{L} \int_0^L \frac{w \, h_1 \, V_1 \, dx}{w \, h_2 + \frac{x \, w}{L} (h_1 - h_2)} = \\ & \quad \frac{h_1 \, V_1}{L} \int_0^L \frac{dx}{h_2 + x} \end{aligned}$$

since $h_1 - h_2 = L$, the average volume of air equals

$$\begin{aligned} & \frac{h_1 \, V_1}{L} \left[\log_e (h_2 + x) \right]_0^L \\ &= \frac{2.303 \, h_1 \, V_1}{h_1 - h_2} \log_{10} \frac{h_1}{h_2} \text{ cubic feet per second.} \end{aligned}$$

The average velocity in the section ab , then equals the above average volume of the air plus the flow of water in cubic feet per second divided by the area of the pipe. The average ratio of volume of air to volume of water equals the volume of air as obtained from equation (19) divided by flow of water in cubic feet per second.

Diagrams For Solving the Air Orifice Equation.—The following diagrams have been prepared to facilitate the solution of the air orifice equation.

$$W = .1639 \, C \, d^2 \, (ip/T)^{1/2}.$$

The value of the coefficients of the orifices have been taken from the curves of figure 7 in the preparation of these diagrams. This graphical solution was not used in working up

the data for this bulletin. However, the results obtained from the diagrams should be sufficiently accurate for preliminary field computations. The method of using the diagram is illustrated in each figure by the heavy broken lines. For example, in figure 58 the observed quantities are as follows:

$i = 7.3$ inches

$d = \frac{1}{2}$ inch

$p = 90$ pounds per square inch, gage.

$t = 200$. degrees (F)

W as read from the lower scale equals .0275 pounds per second, and from the upper scale the quantity of free air flowing per minute equals 21.5 cubic feet.

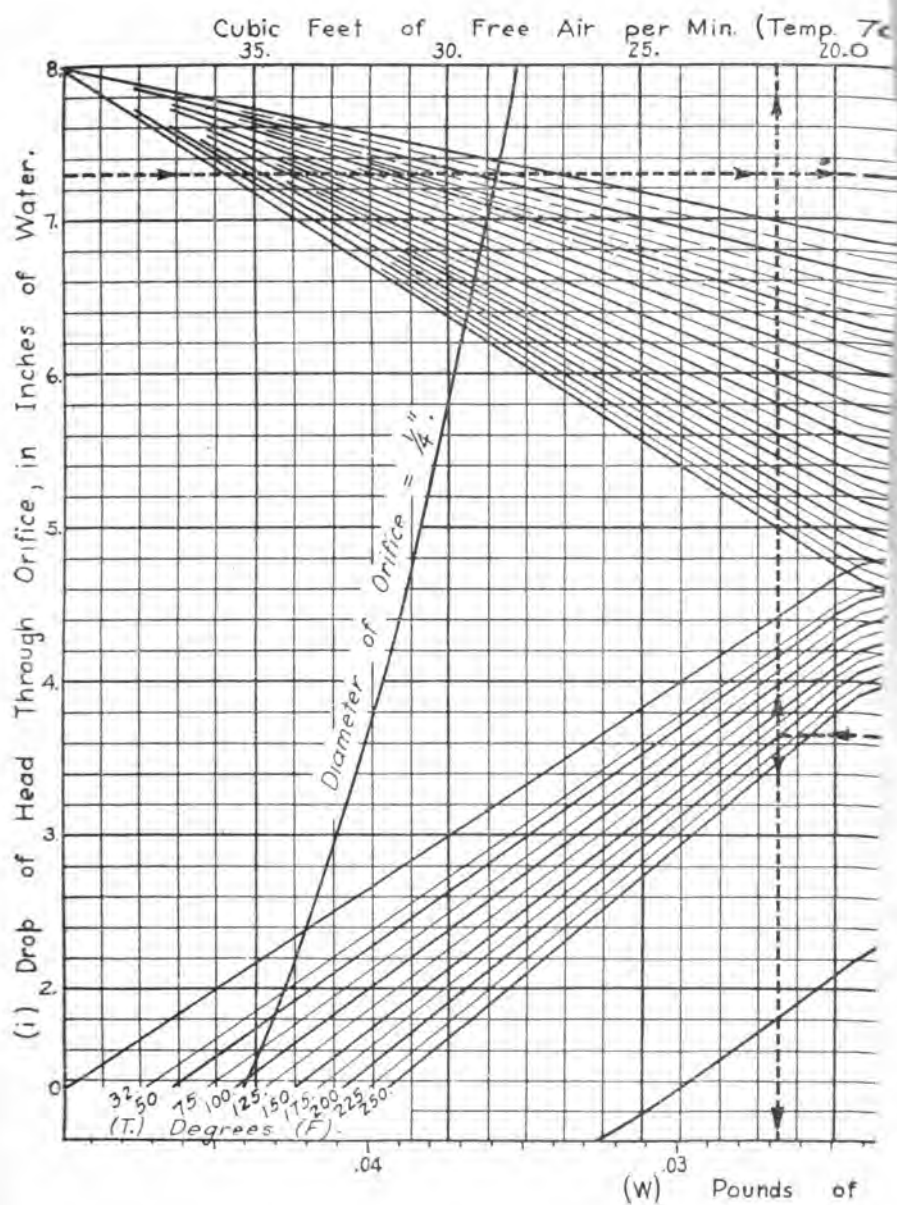
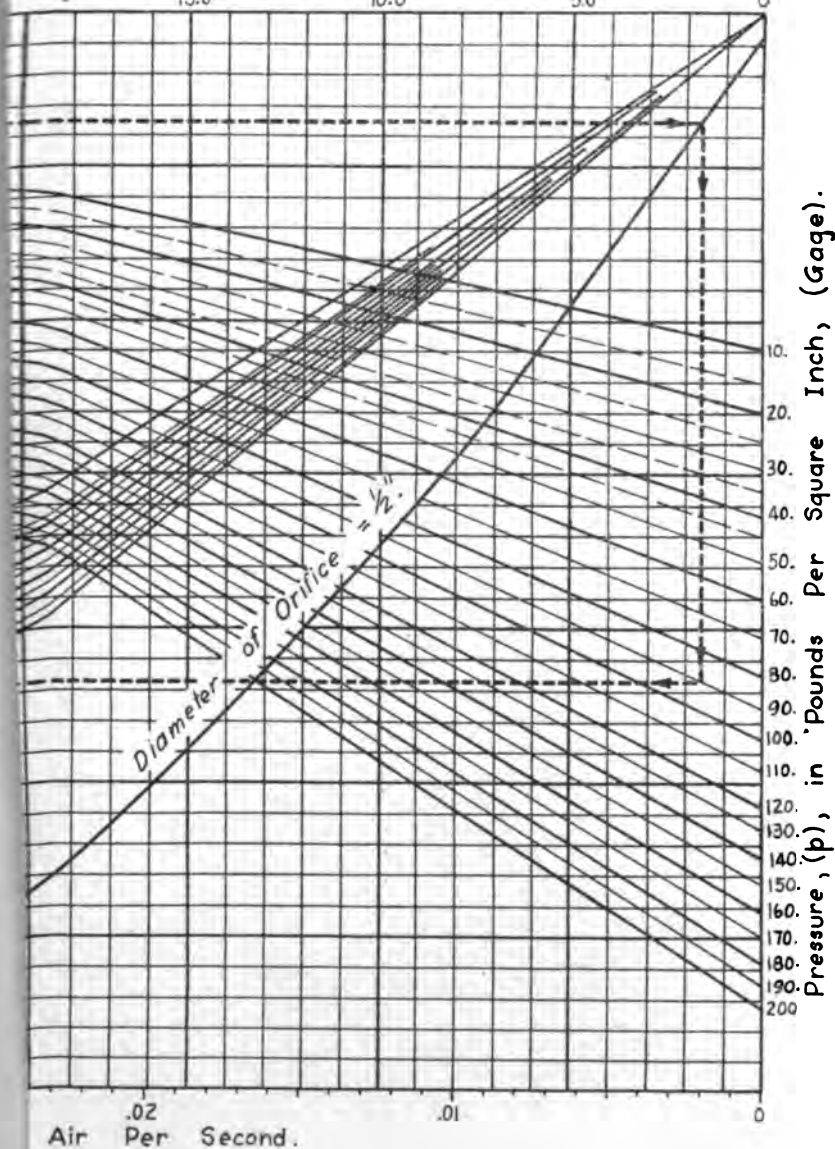


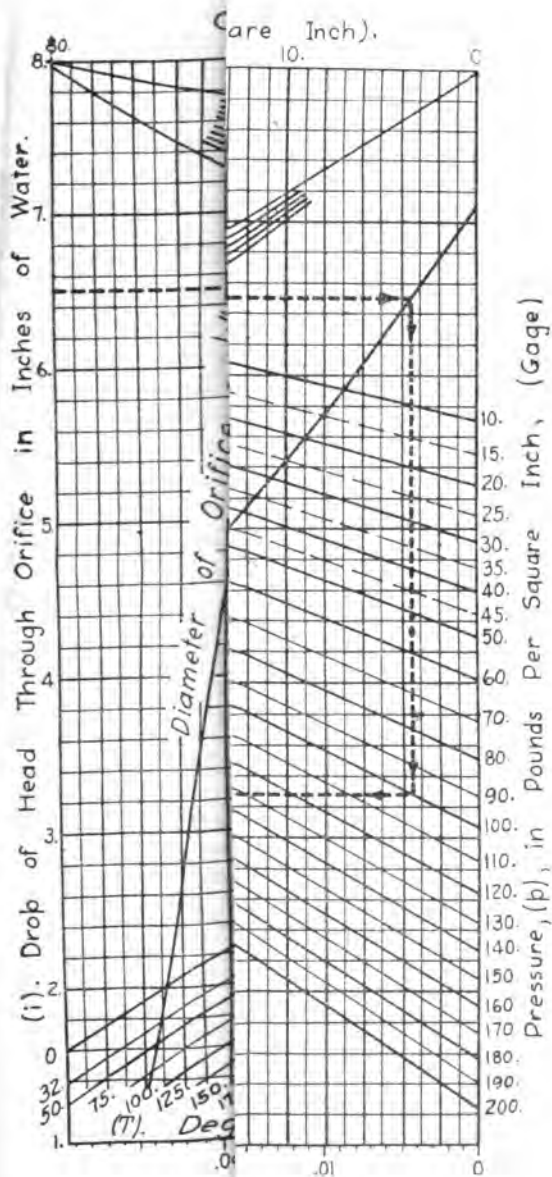
Fig. 58—Graphical solution of
 $W = .1639 C_d$

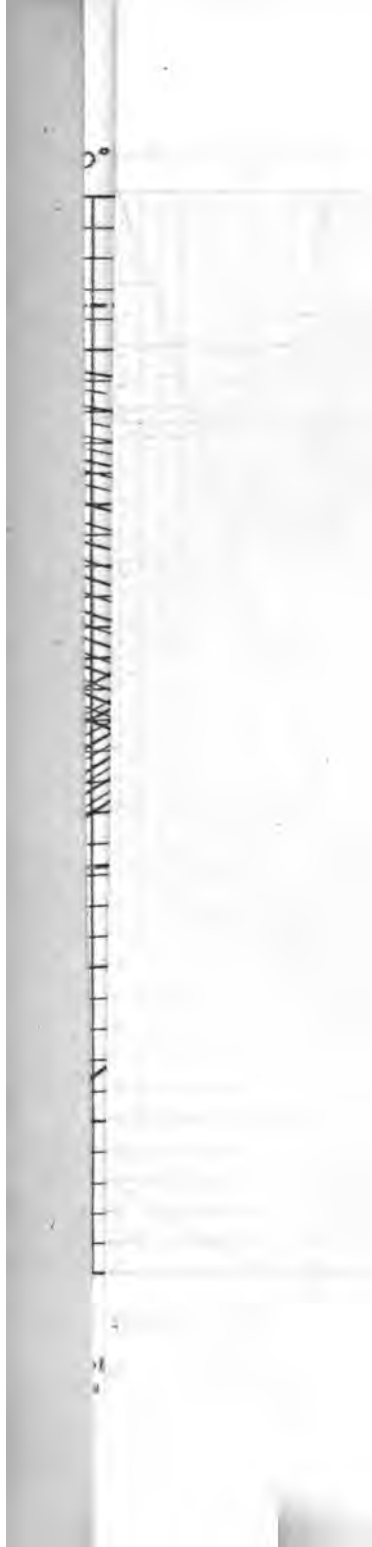
°F. Pressure 14.7 Pounds Per Square Inch.)
 15.0 10.0 5.0

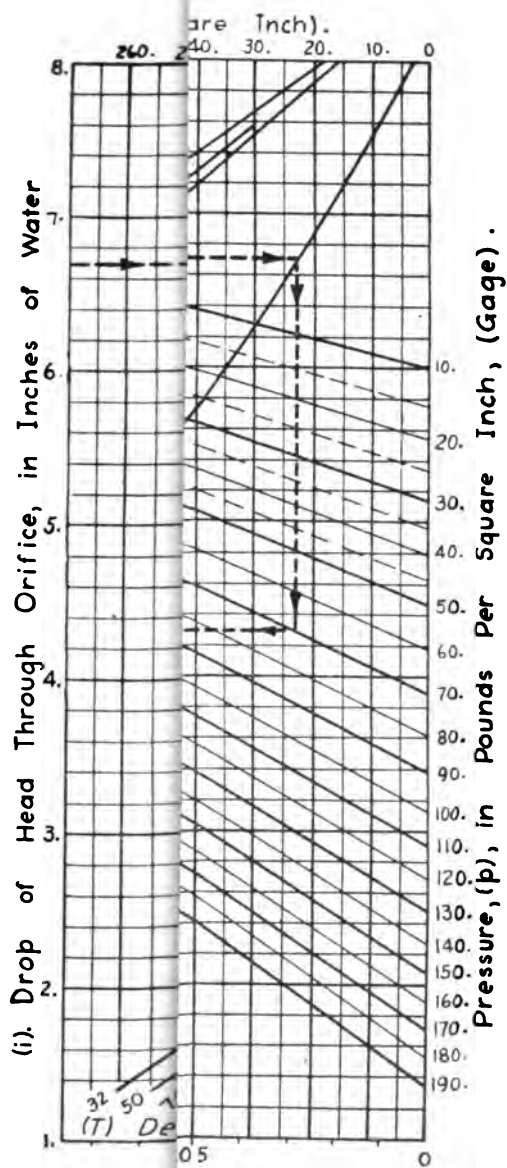


of the air orifice formula,
 $= (p/T)^{1/2}$

Pressure Inch, (Gage).







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APPENDIX B

TABLE I
Summary of Conditions Investigated in Experimental Study

Series number	Diameter of eduction pipe	Length of eduction pipe	Type of foot-piece used. See fig.	General arrange- ment of apparatus, as in fig. No. below	Number of runs made	REMARKS
1	In.	Ft.	A	2	38	Brass rings removed above (aa) see fig. 1.
2	2 1/2	61.17	A	2	126	Brass rings used as shown in fig. 1.
3	2 1/2	61.17	A	2	57	
4	2 1/2	60.88	C	3	45	
5	2 1/2	60.88	C	3	58	
6	1 1/2	60.88	C	3	90	
7	3	60.88	C	3	51	
8	3	81.63	C	3	45	
9	3	100.64	C	3	51	
10	3	37.85	C	3	95	
11	3	61.26	D	3	110	Length of 2 1/2" pipe, 42.36'; of 3" pipe, 19.08'.
12	2 1/2-3	62.04	C	3	150	Length of 2" pipe, 20.38'; 2 1/2" pipe, 21.62'; 3" pipe, 19.08'.
13	1	61.08	C	3	72	
		62.43	C	3		

TABLE II
 SERIES NO. 1
 Diameter of Education Pipe 2 1/2 inches. Foot-piece A, with brass rings removed. Length of Education Pipe 61.17 feet. General Arrangement of Apparatus as in Fig. 2.
 Average Temperature of Water 5° Cent.

1	2	3	4	5	6	7	8	9
Run number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet	Ratio of expansion of air	Energy of expan- sion of air used per pound of wa- ter pumped, foot- pounds	Submergence, per cent	Efficiency of pump, per cent	Volume free air per foot of lift, cubic feet
	G	W	h _d	r	H''	S	E	Q
1	11.5	.04775	46.6	1.47	307.0	28.9	15.2	.07120
2	11.1	.01069	36.9	1.68	96.5	39.6	38.2	.02150
3	16.4	.05385	46.2	1.63	268.0	24.6	17.2	.00665
4	25.8	.01132	25.0	2.03	61.8	59.2	40.4	.01455
5	26.9	.05105	42.7	1.64	180.6	30.2	23.6	.03560
6	20.6	.01486	34.9	1.79	81.6	43.0	42.7	.01655
7	27.7	.04070	39.0	1.69	148.2	36.2	26.3	.03020
8	22.4	.01486	31.6	1.89	81.2	48.4	38.9	.01680
9	29.3	.01345	30.8	1.96	59.4	49.6	51.8	.01195
10	37.4	.01315	24.7	2.06	49.1	59.7	50.3	.01190
11	37.6	.02766	33.2	1.84	86.3	44.7	38.5	.01765
12	38.2	.05475	36.6	1.80	162.1	40.2	21.9	.03140
13	49.6	.08160	32.7	1.96	212.2	46.4	16.4	.04020
14	48.2	.02187	26.6	2.01	59.2	66.8	44.8	.01335
15	47.7	.01410	21.1	2.16	43.9	65.5	48.0	.01125
16	13.2	.01706	40.2	1.57	112.0	34.0	36.0	.02560
17	20.8	.04400	40.5	1.67	135.5	33.8	22.1	.04190
18	47.0	.01178	20.9	2.19	37.7	63.8	65.4	.00963
19	38.8	.00541	10.1	2.66	26.2	33.4	40.4	.01096
20	47.0	.00849	16.1	2.34	29.6	73.6	64.3	.00905

TABLE II
SERIES NO. 1 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second	Delivery head, feet	Ratio of expansion of air.	Energy of expan- sion of air used per pound of wa- ter pumped, foot- pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.
	G	W	H _d	r	H"	S	E	Q
21	37.6	.00889	20.7	2.20	35.9	66.2	57.6	.00919
22	37.1	.00882	16.0	2.39	36.2	73.6	56.0	.00854
23	27.8	.00571	16.0	2.36	36.2	73.6	47.2	.01020
24	28.4	.00730	26.7	2.38	36.5	68.7	49.7	.00985
25	20.1	.00718	25.8	2.05	49.2	58.6	51.4	.01131
26	19.8	.00544	21.5	2.16	40.7	64.9	52.8	.01022
27	20.6	.00918	29.7	1.94	56.8	55.3	52.3	.01205
28	12.0	.00694	31.1	1.91	77.9	49.2	43.3	.01490
29	11.4	.00312	16.4	2.34	44.7	73.2	36.8	.01831
30	11.1	.00500	26.3	2.02	60.5	57.0	43.4	.01867
31	10.9	.00368	21.2	2.19	50.8	65.4	41.7	.01275
32	19.4	.00534	20.8	2.18	41.4	66.0	50.2	.01065
33	19.1	.00282	10.7	2.55	26.6	82.5	40.1	.01110
34	19.8	.00375	15.9	2.36	31.6	74.1	50.8	.00954
35	27.0	.00358	11.5	2.53	23.7	81.4	48.4	.00930
36	45.6	.00351	6.7	2.65	14.4	89.0	46.3	.00924
37	46.1	.00523	10.8	2.55	20.4	82.2	53.1	.00838
38	12.5	.01028	36.1	1.76	89.7	41.0	40.2	.018205

TABLE III
SERIES NO. 2

Diameter of Education Pipe 2½ inches. Foot-piece A. Length of Education Pipe 61.17 feet. General Arrangement of Apparatus as in Fig. 2. Average Temperature of Water 5.5° C.

1	2	3	4	5	6	7	8	9
Run number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head feet	Ratio of expansion of air	Energy of expansion of air used per pound of water pumped, foot pounds	Submergence, per cent	Efficiency of pump per cent	Volume free air per gallon water per foot of lift, cubic feet
	G	W	Pq	r	H"	S	E	Q
1	12.1	.00955	35.8	1.73	83.0	41.5	43.1	.01765
2	11.9	.00678	31.2	1.89	69.6	49.0	44.7	.01460
3	11.9	.00487	26.3	2.04	56.1	57.1	46.8	.01240
4	11.9	.00364	21.0	2.24	47.3	65.5	44.5	.01160
5	20.5	.01430	36.0	1.77	76.6	41.2	47.0	.01555
6	20.5	.00993	31.2	1.87	58.5	49.0	53.2	.01250
7	20.3	.00670	25.7	2.04	46.4	59.6	56.6	.01032
8	20.1	.00515	21.1	2.19	38.6	65.6	54.6	.00976
9	19.6	.00355	15.7	2.37	30.0	74.3	52.3	.00923
10	29.6	.01456	31.2	1.89	60.1	49.0	51.9	.01262
11	30.4	.01034	25.9	2.04	46.5	57.7	55.7	.01048
12	29.4	.00727	20.9	2.21	37.7	65.9	55.4	.00946
13	28.3	.00513	16.1	2.35	29.8	73.7	55.8	.00968
14	38.5	.01365	26.0	2.03	48.5	57.5	53.5	.01092
15	38.2	.00926	20.7	2.21	40.9	66.2	50.6	.00940
16	37.7	.00710	16.4	2.37	31.2	73.2	52.5	.00920
17	36.4	.00412	11.2	2.54	21.5	81.7	51.9	.00903
18	48.1	.01250	20.9	2.19	35.0	65.9	53.5	.00995
19	48.5	.00776	15.1	2.41	28.2	75.4	53.4	.00935
20	40.6	.00524	10.5	2.57	20.4	83.0	51.3	.00863

TABLE III
SERIES NO. 2 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second	Delivery head, feet	Ratio of expansion of air.	Energy of expan- sion of air used per pound of wa- ter pumped, foot- pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet
G	W	pd	r	H"	S	E	Q	
21	45.2	.00314	5.8	2.72	13.3	90.5	43.4	.00964
22	28.4	.00343	11.2	2.56	21.7	81.7	51.5	.00865
23	43.2	.01995	26.1	2.17	61.2	57.3	42.6	.01265
24	48.7	.04335	31.6	1.92	111.5	48.4	28.3	.02260
25	37.4	.05910	35.8	1.80	179.2	41.5	20.0	.03540
26	38.5	.02286	31.1	1.94	75.5	49.2	41.2	.01530
27	30.8	.03130	36.0	1.80	114.7	41.2	31.4	.02260
28	30.8	.05905	38.7	1.73	201.2	36.8	19.2	.03970
29	20.6	.04665	40.9	1.63	212.0	33.2	19.3	.04430
30	11.7	.01771	40.8	1.58	133.8	33.3	30.5	.02970
31	65.8	.06265	26.5	1.99	125.6	56.7	21.1	.02880
32	31.1	.02033	33.2	1.85	77.9	45.5	42.7	.01575
33	28.9	.02081	33.2	1.81	82.4	45.5	40.4	.01730
34	23.6	.01834	32.6	1.85	67.1	46.5	48.7	.01380
35	16.8	.01034	32.7	1.81	70.4	46.2	46.5	.01490
36	50.2	.02248	30.2	1.88	54.8	50.0	55.3	.01180
37	39.0	.01939	29.4	1.92	62.8	51.6	47.0	.01350
38	32.9	.01605	29.7	1.87	59.2	51.1	50.8	.01310
39	23.7	.01080	29.6	1.90	56.5	51.4	52.6	.01220
40	13.8	.00695	29.7	1.92	63.6	52.1	45.9	.01380

TABLE III
SERIES NO. 2 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	G Discharge of water in gallons/min.	W Quantity of Air supplied, pounds per second	h _a Delivery head, feet	r Ratio of expansion of air.	H' Energy of expan- sion of air used per pound of wa- ter pumped, foot pounds.	S Submergence, per cent.	E Efficiency of pump, per cent.	Q Volume free air per gallon water per foot of lift, cubic feet.
41	15.8	.00550	23.4	2.08	49.0	61.4	47.8	.01180
42	32.1	.00983	23.3	2.08	43.3	61.5	54.0	.01045
43	36.7	.01093	23.7	2.08	42.1	61.0	56.6	.00897
44	38.9	.01186	23.6	2.08	43.2	61.2	54.9	.01025
45	44.2	.01409	23.5	2.09	45.6	61.3	51.5	.01080
46	59.9	.02383	23.6	2.10	57.1	61.2	41.5	.01340
47	19.1	.00462	18.4	2.26	38.2	69.6	48.4	.01045
48	31.1	.00681	18.4	2.26	34.6	69.6	53.5	.00848
49	39.0	.00787	18.5	2.27	32.1	70.4	56.4	.00890
50	42.9	.00931	18.5	2.28	34.5	69.4	53.9	.00832
51	60.4	.01364	17.8	2.30	36.4	70.6	49.2	.01010
52	76.5	.02488	17.6	2.26	51.2	70.8	34.6	.01460
53	18.5	.00310	16.5	2.34	37.3	72.6	43.9	.01110
54	11.5	.00409	23.2	2.07	50.0	61.8	46.6	.01220
55	30.4	.00920	23.5	2.08	42.3	61.2	55.1	.01020
56	16.5	.00896	31.4	1.91	66.0	48.4	47.7	.01390
57	28.7	.01280	29.1	1.97	58.5	52.1	49.8	.01220
58	35.1	.01689	29.6	1.91	60.2	51.8	49.3	.01295
59	39.2	.01945	29.2	1.92	62.7	51.9	46.7	.01360
60	44.5	.02420	29.1	1.97	71.2	52.1	41.0	.01482

TABLE III
SERIES NO 2 (Continued)

1	2	3	4	5	6	7	8	9
Run number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head feet	Ratio of expansion of air	Energy of expan- sion of air used per pound of wa- ter pumped, foot pounds	Submergence, per cent	Efficiency of pump, per cent	Volume free air per foot of lift, cubic feet
	G	W	H _d	r	H"	S	E	Q
61	52.0	.02416	26.6	1.97	61.2	56.3	43.7	.01395
62	41.8	.01769	28.7	2.00	56.7	56.1	47.3	.01260
63	37.6	.01533	28.7	1.97	53.5	56.1	50.1	.01215
64	36.3	.01371	28.2	2.01	50.8	56.8	51.8	.01190
65	29.4	.01152	28.5	1.98	51.7	55.9	52.0	.01165
66	20.7	.00791	26.5	2.08	54.2	56.3	49.0	.01145
67	21.1	.00831	12.8	2.45	27.2	78.6	47.4	.00981
68	31.5	.00493	13.9	2.42	26.9	77.0	52.0	.02500
69	37.2	.00593	13.7	2.42	27.2	77.2	50.7	.02550
70	38.8	.00680	14.4	2.40	29.6	76.1	49.0	.01890
71	44.2	.00756	14.2	2.40	29.1	76.4	49.2	.013600
72	55.0	.00962	14.7	2.42	29.8	76.5	47.7	.013930
73	65.8	.01239	14.1	2.43	32.3	76.5	44.0	.015010
74	77.2	.01640	14.0	2.43	36.6	76.8	38.5	.017050
75	86.7	.02359	14.0	2.42	46.6	76.8	30.3	.021600
76	17.1	.01108	33.2	1.83	76.7	45.4	44.0	.01800
77	28.0	.01764	33.1	1.83	74.0	45.5	44.8	.05000
78	32.2	.01426	32.8	1.81	70.6	46.1	46.5	.09100
79	35.4	.02470	32.4	1.84	82.2	46.8	39.5	.05800
80	25.9	.02461	36.2	1.74	101.7	40.5	35.7	.076000

TABLE III
SERIES NO. 2 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second	Delivery head, feet	Ratio of expansion of air.	Energy of expan- sion of air used per pound of wa- ter pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic gallon water per foot of lift, cubic
G	W	h _d	r	H''	S	E	Q	
81	18.7	.01472	35.2	1.70	80.8	42.1	43.7	.62900
82	13.4	.01144	35.7	1.81	97.7	41.4	36.7	.68100
83	13.4	.00390	19.3	2.24	45.4	68.1	42.8	.23300
84	24.4	.00550	20.2	2.20	40.6	66.6	50.0	.21309
85	30.9	.00794	20.2	2.19	38.9	66.6	52.2	.20500
86	35.2	.00875	19.7	2.20	36.9	67.5	53.7	.19250
87	39.7	.00954	20.2	2.19	36.4	66.6	55.8	.19180
88	44.3	.01149	20.2	2.20	39.6	66.6	51.3	.20700
89	52.8	.01460	20.0	2.30	44.5	67.0	45.2	.22100
90	67.5	.02447	20.2	2.29	57.8	66.6	35.2	.28900
91	19.3	.01107	32.9	1.85	67.7	45.9	48.7	.45700
92	28.1	.01604	32.8	1.85	68.0	46.0	48.4	.45500
93	39.5	.03056	32.6	1.84	91.0	46.4	36.0	.61700
94	42.7	.03835	32.6	1.84	110.6	46.4	29.6	.71700
95	19.4	.02092	35.4	1.70	111.1	42.1	31.9	.86100
96	34.0	.04479	35.9	1.77	144.8	41.0	24.9	1.05000
97	50.9	.04269	30.0	1.92	106.8	50.6	28.5	.67000
98	59.9	.04180	26.1	2.00	93.3	56.0	28.7	.56600
99	54.6	.03005	26.6	2.00	74.0	56.1	36.1	.44000
100	53.7	.01501	20.2	2.20	42.4	65.0	47.9	.22350

TABLE III
SERIES NO. 2 (Continued)

1	2	3	4	5	6	7	8	9
Run number	Discharge of water mln.	Quantity of Air supplied, pounds per second.	Delivery head feet	Ratio of expansion	Energy of expansion per pound wa- ter pumped, foot pounds	Submergence, per cent	Efficiency of pump per cent	Volume free air per gallon water per foot of lift, cubic feet
G	W	H _d	r	H _e	S	E	Q	
101	64.0	.02004	20.8	2.31	50.2	65.6	41.7	.24900
102	78.3	.04160	20.5	2.22	81.9	66.1	26.2	.42400
103	19.0	.00306	14.0	2.43	27.7	76.7	51.0	.00918
104	11.0	.00215	14.2	2.41	33.2	76.5	43.1	.01095
105	11.0	.00283	17.9	2.34	42.4	70.7	42.2	.01165
106	11.1	.00316	20.1	2.24	44.6	66.8	45.3	.01135
107	10.9	.00411	23.8	2.12	54.8	60.7	43.7	.01265
108	11.1	.00515	26.7	2.08	65.5	56.0	40.9	.01380
109	11.0	.00594	29.5	2.00	72.1	51.5	41.1	.01460
110	11.2	.00812	33.2	1.93	89.8	45.4	37.1	.01740
111	31.2	.00721	19.5	2.20	35.3	67.8	55.5	.00846
112	36.8	.00902	20.5	2.20	37.4	66.1	55.1	.00857
113	40.4	.01007	20.7	2.16	37.0	65.7	56.2	.00861
114	49.5	.01301	20.2	2.16	39.2	66.6	51.8	.01040
115	55.0	.01657	19.6	2.16	41.8	67.6	47.2	.01130
116	65.1	.01990	19.7	2.29	48.8	67.5	40.6	.01240
117	30.0	.00534	24.4	2.24	39.6	59.6	49.7	.01030
118	32.8	.00691	23.7	2.22	43.2	60.6	53.0	.00884
119	38.3	.01225	24.2	2.07	48.5	60.0	54.6	.01042
120	38.9	.01400	24.0	2.07	50.8	60.4	47.5	.01200

TABLE III
SERIES NO. 2 (Continued)

1	2	3	4	5	6	7	8	9
Run number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet	Ratio of expansion of air	Energy of expan- sion of air used per pound of wa- ter pumped, foot pounds	Submergence, per cent	Efficiency of pump per cent	Volume free air per foot of lift, cubic feet
	G	W	H _d	r ₁	H''	S	E	Q
121	48.6	.01641	23.6	2.10	48.6	61.1	48.3	.01145
122	28.6	.00550	17.2	2.28	30.7	71.5	56.4	.00895
123	35.2	.00704	16.8	2.26	31.6	72.3	53.5	.00954
124	27.2	.00682	16.8	2.30	40.6	72.3	41.6	.01185
125	40.3	.00870	18.1	2.30	34.8	70.0	52.3	.00955
126	44.8	.00805	17.7	2.28	32.3	70.6	55.2	.00913

TABLE IV
SERIES 2
Special Velocity-Friction Study

See figure 2 for location of sections 1, 2, and 3.

Run Number	Pressure head at Sections				Ratio of average volume of air to volume of water between sections			Average velocity of mixture between sections			Loss of head per foot of pipe between sections		
	2			Gage	1 & 2	2 & 3	3 & 4	1 & 2	2 & 3	3 & 4	1 & 2	2 & 3	3 & 4
	1	2	3										
	Feet	of water							Feet per second			Feet of Water	
32	26.7	16.4	8.5	0.0	2.34	2.81	3.45	6.98	7.96	9.30	.70	.51	1.06
33	24.4	15.8	7.9		2.63	3.12	3.80	7.07	8.02	9.37	.56	.68	1.07
34	27.7	17.2	8.7		1.99	2.39	2.91	4.75	5.40	6.31	.55	.44	.88
35	27.0	16.6	8.6		2.18	2.62	3.23	3.61	4.10	4.79	.65	.60	.99
36	28.0	16.9	8.2		1.58	1.92	2.37	8.71	9.86	11.37	.40	.27	.46
37	29.9	18.5	9.0		1.71	2.08	2.61	7.11	8.08	9.45	.39	.46	.75
38	28.7	17.6	8.4		1.69	2.08	2.58	5.97	6.81	7.91	.49	.41	.61
39	29.4	18.0	8.7		1.57	1.92	2.40	4.09	4.65	5.41	.46	.36	.59
40	30.1	18.4	9.0		1.72	2.11	2.64	2.52	2.88	3.37	.53	.48	.79
41	35.0	21.6	10.5		1.11	1.38	1.79	2.24	2.54	2.97	.41	.33	.59
42	35.3	21.9	10.5		.97	1.22	1.58	4.25	4.79	5.57	.31	.28	.50
43	35.2	21.9	10.4		.94	1.19	1.54	4.80	5.40	6.27	.28	.26	.40
44	35.6	22.4	10.9		.96	1.20	1.57	5.14	5.78	6.72	.28	.27	.49
45	35.6	21.6	10.2		1.01	1.28	1.65	5.98	6.77	7.88	.40	.29	.44
46	34.1	21.2	9.9		1.28	1.61	2.07	9.19	10.50	12.34	.46	.45	.69
47	40.4	25.5	12.2		.92	.92	1.23	2.20	2.46	2.86	.28	.28	.45
48	40.1	25.4	12.2		.65	.83	1.11	3.45	3.82	4.41	.20	.22	.37
49	41.0	27.2	12.5		.59	.75	1.02	4.16	4.60	5.30	.09	.29	.34
50	40.0	25.9	12.1		.65	.82	1.10	4.75	5.26	6.06	.19	.22	.34
51	40.3	25.4	11.9		.67	.86	1.15	6.79	7.56	8.73	.24	.25	.34
52	40.1	25.4	12.2		.96	1.23	1.65	10.11	11.49	13.62	.42	.45	.69
53	42.7	27.1	13.2		.62	.85	1.16	1.51	1.68	1.95	.29	.29	.53
54	35.8	22.4	10.9		1.12	1.40	1.82	1.64	1.86	2.19	.41	.40	.67
55	35.8	22.6	11.2		.95	1.19	1.55	3.99	4.48	5.22	.28	.25	.53

TABLE IV
SERIES 2
Special Velocity-Friction Study

See figure 2 for location of sections 1, 2, and 3.

Run Number	Pressure head at Sections				Ratio of average volume of air to volume of water between sections				Average velocity of mix- between sections				Loss of head per foot of pipe be- tween sections			
	1 Feet	2 of Water		4	1 & 2	2 & 3	3 & 4	Feet	1 & 2	2 & 3 per S	3 & 4 second	Feet	1 & 2	2 & 3 of	3 & 4	Water
56	28.0	17.2	8.5		2.92	2.33	2.83	3.21	3.66	4.27	4.27	.57	.47	.47	.80	
57	30.0	18.3	9.2		1.52	1.87	2.33	4.87	5.52	6.43	6.43	.46	.27	.27	.72	
58	30.0	19.6	9.2		1.64	2.01	2.52	6.23	7.09	8.30	8.30	.49	.41	.41	.75	
59	29.9	18.4	8.9		1.69	2.07	2.60	7.10	8.11	9.49	9.49	.52	.47	.47	.72	
60	30.3	18.8	9.4		1.84	2.26	2.84	8.52	9.76	11.50	11.50	.61	.52	.52	.94	
61	32.6	20.5	10.2		1.52	1.88	2.41	8.82	10.06	11.90	11.90	.50	.47	.47	.85	
62	32.3	20.1	9.9		1.39	1.73	2.20	6.74	7.67	8.99	8.99	.45	.38	.38	.70	
63	32.7	20.5	10.2		1.33	1.65	2.11	5.91	6.70	7.87	7.87	.41	.37	.37	.71	
64	33.4	20.8	10.2		1.23	1.52	1.95	5.44	6.16	7.21	7.21	.38	.34	.34	.65	
65	32.3	20.0	9.8		1.29	1.60	2.04	4.54	5.15	6.01	6.01	.40	.33	.33	.60	
66	32.9	20.5	10.0		1.25	1.55	1.98	3.13	3.55	4.15	4.15	.39	.34	.34	.61	
67	46.0	29.5	14.2		.44	.57	.78	2.03	2.22	2.53	2.53	.18	.20	.20	.34	
68	45.2	29.0	13.9		.44	.56	.78	3.04	3.32	3.78	3.78	.16	.19	.19	.31	
69	44.3	28.6	13.7		.45	.58	.80	3.62	3.95	4.50	4.50	.16	.18	.18	.30	
70	44.4	28.5	11.4		.49	.66	.89	3.90	4.33	4.95	4.95	.18	.41	.41	.13	
71	43.9	27.9	11.7		.49	.64	.87	4.41	4.87	5.56	5.56	.18	.33	.33	.14	
72	43.3	27.3	11.4		.50	.66	.89	5.56	6.15	7.01	7.01	.19	.32	.32	.12	
73	44.2	28.3	11.7		.53	.70	.96	6.79	7.55	8.68	8.68	.19	.41	.41	.19	
74	44.6	28.4	11.7		.60	.80	1.09	8.23	9.30	10.80	10.80	.25	.43	.43	.26	
75	44.1	28.2	11.4		.77	1.02	1.33	10.33	11.80	14.00	14.00	.37	.66	.66	.40	
76	26.4	16.1	7.9		2.34	2.82	3.45	3.33	4.38	5.10	5.10	.70	.57	.57	.92	
77	26.9	16.6	8.4		2.24	2.69	3.31	3.10	4.05	4.81	4.81	.61	.52	.52	.93	
78	26.4	16.0	7.9		2.23	2.67	3.27	3.01	3.73	4.43	4.43	.66	.43	.43	.83	
79	27.1	16.8	8.4		2.48	3.00	3.69	3.29	4.22	5.02	5.02	.78	.68	.68	1.21	
80	24.2	16.0	7.9		3.53	4.17	5.03	7.88	9.01	10.52	10.52	1.05	1.35	1.35	2.44	

TABLE IV
SERIES 2
Special Velocity-Friction Study

See figure 2 for location of sections 1, 2, and 3.

Run Number	Pressure head at Sections				Ratio of average volume of air to volume of water between sections		Average velocity of mix- ture between sections				Loss of head per foot of pipe be- tween sections			
	Feet				1 & 2	2 & 3	1 & 2	2 & 3	per Second	Feet	1 & 2	2 & 3	3 & 4	of Water
	1	2	3	4										
81	24.1	14.5	7.2	0	2.93	3.59	4.21	4.94	5.65	6.54	.86	.65	.92	
82	24.2	14.7	7.4		3.16	3.76	4.33	3.76	4.30	5.00	1.02	.71	1.25	
83	38.6	25.0	12.2		.87	1.11	1.33	1.69	1.90	2.23	.33	.33	.63	
84	39.1	24.8	12.2		.70	1.02	1.35	2.23	3.31	3.87	.23	.27	.54	
85	38.8	24.5	11.9		.77	.98	1.30	3.70	4.13	4.80	.35	.25	.47	
86	39.7	25.2	12.2		.72	.92	1.22	4.20	4.67	5.42	.24	.25	.44	
87	39.6	25.2	12.5		.72	.91	1.22	4.60	5.10	5.92	.24	.22	.47	
88	39.0	24.8	12.2		.84	.99	1.32	5.31	5.83	6.91	.35	.25	.50	
89	38.1	24.8	12.2		.84	1.03	1.41	6.50	7.23	8.51	.30	.23	.56	
90	38.6	24.6	12.2		1.09	1.33	1.83	9.50	10.80	12.85	.44	.46	.82	
91	26.3	16.9	8.5		2.03	2.44	3.00	3.92	4.48	5.23	.45	.47	.87	
92	26.7	16.5	8.2		2.04	2.45	3.01	6.75	6.53	7.59	.38	.42	.78	
93	26.8	18.0	9.4		2.71	3.23	4.03	9.87	11.25	13.08	.62	.63	1.41	
94	26.6	18.0	9.7		3.16	3.73	4.66	11.93	13.59	16.25	.76	.94	1.99	
95	24.2	14.6	7.2		3.99	4.77	5.75	6.51	7.51	8.81	1.38	1.16	1.72	
96	22.9	15.8	8.5		4.88	5.69	6.94	13.45	15.29	18.16	1.06	1.44	2.60	
97	29.5	19.1	9.9		2.89	3.45	4.36	13.24	15.24	18.34	.95	1.02	1.85	
98	31.6	20.3	10.2		2.31	2.83	3.61	12.74	14.82	17.97	.82	.88	1.49	
99	32.1	20.3	10.2		1.81	2.22	2.85	10.34	11.87	14.16	.63	.61	1.10	
100	37.4	24.2	12.2		.83	1.07	1.42	6.71	7.32	8.78	.15	.25	.63	
101	38.3	24.4	12.2		.95	1.19	1.58	8.50	9.57	11.17	.33	.33	.64	
102	38.2	24.6	12.5		1.61	2.02	2.68	13.73	15.92	19.44	.73	.80	1.40	
103	46.2	29.2	14.2		.45	.55	.81	1.83	2.02	2.30	.22	.19	.36	

TABLE IV
SERIES 2

Special Velocity-Friction Study

See figure 2 for location of sections 1, 2, and 3.

Run Number	Pressure head of Sections				Ratio of average volume of air to volume of water between sections			Average velocity of mi- ture between sections			Loss of head per foot of pipe be- tween sections		
	Feet				1 & 2	2 & 3	3 & 4	1 & 2 Feet	2 & 3 per Second	3 & 4 Second	1 & 2 Feet	2 & 3 of Water	3 & 4
	1	2	3	4									
104	45.1	29.0	14.2	0.0	.54	.70	.97	1.14	1.26	1.46	.19	.27	.50
105	40.5	25.5	12.2		.76	.98	1.31	1.30	1.46	1.70	.32	.33	.52
106	39.2	24.7	12.0		.85	1.09	1.45	1.39	1.56	1.83	.34	.33	.59
108	32.8	20.4	10.2		1.52	1.87	2.40	1.88	2.15	2.54	.56	.48	.90
109	30.6	18.8	9.4		1.82	2.23	2.80	2.10	2.40	2.83	.64	.52	.96
110	26.7	16.3	8.2		2.50	3.13	3.83	2.71	3.11	3.64	.85	.66	1.18
111	39.6	25.1	12.9		.69	.87	1.17	3.54	3.92	4.53	.22	.15	.49
112	38.8	24.3	12.2		.74	.95	1.25	4.31	4.79	5.56	.26	.18	.47
113	38.8	24.6	12.2		.75	.95	1.27	4.77	5.31	6.16	.24	.20	.47
114	38.7	24.4	11.9		.80	1.01	1.34	5.98	6.69	7.79	.27	.26	.48
115	39.1	24.2	11.9		.85	1.08	1.43	7.34	8.25	9.83	.37	.28	.53
116	39.7	24.5	12.5		.82	1.06	1.85	8.41	9.47	11.15	.44	.35	.68
117	35.6	22.2	11.5		.89	1.24	1.82	9.77	10.84	12.71	.33	.24	.57
118	36.5	22.5	11.5		.82	1.14	1.49	4.11	4.84	5.75	.33	.24	.52
119	35.9	22.4	11.5		.99	1.24	1.61	5.20	5.84	6.82	.33	.22	.60
120	35.6	21.8	10.8		1.14	1.43	1.85	5.60	6.36	7.47	.46	.34	.74
121	36.8	22.5	11.5		1.06	1.32	1.79	6.78	7.61	8.83	.45	.37	.67
122	42.5	27.2	13.3		.56	.71	1.07	9.98	10.91	12.78	.45	.37	.67
123	41.6	26.2	12.7		.59	.75	1.01	8.75	9.78	11.19	.45	.37	.67
124	42.7	27.3	13.2		.72	.98	1.27	8.14	9.51	11.12	.42	.36	.61
125	41.1	27.2	12.8		.63	.81	1.09	4.40	4.88	5.66	.13	.11	.42
126	41.8	26.7	13.2		.59	.75	1.02	4.78	5.27	6.07	.19	.18	.41

TABLE V
 SERIES NO. 3
 Diameter of Education Pipe 2 1/2 inches. Foot-piece B. Length of Education Pipe 50.85 feet. General Arrangement of Apparatus as in Fig. 3.
 Average Temperature of Water 41.1° C.

1	2	3	4	5	6	7	8	9	10	11	12	13	
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in eduction pipe, cubic feet per second.	Average velocity of mixture in educ- tion pipe, feet per second.	Average volume of air	Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H"	S	E	Q	V _{av}	V _{av}	G	H	
1	31.4	.00775	19.9	2.35	336	67.7	49.5	.00998	.0693	4.1	.942	.331	
2	33.6	.00901	21.6	2.32	360	65.0	50.3	.00997	.0804	4.6	1.082	.349	
3	35.9	.00980	21.9	2.29	364	64.4	50.3	.01010	.0895	5.0	1.062	.350	
4	38.6	.00988	20.6	2.35	349	66.6	49.3	.00995	.0896	5.1	.977	.342	
5	49.6	.01331	20.6	2.35	381	66.6	45.4	.01080	.1250	6.9	1.062	.399	
6	17.9	.00585	21.3	2.29	431	63.8	41.2	.01230	.0523	2.7	1.259	.331	
7	24.2	.00958	26.9	2.20	489	56.4	45.3	.01180	.0833	4.2	1.592	.516	
8	29.8	.01217	28.6	2.10	494	53.6	48.4	.01140	.1188	5.3	1.678	.495	
9	33.0	.01408	27.6	2.10	518	55.2	44.4	.01240	.1378	6.0	1.703	.557	
10	36.0	.01545	27.6	2.14	522	55.2	44.2	.01250	.1516	6.6	1.738	.563	
11	39.8	.01670	27.3	2.16	516	55.7	44.0	.01230	.1544	7.2	1.708	.553	
12	46.4	.02250	27.6	2.17	597	55.2	38.6	.01410	.2273	9.3	2.095	.700	
13	14.3	.00703	27.9	2.11	584	64.8	39.8	.01410	.0678	3.1	2.010	.657	
14	23.8	.00431	17.6	2.43	287	71.5	51.3	.00922	.0424	2.8	.760	.274	
15	29.6	.00608	17.6	2.41	289	71.4	50.8	.00942	.0530	3.5	.777	.276	
16	31.1	.00720	17.6	2.41	325	71.4	45.1	.01050	.0628	3.9	.873	.348	
17	39.2	.00792	18.1	2.37	279	71.4	54.1	.00998	.0684	4.6	.759	.249	
18	46.6	.00972	17.6	2.41	289	71.4	49.1	.00973	.0846	5.5	.798	.278	
19	24.3	.00754	23.8	2.24	400	61.4	49.7	.01040	.0703	3.6	1.222	.392	
20	28.8	.00893	23.6	2.25	401	61.7	49.2	.01050	.0830	4.3	1.221	.396	

TABLE V
SERIES NO. 3 (Continued)

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per gallon water lift, cubic feet.	Average volume of air in reduction pipe, cubic feet per second.	Average velocity of mixture in reduc- tion pipe, feet per second.	Average volume of air 449V _{av} G	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'
21	31.8	.01020	23.8	2.24	413.	61.4	48.1	.0108	.0952	4.8	1.266	.416
22	35.3	.01083	23.6	2.25	398.	61.7	49.6	.0104	.1012	5.2	1.202	.389
23	39.8	.01271	23.6	2.25	412.	61.7	47.9	.0109	.1180	6.0	1.263	.415
24	54.4	.01303	23.3	2.25	310.	62.2	62.6	.00820	.1205	7.0	1.929	.218
25	49.1	.01753	23.8	2.25	463.	61.4	42.9	.0120	.1632	7.7	1.405	.506
26	40.4	.01277	23.4	2.24	407.	62.0	47.9	.0108	.1172	6.0	1.238	.405
27	49.9	.01520	23.4	2.25	395.	62.0	49.4	.0104	.1399	7.3	1.198	.381
28	54.2	.02070	23.6	2.25	495.	61.7	39.8	.0130	.1332	8.1	1.602	.565
29	38.4	.02125	29.9	2.07	645.	51.5	38.6	.0148	.2140	9.5	2.300	.760
30	35.5	.01975	30.3	2.06	642.	50.8	39.2	.0147	.2005	7.9	2.310	.751
31	30.2	.01985	30.4	2.04	638.	50.7	39.8	.0147	.1890	6.7	2.325	.745
32	24.2	.01170	29.8	2.05	556.	51.6	44.7	.0180	.1188	4.9	2.000	.699
33	16.8	.00856	30.8	2.06	589.	50.8	42.9	.0135	.1070	3.5	2.132	.659
34	16.9	.01007	32.8	1.98	652.	46.8	41.9	.0145	.1044	4.0	2.620	.740
35	27.7	.01966	33.1	1.93	744.	46.3	37.1	.0172	.1985	7.4	3.015	.907
36	24.7	.00409	14.6	2.50	240.	76.4	50.5	.0091	.0347	2.7	0.609	.232
37	18.5	.00276	14.0	2.53	220.	77.2	63.8	.0088	.0223	1.9	0.641	.203
38	8.1	.00201	17.1	2.43	351.	72.2	40.6	.0116	.0168	1.0	0.920	.410
39	17.8	.00292	15.1	2.51	241.	52.3	52.3	.00572	.0238	1.9	0.600	.226
40	22.5	.00364	14.6	2.51	238.	76.3	51.3	.00868	.0293	2.4	0.591	.227

TABLE V
SERIES NO. 8 (Continued)

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in suction pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	h_d	r	H'''	S	E	Q	V_{av}	V_{av}	$449V_{av}$	H'
											G	
41	26.5	.00427	14.6	2.51	236	76.3	51.7	.00883	.0349	2.8	0.592	.293
42	28.5	.00451	14.6	2.51	237	76.3	51.6	.00883	.0376	3.0	0.592	.293
43	32.5	.00574	14.6	2.51	244	76.3	50.1	.00913	.0469	3.7	0.612	.297
44	43.0	.00764	15.1	2.49	264	75.8	47.8	.00964	.0826	4.7	0.639	.265
45	55.8	.01047	14.6	2.51	276	76.3	44.5	.01026	.0853	6.3	0.687	.294
46	69.9	.01480	14.1	2.51	311	77.2	39.2	.01203	.1200	8.3	0.771	.366
47	81.7	.02100	13.6	2.51	376	77.9	30.1	.01510	.1700	10.6	0.985	.487
48	98.5	.02182	12.1	2.54	520	80.4	19.5	.01630	.1750	11.2	0.888	.793
49	18.6	.00849	29.1	2.10	541	52.8	44.8	.01260	.0781	3.6	1.882	.583
50	24.8	.01258	30.4	2.05	583	50.6	43.5	.01340	.1170	5.2	2.125	.640
51	29.5	.01415	30.3	2.10	567	50.8	44.5	.01270	.1310	5.9	1.998	.610
52	37.1	.01779	29.8	2.08	558	51.6	44.6	.01290	.1655	7.5	2.005	.577
53	43.1	.02970	30.3	2.08	803	50.8	31.5	.01830	.2770	11.2	2.855	1.051
54	69.8	.00730	32.6	2.01	698	47.1	39.0	.01540	.0689	2.9	2.650	.837
55	22.3	.01221	32.6	2.01	612	47.1	44.4	.01350	.1155	5.0	2.325	.664
56	26.4	.01548	32.9	2.02	661	46.3	41.6	.01430	.1455	6.2	2.475	.751
57	36.4	.02815	32.4	1.98	845	47.4	32.0	.01910	.2650	10.1	3.270	1.103

TABLE VI
 SERIES NO. 4
 Diameter of Education Pipe 2 1/2 inches. Foot-piece C. Length of Education Pipe 60.88 feet. General Arrangement of Apparatus as in Figure 3.
 Average Temperature of Water 5.5° C.

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.	Average volume of air in education pipe, cubic feet per second.	Average velocity of mixture in educa- tion pipe, feet per second.	Average volume of air 449V _{av}	Volume of water of pipe, feet of water.
H'	G	W	h _a	r	H''	S	E	Q	V _{av}	V _{av}	G	H'
1	39.3	.01070	20.8	2.28	359.	65.7	48.5	.01050	.0942	5.5	1.08	.36
2	26.7	.00707	20.6	2.29	349.	66.1	49.2	.01025	.0623	3.7	1.05	.35
3	35.5	.00979	20.9	2.26	358.	65.6	48.7	.01063	.0855	5.0	1.08	.36
4	50.8	.01340	20.8	2.31	364.	67.3	49.2	.01016	.1196	7.0	1.06	.35
5	45.8	.01290	20.9	2.28	371.	65.6	47.0	.01076	.1132	6.5	1.11	.38
6	60.1	.01760	19.3	2.33	396.	68.3	40.7	.01217	.1530	8.7	1.14	.45
7	63.8	.02320	20.4	2.29	480.	66.5	35.4	.01430	.2023	10.4	1.43	.58
8												
9												
10	33.3	.00735	17.9	2.35	302.	70.5	49.6	.00938	.0637	4.1	.35	.30
11	42.3	.00902	17.6	2.38	295.	71.0	49.7	.00971	.0774	5.2	.32	.29
12	54.4	.01201	16.8	2.39	309.	72.2	45.5	.01063	.1050	6.7	.34	.32
13	62.4	.01720	17.6	2.38	331.	71.0	38.5	.01255	.1473	8.6	1.06	.44
14	72.6	.02310	17.2	2.40	444.	71.8	32.2	.01438	.1980	10.8	1.22	.56
15	43.6	.02150	26.9	2.20	556.	56.8	40.5	.01460	.1906	8.7	2.00	.63
16	39.2	.01740	27.3	2.07	518.	55.1	43.8	.01300	.1618	7.5	1.85	.56
17	33.7	.01420	26.9	2.09	497.	55.8	45.8	.01260	.1318	6.2	1.76	.53

TABLE VI
SERIES No. 4 (Continued)

Run Number	1	2	3	4	5	6	7	8	9	10	11	12	13	
		Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	r	H'''	S	E	Q	V _{av}	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume air	Volume of water	Loss of head per ft. of pipe, feet of water.
		G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'	
18		23.4	.00998	27.3	2.08	485.	55.1	45.3	.01250	.0837	4.4	1.80	.50	
19		14.5	.00524	26.8	2.12	513.	56.0	48.6	.00310	.0590	2.7	1.83	.57	
20		16.2	.00482	23.6	2.18	371.	61.2	53.1	.01010	.0432	2.4	1.20	.34	
21		17.1	.00493	20.5	2.25	1041.	66.3	16.4	.01125	.0425	2.4	1.12	1.72	
22		18.3	.00462	18.3	2.27	331.	70.0	45.1	.01100	.0381	2.4	.94	.36	
23		19.2	.00773	27.0	2.06	464.	55.6	48.5	.01190	.0705	3.4	1.65	.47	
24		26.3	.01100	27.9	2.06	482.	54.1	48.1	.01200	.1040	4.8	1.85	.49	
25		30.3	.01313	27.7	2.06	498.	54.1	46.4	.01250	.1222	5.7	1.81	.52	
26		34.9	.01538	28.0	2.10	523.	54.0	44.6	.01260	.1487	6.8	1.91	.56	
27		41.3	.02102	27.8	2.07	594.	54.3	39.1	.01450	.1981	8.8	2.15	.69	
28		81.6	.02247	14.0	2.49	403.	77.0	29.0	.01560	.1870	11.1	1.03	.53	
29		67.3	.01443	14.4	2.48	310.	76.2	38.8	.01190	.1210	8.2	.81	.36	
30		58.9	.01277	12.9	2.49	315.	73.6	34.2	.01340	.1042	7.1	.80	.40	
31		50.2	.00825	14.1	2.48	239.	76.8	49.4	.00924	.0873	5.4	.60	.23	
32		41.9	.00703	14.4	2.48	244.	76.2	49.4	.00937	.0592	4.6	.63	.24	
33		32.7	.00513	14.7	2.49	228.	75.9	53.7	.00854	.0430	3.5	.60	.21	
34		21.0	.00349	12.9	2.45	238.	78.7	45.3	.01030	.0289	2.3	.62	.21	
35		43.9	.01850	24.4	2.13	456.	60.0	44.3	.01240	.1655	8.3	1.52	.49	

TABLE VI
SERIES NO. 4 (Continued)

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, percent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in eduction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air 449V _a G	Loss of head per ft. of pipe, feet of water.
36	68.5	.02930	23.6	2.15	567.	61.2	84.8	.01560	.2590	12.1	1.83	.69
37	68.2	.04200	27.4	2.04	567.	55.0	28.4	.02220	.3540	15.3	3.12	1.19
38	72.6	.02975	27.6	2.03	536.	64.6	48.0	.01380	.1900	8.6	2.00	.86
39	72.3	.04210	31.6	2.22	745.	64.5	28.2	.02180	.3670	15.5	2.28	.87
40	19.3	.00956	30.3	1.95	527.	50.2	49.5	.01310	.0688	4.0	2.06	.54
41	31.1	.01625	30.8	1.96	560.	49.4	46.9	.01360	.1545	6.7	2.23	.59
42	37.8	.02890	30.6	1.95	673.	49.7	39.1	.0165	.2260	9.3	2.68	.80
43	43.0	.02760	30.3	1.99	703.	50.1	37.0	.0169	.2660	11.0	2.71	.86
44	48.1	.04490	30.4	1.99	1000.	50.0	28.2	.0241	.4370	16.1	3.94	1.38
45	25.6	.01635	30.4	1.98	721.	50.0	36.3	.0173	.1623	6.6	2.85	.91

TABLE VII
 SERIES NO. 5
 Diameter of Education Pipe 1 1/4 inches. Foot-pieces C. Length of Education Pipe, 60.88 feet. General Arrangement of Apparatus as in Figure 3.
 Average Temperature of Water 8.0° C.

Run Number	2	3	4	5	6	7	8	9	10	11	12	13	
	Discharge of water in gallons per min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.	Average volume of air in reduction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air	Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	H _d	r	H''	S	E	Q	V _{av}	V _{av}	449V _{av}	H'	
1	6.20	.00478	37.3	1.74	682	38.7	45.6	.01665	.0487	4.4	3.53	0.75	
2	11.62	.01401	35.3	1.77	949	40.3	31.9	.02660	.1409	11.8	5.44	1.57	
3	11.80	.02190	35.9	1.80	1725	41.0	17.4	.04150	.2205	17.4	8.39	2.81	
4	6.88	.00574	36.6	1.74	745	39.9	40.9	.01840	.0572	5.1	3.76	0.88	
5	7.38	.00447	33.8	1.82	581	44.5	48.5	.01410	.0430	4.2	2.6	0.60	
6	14.17	.01845	33.6	1.83	1280	44.8	22.3	.03100	.1772	14.8	5.6	1.89	
7	12.90	.02360	34.9	1.84	1775	44.3	16.4	.04180	.2390	19.0	8.3	2.89	
8	16.70	.02080	30.1	1.83	1175	50.5	21.4	.03240	.1722	14.9	4.6	1.76	
9	10.70	.00560	30.8	1.94	564	49.4	45.4	.01860	.0535	5.5	2.2	.59	
10	9.22	.00354	27.6	2.04	437	54.7	52.7	.01110	.0328	3.7	1.6	.41	
11	15.10	.00677	27.6	2.07	521	54.7	44.1	.01300	.0540	6.9	1.9	.57	
12	18.37	.01100	27.3	2.04	683	51.1	33.1	.01760	.1901	10.0	2.4	.90	
13	19.26	.02280	24.4	2.08	1358	55.0	17.6	.03800	.1942	16.8	4.6	2.21	
14	22.22	.01960	24.3	2.13	1075	60.0	18.9	.02910	.1760	16.0	8.6	1.65	
15	16.87	.00590	24.8	2.12	486	59.3	47.5	.01160	.0533	6.4	1.6	.46	
16	9.22	.00808	24.9	2.11	397	59.1	52.1	.01070	.0276	3.4	1.3	.40	
17	11.62	.00285	21.3	2.22	325	65.0	54.7	.00958	.0261	3.7	1.0	.30	
18	18.37	.00586	21.2	2.21	472	65.2	47.6	.01160	.0463	6.2	1.1	.57	
19	23.47	.00850	21.0	2.26	470	65.5	37.2	.01380	.0752	9.0	1.4	.37	
20	25.30	.02330	21.4	2.23	1175	64.9	15.2	.03500	.2040	18.4	3.6	1.88	

TABLE VII
SERIES NO. 5 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air 449V _a ² G	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H	S	E	Q	V _a	V _a	G	H'
21	28.67	.02320	18.2	2.32	1090.	70.1	13.9	.03560	.1982	18.6	3.39	1.74
22	20.20	.00517	18.7	2.29	337.	69.4	46.3	.01100	.0441	6.8	0.98	0.86
23	14.55	.00304	18.3	2.30	278.	70.0	55.0	.00914	.0258	4.1	0.80	0.25
24	30.87	.00214	15.6	2.38	960.	74.4	13.5	.03570	.0177	6.1	0.25	
25	26.77	.00720	15.1	2.40	352.	75.2	34.8	.01480	.0594	8.4	1.00	0.48
26	19.24	.00358	14.8	2.44	265.	75.6	46.3	.01010	.0800	5.1	0.70	0.28
27	22.55	.00346	11.9	2.54	228.	80.4	43.4	.01080	.0284	5.6	0.57	0.25
28	31.07	.00687	11.4	2.55	331.	81.2	28.7	.01560	.0662	8.9	0.81	0.60
29	36.00	.00737	10.6	2.57	308.	82.5	28.6	.01560	.0599	9.9	0.75	0.60
30	29.57	.00648	13.1	2.46	316.	78.5	34.5	.01340	.0632	8.4	0.77	0.40
31	25.80	.00438	11.6	2.54	258.	80.9	37.6	.01200	.0357	6.5	0.68	0.32
32	20.85	.00256	11.1	2.55	190.	81.7	48.8	.00910	.0210	4.7	0.46	0.19
33	12.80	.00185	10.8	2.60	168.	82.2	53.5	.00815	.0111	2.7	0.41	0.16
34	8.3	.00132	14.3	2.43	226.	76.5	52.6	.00895	.0109	2.1	0.59	0.21
35	17.36	.00299	14.8	2.41	244.	75.6	50.6	.00932	.0247	4.5	0.64	0.24
36	23.30	.00484	15.1	2.41	292.	75.2	43.1	.01100	.0403	6.5	0.78	0.33
37	23.67	.00769	14.4	2.43	282.	76.2	42.6	.01480	.0638	9.0	1.00	0.50
38	24.00	.00769	18.4	2.80	427.	69.7	36.8	.01390	.0650	8.4	1.21	0.63
39	16.63	.00364	18.6	2.29	317.	69.1	49.5	.00994	.0308	4.6	0.89	0.31
40	10.82	.00220	18.9	2.30	284.	69.0	55.3	.00905	.0187	3.0	0.82	0.25

TABLE VII
SERIES NO. 5 (Continued)

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per gallon water lifted, cubic feet.	Average volume of air in reduction pipe, cubic feet per second.	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume of air 449V _{av} G	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'
41	15.78	.00438	20.9	2.23	353.	65.6	47.6	.01070	.0331	5.2	1.08	0.36
42	9.22	.00232	20.4	2.24	325.	66.4	52.3	.00991	.0205	2.9	1.09	0.31
43	5.72	.00154	21.3	2.22	368.	65.0	48.1	.01085	.0143	1.9	1.12	0.39
44	3.88	.00160	24.1	2.12	502.	60.3	40.1	.01370	.0141	1.6	1.64	0.60
45	10.67	.00346	24.9	2.11	338.	59.0	53.3	.01043	.0313	3.9	1.31	0.38
46	15.04	.00575	25.1	2.10	454.	58.7	46.0	.01220	.0517	6.1	1.54	0.48
47	14.70	.00684	27.6	2.02	522.	54.6	44.0	.01350	.0625	6.7	1.31	0.58
48	8.50	.00350	27.3	2.03	485.	55.0	48.7	.01215	.0318	3.6	1.68	0.48
49	5.72	.00226	27.0	2.04	451.	55.6	49.9	.01170	.0206	2.4	1.62	0.45
50	4.98	.00266	30.4	1.94	586.	50.0	44.8	.01410	.0250	2.6	2.25	0.63
51	7.57	.00364	29.8	1.96	520.	51.0	47.7	.01290	.0342	3.6	2.00	0.54
52	13.43	.00702	30.0	1.84	553.	50.7	45.2	.01395	.0622	6.7	2.18	0.60
53	10.85	.00696	33.3	1.83	621.	45.2	44.6	.01640	.0633	6.3	2.71	0.68
54	6.47	.00362	33.1	1.84	547.	45.5	50.6	.01360	.0343	3.5	2.38	0.56
55	2.79	.00221	38.4	1.81	782.	45.1	37.9	.01900	.0206	1.9	3.32	0.95
56	1.67	.00222	36.1	1.75	1190.	40.6	25.3	.02950	.0218	1.8	5.87	1.78
57	4.98	.00376	36.6	1.72	659.	39.8	46.3	.01680	.0387	3.4	3.31	0.71
58	8.50	.00677	36.1	1.75	703.	40.6	42.9	.01770	.0665	6.1	3.52	0.80

TABLE VIII
 SERIES NO. 6
 Diameter of Education Pipe 3 inches. Foot-piece C. Length of Education Pipe 60.875 feet. General Arrangement of Apparatus as in Figure 3.
 Average Temperature of Water 11.0° C.

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.	Average volume of air in education pipe, cubic feet per second.	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume of air 449 V _{av}	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H"	S	E	Q	V _{av}	V _{av}	G	H'
1	18.2	0.222	37.4	1.72	1100	38.6	28.4	0.2510	2205	5.1	5.43	1.55
2	27.3	0.278	35.6	1.76	920	39.9	33.2	0.2230	2730	6.5	4.50	1.20
3	5.2	0.108	36.5	1.77	1898	40.0	16.0	0.4.10	1065	2.3	9.28	3.15
4	37.3	0.360	36.4	1.78	891	40.2	34.0	0.2120	3530	8.5	4.25	1.14
5	7.6	0.107	33.3	1.79	1320	40.3	22.8	0.3140	1050	2.4	6.22	1.99
6	48.8	0.4.4	35.8	1.82	930	41.1	32.0	0.2170	4340	11.2	4.27	1.21
7	19.1	0.206	35.8	1.78	996	41.1	30.0	0.2410	2038	4.8	4.80	1.37
8	10.0	0.133	35.4	1.74	1176	41.9	25.1	0.3020	1293	3.0	5.76	1.74
9	42.8	0.347	34.4	1.83	782	43.5	36.7	0.1890	3320	8.3	3.40	0.94
10	59.3	0.4.7	33.6	1.90	808	44.8	34.7	0.1870	5130	12.6	3.88	1.01
11	30.3	0.221	33.6	1.84	711	44.8	39.4	0.1740				
12	17.5	0.135	33.6	1.84	751	44.8	37.3	0.1840	1375	3.2	3.27	0.88
13	13.6	0.128	33.6	1.80	881	44.8	31.0	0.2250	1328	3.0	4.05	1.18
14	26.8	0.206	33.4	1.84	751	45.1	37.8	0.1860	1973	5.0	3.30	0.92
15	13.9	0.121	33.3	1.87	869	45.2	31.9	0.2090	1159	2.8	3.74	1.16
16	6.8	0.092	32.9	1.88	1359	45.9	20.2	0.3290	0871	2.0	5.73	2.14
17	10.3	0.093	31.3	1.92	943	48.6	27.7	0.2320	0877	2.0	3.81	1.34
18	44.5	0.244	30.9	1.93	575	49.2	44.9	0.1420	2275	6.4	2.29	0.60
19	16.7	0.112	30.8	1.88	678	49.4	37.9	0.1750	1052	2.8	2.82	0.84
20	55.1	0.316	30.8	1.93	600	49.4	42.9	0.1490	2930	8.1	2.41	0.66

TABLE VIII
SERIES NO. 6 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume of air 449V _{av}	Loss of head per ft. of pipe, feet of water.
	G	W	p _q	r	H ¹	S	E	Q	V _{av}	V _{av}	G	H'
21	68.1	.0415	30.6	1.96	654	49.7	39.0	.01595	3875	10.5	2.55	0.76
22	36.1	.0193	30.4	1.93	531	50.0	47.7	.01325	1702	4.9	2.11	0.54
23	20.5	.0155	30.3	1.94	635	50.2	39.7	.01595	1152	3.1	2.46	0.75
24	37.1	.0205	30.3	1.95	538	50.2	42.9	.01460	1903	5.3	2.30	0.65
25	30.4	.0153	29.8	1.97	542	51.0	45.8	.01350	1416	4.1	2.09	0.53
26	56.6	.0254	28.4	1.99	493	53.4	48.1	.01265	2328	7.0	1.84	0.49
27	33.1	.0150	28.1	1.98	494	53.8	47.4	.01288	1369	4.1	1.85	0.51
28	37.0	.0178	28.1	2.00	532	53.8	44.0	.01370	1624	4.8	1.95	0.53
29	51.2	.0220	27.9	2.02	485	54.1	48.1	.01285	2005	6.1	1.76	0.49
30	14.0	.0085	27.8	2.02	681	54.3	34.0	.01740	0773	2.1	2.48	0.89
31	19.6	.0092	27.4	2.04	536	55.0	42.6	.01370	0834	2.5	1.91	0.61
32	74.4	.0350	27.4	2.05	538	55.0	42.5	.01370	3175	9.4	1.91	0.59
33	48.9	.0204	27.3	2.06	483	55.1	47.1	.01280	1868	5.8	1.71	0.49
34	29.6	.0128	27.3	2.04	496	55.1	45.9	.01275	1165	3.6	1.76	0.53
35	18.9	.0092	26.8	2.07	568	56.0	39.3	.02430	0834	2.4	1.98	0.68
36	54.9	.0186	25.1	2.11	403	58.8	51.9	.01080	1658	5.6	1.35	0.37
37	24.5	.0097	24.8	2.10	467	59.2	44.3	.01275	0858	3.1	1.55	0.51
38	67.6	.0225	24.6	2.12	400	59.6	51.3	.01080	1990	6.8	1.32	0.37
39	13.8	.0070	24.6	2.10	601	59.6	34.1	.01650	0617	1.8	2.00	0.78
40	48.0	.0162	24.6	2.12	404	59.6	50.7	.01100	1429	4.9	1.33	0.39

TABLE VIII
SERIES NO. 6 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in eduction per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air 449V ^{av} G	Loss of head per ft. of pipe, feet of water.
	G	W	H _q	r	H ^{'''}	S	E	Q	V _{av}	V _{av}	G	H'
41	88.6	.0350	24.4	2.13	478.	59.9	42.7	.01800	.2905	9.5	1.47	0.52
42	20.2	.0082	24.4	2.12	485.	60.0	42.1	.01380	.0720	2.8	1.60	0.56
43	45.0	.0146	24.4	2.12	388.	60.0	52.6	.01065	.1872	4.5	1.86	0.36
44	80.1	.0277	24.1	2.13	418.	60.4	48.0	.01150	.2445	8.2	1.87	0.41
45	33.0	.0105	23.8	2.13	386.	60.9	51.6	.01075	.0929	3.2	1.26	0.37
46	58.3	.0158	22.1	2.16	334.	63.7	55.0	.00987	.1872	5.2	1.06	0.30
47	77.2	.0217	21.9	2.19	352.	64.0	52.0	.01028	.1881	7.0	1.09	0.32
48	48.7	.0127	21.8	2.18	325.	64.1	56.0	.00957	.1104	4.3	1.02	0.28
49	65.6	.0175	21.8	2.18	388.	64.1	54.7	.00983	.1516	5.8	1.04	0.29
50	57.0	.0148	21.8	2.22	332.	64.2	54.8	.00954	.1278	5.0	1.00	0.20
51	40.4	.0105	21.6	2.19	326.	64.5	55.2	.00973	.0993	3.7	1.10	0.28
52	94.0	.0278	21.6	2.20	372.	64.5	48.4	.01098	.2440	8.8	1.16	0.36
53	26.1	.0078	21.4	2.19	373.	64.9	47.7	.01115	.0688	2.4	1.15	0.38
54	73.4	.0207	21.3	2.22	361.	65.0	49.1	.01062	.1642	6.4	1.00	0.35
55	17.1	.0059	21.3	2.21	436.	65.0	40.6	.01292	.0507	1.7	1.33	0.51
56	50.8	.0129	21.1	2.22	324.	65.3	54.2	.00968	.1107	4.4	0.97	0.29
57	40.9	.0102	20.6	2.32	337.	66.1	51.0	.00973	.0875	3.6	0.96	0.32
58	44.6	.0100	19.1	2.25	291.	68.6	54.6	.00940	.0846	3.6	0.85	0.35
59	96.3	.0237	18.9	2.28	323.	68.9	48.0	.01068	.2014	8.1	0.86	0.32
60	66.1	.0143	18.9	2.28	234.	68.9	55.5	.00918	.1211	5.3	0.82	0.34

TABLE VIII
SERIES NO. 6 (Continued)

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, percent.	Volume of free air per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	H _d	r	H ^{'''}	S	E	Q	V _a	V _a v	449V _a v G	H'
61	64.4	.0140	18.9	2.28	289	68.9	54.5	.00927	.1183	5.1	0.82	0.25
62	77.8	.0174	18.8	2.38	296	69.1	53.1	.00958	.1273	6.3	0.86	0.27
63	56.3	.0118	18.8	2.28	276	69.1	53.7	.00936	.1092	4.4	0.79	0.23
64	92.5	.0223	18.6	2.38	317	69.4	48.9	.01038	.1865	7.7	0.91	0.31
65	57.2	.0118	18.3	2.29	266	70.0	57.3	.00904	.1002	4.4	0.77	0.22
66	24.2	.0058	18.1	2.32	324	70.3	46.6	.01065	.0488	2.0	0.90	0.35
67	31.3	.0069	17.8	2.32	295	70.8	50.2	.00989	.0577	2.5	0.83	0.29
68	91.7	.0171	15.9	2.38	258	73.9	51.4	.00938	.1008	5.2	0.83	0.24
69	68	.0124	15.8	2.37	236	74.1	55.9	.00868	.1028	5.2	0.83	0.20
70	77.4	.0135	15.8	2.37	241	74.1	54.8	.00887	.1116	5.5	0.66	0.21
71	83.3	.0153	15.8	2.38	255	74.1	51.8	.00980	.1257	6.1	0.68	0.24
72	97.7	.0198	15.6	2.38	280	74.4	45.3	.01039	.1827	7.4	0.76	0.29
73	93.6	.0179	15.6	2.39	267	74.4	48.6	.00984	.1482	7.0	0.71	0.26
74	23.8	.0049	15.4	2.38	289	74.6	44.5	.01084	.0403	1.8	0.76	0.31
75	39.0	.0068	15.4	2.41	245	74.7	52.2	.00909	.0565	2.8	0.65	0.23
76	52.7	.0089	15.3	2.41	236	75.0	54.0	.00883	.0731	3.7	0.62	0.21
77	61.4	.0101	15.1	2.41	231	75.1	54.4	.00875	.0832	4.3	0.61	0.20
78	45.6	.0076	15.0	2.39	214	75.3	54.1	.00887	.0819	3.2	0.61	0.21
79	61.0	.0093	14.9	2.40	231	75.5	58.2	.00826	.0767	4.1	0.56	0.17
80	30.9	.0056	14.9	2.39	251	75.5	49.5	.00967	.0460	2.2	0.67	0.25

TABLE VIII
SERIES NO. 6 (Continued)

Run Number	1	2	3	4	5	6	7	8	9	10	11	12	13
		Discharge of water in gallons/min.	Quantity of air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expansion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in suction pipe, feet per second.	Average volume of air	Volume of water
		G	W	P _d	r	H''	S	E	Q	V _{av}	v _{av}	449V _{av}	H''
81		37.0	.0061	14.4	2.42	234.	76.2	51.4	.00924	.0502	2.6	0.61	0.22
82		84.4	.0123	13.2	2.43	206.	78.4	53.1	.00887	.1000	5.6	0.53	0.18
83		89.4	.0128	13.1	2.45	206.	78.5	52.9	.00879	.1042	5.9	0.52	0.18
84		48.9	.0067	12.9	2.45	196.	78.8	54.9	.00846	.0538	3.2	0.49	0.17
85		100.2	.0155	12.9	2.38	214.	78.8	50.2	.00862	.1253	6.8	0.56	0.20
86		79.2	.0109	12.8	2.46	197.	79.0	54.3	.00861	.0881	5.2	0.50	0.17
87		75.8	.0103	12.8	2.47	196.	79.1	54.3	.00851	.0841	4.9	0.50	0.17
88		97.4	.0135	12.4	2.48	192.	79.5	53.9	.00896	.1091	6.4	0.50	0.17
89		35.0	.0049	12.5	2.48	205.	79.5	50.8	.00808	.0398	2.3	0.51	0.20
90		60.6	.0076	12.3	2.48	183.	79.8	56.0	.00823	.0618	3.8	0.46	0.16

TABLE IX
SERIES NO. 7
Diameter of Education Pipe 3 inches. Foot-piece C. Length of Education Pipe 81.53 feet. General Arrangement of Apparatus as in Figure 3.
Average Temperature of Water 50° C.

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expansion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in reduction pipe, cubic feet.	Average velocity of mixture in education pipe, feet per second.	Average volume of air	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	449V _{av}	H'
1	16.10	.01573	48.9	1.94	1037	40.1	39.4	.01600	1380	3.4	3.85	.92
2	23.80	.02126	48.7	1.96	954	40.3	42.3	.01470	1380	4.7	3.55	.81
3	32.18	.03760	49.8	1.92	1216	39.0	34.2	.01880	3320	7.9	4.63	1.16
4	38.18	.04630	49.9	1.94	1289	38.9	32.3	.01950	4175	9.8	4.91	1.26
5	68.23	.07200	50.0	2.00	1169	38.8	35.6	.01690	6900	16.4	4.54	1.06
6	52.50	.06430										
7	24.12	.01248	40.7	2.24	666	50.2	51.0	.01020	1050	3.1	1.95	.48
8	25.80	.01594	40.7	2.24	796	50.2	42.7	.01215	1348	3.8	2.34	.67
9	47.87	.02400			646		52.5					
10	37.10	.01937	40.6	2.21	661	50.4	52.3	.01030	1605	4.8	1.94	.47
11	16.95	.01136	41.5	2.21	848	49.7	40.6	.01295	.0956	2.6	2.53	.74
12	50.74	.02700	41.2	2.30	685	49.3	48.3	.01032	2320	6.7	2.05	.49
13	64.40	.03575	39.7	2.30	738	49.5	44.3	.01120	3160	9.0	2.20	.56
14	95.13	.08660	39.2	2.32	1223	51.5	28.3	.01860	7420	18.6	3.50	1.23
15	78.95	.06320	41.5	2.24	1053	49.5	32.7	.01585	5410	13.0	3.16	1.00
16	65.70	.02495	33.3	2.45	545	59.1	51.0	.00913	1995	6.7	1.86	.38
17	88.30	.03640	31.5	2.49	600	60.2	43.8	.01045	2910	9.5	1.48	.46
18	37.00	.01130	28.3	2.53	452	62.7	52.2	.00863	.0884	3.3	1.07	.29
19	44.80	.01436	31.1	2.45	468	60.7	56.5	.00826	1110	4.1	1.11	.28
20	50.55	.01700	31.4	2.46	483	59.0	54.3	.00861	1360	4.9	1.21	.30

TABLE IX
SERIES NO. 7 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in eduction pipe, cubic feet per second.	Average velocity of mixture in educ- tion pipe, feet per second.	Average volume of air 449V _{av}	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'
21	59.00	.02005	32.3	2.47	491.	60.2	54.8	.00843	.1595	5.7	1.21	.32
22	26.60	.00896	31.0	2.57	507.	60.7	51.1	.00872	.0895	2.9	1.51	.35
23	19.40	.00728	31.0	2.57	565.	60.7	45.8	.00971	.0596	2.0	1.38	.44
24	14.40	.00533	31.0	2.58	560.	60.7	46.3	.00955	.0438	1.5	1.37	.43
25	27.80	.00999	23.6	2.56	573.	69.8	34.4	.01220	.0754	2.7	1.22	.54
26	43.70	.00955	23.6	2.72	348.	69.8	56.5	.00742	.0722	3.3	.74	.21
27	51.10	.01116	23.8	2.71	347.	69.6	57.0	.00736	.0844	3.9	.74	.20
28	62.50	.01103	23.8	2.71	282.	69.6	70.2	.00593	.0833	4.3	.60	.11
29	39.60	.00898	23.8	2.68	352.	69.6	56.3	.00757	.0744	3.2	.84	.21
30	14.80	.00457	24.0	2.70	491.	69.4	40.7	.01030	.0345	1.3	1.04	.42
31	83.50	.01980	24.0	2.70	376.	69.4	53.2	.00792	.1495	6.6	.80	.24
32	21.80	.00455	20.6	2.84	356.	75.0	48.3	.00831	.0384	1.6	.70	.27
33	30.00	.00565	20.9	2.82	311.	74.5	56.0	.00723	.0414	2.1	.82	.20
34	40.55	.00661	19.9	2.83	281.	75.2	59.0	.00679	.0502	2.7	.86	.16
35	50.80	.00861	20.6	2.84	283.	75.0	60.8	.00660	.0631	3.4	.56	.16
36	61.40	.01080	20.6	2.83	292.	75.0	58.9	.00684	.0791	4.2	.58	.18
37	75.60	.01353	19.9	2.84	298.	75.2	56.7	.00718	.0988	5.2	.59	.19
38	89.60	.01743	20.6	2.84	325.	73.0	53.0	.00758	.1250	6.4	.56	.22
39	41.10	.00580	16.9	2.92	242.	79.4	58.8	.00669	.0413	2.6	.45	.16
40	26.50	.00406	16.0	2.97	266.	80.4	50.0	.00769	.0289	1.7	.49	.19

TABLE IX

SERIES NO. 7 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume of air 449V _{av} G	Volume of water Loss of head per ft. of pipe, feet of water.
	G	W	P _q	r	H	S	E	Q	V _{av}	V _{av}	G	H
41	51.15	.00691	16.5	2.93	232.	79.9	59.5	.00655	.0489	3.2	.43	.14
42	62.40	.00828	16.4	2.95	230.	80.3	59.5	.00647	.0589	3.9	.42	.14
43	79.90	.01173	16.3	2.95	240.	80.0	56.6	.00588	.0793	5.0	.45	.16
44	91.00	.01374	16.6	2.94	260.	79.9	53.1	.00729	.0976	5.9	.43	.16
45	93.20	.04510	49.3	2.35	1263.	39.6	32.5	.01880	.4125	9.8	4.74	1.28
46	54.10	.07950	49.8	2.84	1131.	39.0	36.6	.09360	.7820	16.6	6.07	.98
47	89.50	.04390	49.5	1.96	1295.	39.4	34.3	.07760	.4330	9.6	4.91	1.14
48	57.0	.07680	48.6	2.07	1517.	43.5	26.1	.0520	.730	16.7	5.89	1.56
49	22.4	.00579	24.7	2.71	441.	69.8	50.2	.06349	.0435	1.8	.67	.86
50	46.8	.00868	24.8	2.72	332.	69.6	62.4	.00670	.0736	3.5	.70	.18
51	56.5	.01195	24.3	2.72	340.	70.2	59.7	.00696	.0900	4.2	.71	.20

TABLE X

SERIES NO. 8

Diameter of Education Pipe 3 inches. Foot-piece C. Length of Education Pipe 100.64 feet. General Arrangement of Apparatus as in Figure 3.
Average Temperature of Water 7.2° C.

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons/min.	Quantity of air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expansion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in education pipe, cubic feet per second.	Average velocity of mixture in education pipe, feet per second.	Average volume of air	Volume of water
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	449V _{av}	H'
1	22.35	.02330	60.0	2.23	1337.	40.4	37.4	.01390	1972	4.8	3.96	.99
2	38.80	.03580	59.0	2.26	1202.	40.7	40.9	.01255	3050	7.6	3.53	.83
3	49.65	.05700	60.2	2.28	1510.	39.8	33.2	.01530	.4980	11.9	4.50	1.17
4	58.60	.07610	59.6	2.26	1693.	40.0	29.4	.01745	.6610	15.4	5.06	1.37
5												
6	20.15	.00892	41.0	2.82	683.	59.6	50.1	.00808	.0614	2.1	1.37	41
7	38.90	.01402	40.6	2.78	588.	59.2	57.6	.00712	.1038	3.7	1.20	29
8	53.80	.01975	41.0	2.81	574.	59.6	59.5	.00681	.1381	5.0	1.16	28
9	61.75	.02172	41.0	2.81	581.	59.6	58.8	.00698	.1595	5.8	1.16	28
10												
11	72.40	.02730	40.8	2.80	620.	59.4	55.0	.00740	.2010	7.1	1.24	32
12	89.30	.04090	40.9	2.78	748.	59.5	45.7	.00897	.2985	9.7	1.49	47
13	63.40	.01190	26.2	3.25	335.	74.5	65.1	.00546	.0761	4.2	.54	14
14	79.10	.01438	26.1	3.26	347.	74.6	62.8	.00563	.0980	5.4	.55	16
15	86.30	.01538	26.1	3.24	339.	74.6	64.0	.00548	.1042	5.8	.54	15
16	25.60	.00508	25.9	3.26	374.	74.8	57.8	.00614	.0342	1.8	.60	19
17	34.90	.00661	25.8	3.26	357.	74.8	60.5	.00590	.0444	2.4	.57	18
18	62.60	.00940	25.9	3.26	338.	74.7	63.9	.00541	.0634	3.5	.54	15
19												
20	8.88	.00820	50.2	2.49	1429.	50.4	29.3	.01565	.0718	1.8	3.84	1.21

TABLE X
SERIES NO. 8 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used, per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic	Average volume of air in education pipe, cubic feet per second.	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	p_d	r	H''	S	E	Q	V_{av}	V_{av}	$\frac{449V_{av}}{G}$	H'
21	8.43	.00599	41.8	2.77	1156	58.8	30.2	.01863	.0442	1.2	2.35	.97
22	75.50	.01600	39.2	2.40	256	41.4						
23	61.30	.00506	40.1	3.18	153	60.5						
24	53.30	.01010	40.9	3.16	139	59.7						
25	78.60	.00738	41.0	3.16	174	59.6						
26	36.40	.01904	50.0	2.57	788	50.4	53.0	.00838	.1519	4.6	1.87	.44
27	51.10	.02760	50.0	2.53	800	50.4	52.2	.00867	.2165	6.5	1.90	.45
28	61.60	.03710	50.5	2.51	901	49.7	45.7	.00871	.2930	8.4	2.17	.56
29	19.35	.01210	50.0	2.50	917	50.4	45.5	.01001	.0939	2.7	2.18	.69
30	35.40	.01890	50.0	2.50	783	50.4	53.3	.00857	.1468	4.4	1.86	.43
31	74.90	.05170	50.0	2.53	1023	50.4	40.8	.01105	.4050	11.1	2.43	.70
32	94.25	.08330	49.5	2.59	1342	50.8	30.8	.01515	.7040	17.8	3.35	1.02
33	321.60	.00795	30.0	3.09	440	70.0	56.8	.00554	.0545	2.5	3.75	.23
34	18.60	.00525	29.8	3.11	511	70.1	48.5	.00758	.0352	1.5	.87	.31
35	50.00	.01050	28.6	3.15	359	71.5	61.5	.00593	.0725	3.6	.65	.18
36	67.45	.01440	30.0	3.09	386	70.0	64.8	.00571	.0990	4.9	.66	.16
37	83.30	.01890	29.7	3.13	414	70.2	60.0	.00613	.1312	6.2	.71	.19
38	27.4	.00599	20.0	3.44	288	80.4	58.0	.00585	.0282	1.7	.43	.15
39	36.8	.00553	20.5	3.40	293	79.9	58.3	.00585	.0361	2.3	.44	.15
40	51.6	.00710	20.0	3.45	273	80.4	61.2	.00550	.0466	3.2	.41	.13

TABLE X
SERIES NO. 8 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in eduction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air 449V _{av}	Loss of head per ft. of pipe, feet of water.
	G	W	p _q	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'
41	63.80	.00936	20.2	3.42	289.	80.0	58.1	.00584	.0612	4.0	.43	.14
42	82.60	.01229	19.7	3.45	295.	80.5	55.6	.00504	.0906	5.2	.44	.15
43	8.36	.00393	30.3	3.10	850.	69.8	29.8	.01243	.0272	.9	1.46	.71
44	8.34	.00316	25.0	3.26	715.	75.2	29.1	.01215	.02112	.7	1.14	.60
45	10.42	.00240	19.0	3.46	456.	80.2	34.6	.00968	.01586	.8	.68	.36
46	8.67	.01245	61.0	2.17	1776.	39.4	28.6	.01890	.01041	2.4	5.41	1.51

TABLE XI

SERIES NO. 9

Diameter of Eduction Pipe 3 inches. Foot-piece C. Length of Eduction Pipe 37.85 feet. General Arrangement of Apparatus as in Figure 3.
Average Temperature of Water 4.4° C.

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per gallon water lifted, cubic feet.	Average volume of air in eduction pipe, cubic feet per second.	Average velocity of mixture in educ- tion pipe, feet per second.	Average volume of air 449V _{av} G	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'
1	13.7	.0169	21.3	1.49	786.	43.7	22.6	.04640	.1737	4.0	5.72	1.92
2	23.0	.0255	21.2	1.47	688.	44.0	25.7	.04190	.2580	6.0	4.97	1.61
3	33.9	.0395	22.2	1.45	704.	41.5	26.3	.04210	.4100	9.5	5.43	1.62
4	48.2	.0644	22.0	1.49	860.	41.6	21.7	.04870	.6970	15.7	6.48	2.04
5	61.2	.0874	21.2	1.55	999.	44.0	17.7	.05370	.9840	21.9	7.21	2.48
6	7.2	.0055	15.0	1.64	606.	60.4	20.6	.04060	.0513	1.3	3.19	1.52
7	19.1	.0092	15.2	1.64	382.	59.8	33.3	.02540	.0868	2.5	2.03	0.81
8	29.6	.0115	15.2	1.65	312.	59.8	40.7	.02040	.1085	3.4	1.66	0.88
9	52.8	.0197	14.9	1.65	325.	60.5	38.2	.02010	.1890	6.0	1.60	0.82
10	63.4	.0233	14.6	1.65	305.	61.3	40.9	.02010	.2235	7.1	1.58	0.87
11	42.4	.0141	14.4	1.68	276.	61.9	43.5	.01840	.1388	4.5	1.42	0.49
12	99.4	.0392	14.5	1.71	399.	61.6	20.2	.03850	.6940	17.6	3.91	1.92
13	67.0	.0483	15.1	1.69	464.	60.5	27.2	.02860	.3190	13.0	2.44	1.02
14	71.7	.0396	16.0	1.64	437.	57.8	30.4	.02760	.3890	10.6	2.11	0.92
15	99.4	.0235	9.5	1.84	231.	75.3	34.2	.02000	.2187	8.6	0.99	0.46
16	90.3	.0204	9.5	1.83	210.	74.7	37.6	.01910	.1876	7.6	0.98	0.40
17	83.1	.0176	9.6	1.84	200.	74.8	40.0	.01710	.1570	6.7	0.95	0.37
18	73.1	.0142	9.5	1.83	186.	73.7	42.5	.01620	.1307	5.8	0.79	0.33
19	58.7	.0114	10.1	1.80	163.	73.2	45.0	.01530	.1038	4.6	0.79	0.31
20	43.9	.0084	10.0	1.80	179.	73.5	47.0	.01530	.0762	3.4	0.78	0.30

TABLE XI
SERIES NO. 9 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in reduc- tion pipe, feet per second.	Average volume of air 449 V _{av} G	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H ^{'''}	S	E	Q	V _{av}	V _{av}	G	H'
21	26.0	.0057	10.1	1.80	206.	73.2	40.8	.01740	.0522	2.1	0.90	0.38
22	22	.0028	9.7	1.80	440.	74.4	18.4	.03840	.0252	0.7	1.88	1.14
23	14.3	.0037	9.5	1.82	251.	74.7	31.9	.02190	.0399	1.3	1.06	0.64
24	98.4	.0172	7.6	1.91	181.	79.9	34.8	.01845	.1582	7.4	0.72	0.36
25	66.9	.0125	7.3	1.91	192.	80.6	31.8	.02050	.1132	5.1	0.76	0.41
26	71.1	.0100	7.7	1.89	144.	79.6	44.5	.01470	.0908	5.1	0.53	0.25
27	94.3	.0164	7.7	1.90	178.	79.6	36.0	.01810	.1498	7.0	0.71	0.35
28	72.0	.0102	7.7	1.89	145.	79.6	44.2	.01480	.0980	4.9	0.58	0.25
29	51.7	.0067	7.5	1.88	131.	80.1	48.1	.01380	.0601	3.4	0.52	0.21
30	35.4	.0046	7.3	1.91	135.	80.6	45.2	.01440	.0421	2.3	0.53	0.23
31	56.0	.0071	7.6	1.94	136.	80.0	47.0	.01360	.0567	3.4	0.46	0.23
32	62.9	.0085	7.7	1.91	139.	79.6	46.0	.01410	.0785	4.2	0.56	0.25
33	74.8	.0108	7.8	1.89	147.	79.6	44.2	.01480	.0981	6.1	0.59	0.26
34	84	.0054	7.7	1.88	289.	83.3	22.1	.02800	.0213	0.8	1.16	0.71
35	17.2	.0027	6.3	1.91	161.	89.6	32.3	.01980	.0237	1.2	0.62	0.34
36	23.0	.0037	7.6	1.89	164.	80.0	38.4	.01710	.0355	1.6	0.65	0.32
37	17.1	.0091	18.6	1.86	296.	50.5	16.8	.05640	.0807	1.8	6.84	2.47
38	6.7	.0123	18.7	1.86	513.	50.7	30.8	.03120	.1278	3.3	3.24	1.14
39	34.3	.0229	16.9	1.86	466.	50.1	33.9	.02850	.2293	6.0	7.02	0.97
40	80.5	.1100	18.2	1.70	1151.	52.0	13.1	.06080	1.2580	28.0	7.02	2.96

TABLE XI
SERIES NO. 9 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.	Average volume of air in reduction pipe, cubic feet per second.	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H ^{'''}	S	E	Q	V _{av}	V _{av}	G	H'
41	70.5	.0690	18.7	1.60	731	50.7	21.3	.0419	.7220	17.1	4.60	1.73
42	59.1	.0503	19.2	1.56	607	49.4	26.3	.0356	.5160	12.6	3.92	1.87
43	48.5	.0342	19.2	1.56	500	49.4	32.0	.0294	.8470	8.9	3.21	1.05
44	20.9	.0054	11.1	1.78	239	70.7	38.9	.0188	.0507	1.9	1.09	1.46
45	35.9	.0079	11.2	1.76	199	70.6	46.7	.0157	.0716	2.9	.90	.34
46	50.3	.0116	11.5	1.75	207	69.6	46.3	.0161	.1059	4.2	.94	.35
47	60.0	.0139	11.2	1.76	210	70.7	44.3	.0165	.1262	5.1	.94	.37
48	74.6	.0193	11.3	1.76	235	70.1	40.0	.0184	.1781	6.7	1.07	.43
49	94.7	.0295	10.6	1.79	276	71.9	31.9	.0224	.2720	9.6	1.23	.57
50	6.1	.0036	11.7	1.74	533	69.0	18.3 ^a	.0411	.0335	.9	2.46	1.38
51	10.6	.0043	11.7	1.75	368	69.0	26.5	.0284	.0401	1.2	1.70	.86

TABLE XII
SERIES NO. 10

Diameter of Education Pipe 3 inches. Foot-piece D. Length of Education Pipe 61.26 feet. Harris Foot-piece. General arrangement of Apparatus as in Figure 3.
Average Temperature of Water 16.1° C.

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.	Average volume of air in education pipe, cubic feet per second.	Average velocity of mixture in educa- tion pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	v _{av}	449V _{av} G	H'
1	54.50	.06820	37.2	1.74	1118.	39.3	27.7	.02700	.6830	15.7	5.63	1.52
2	44.50	.05860	37.4	1.74	1150.	39.0	27.1	.02690	.5880	13.4	5.88	1.60
3	40.60	.04800	37.5	1.73	1043.	38.8	29.8	.02550	.4820	11.2	5.33	1.40
4	42.20	.04900	37.0	1.75	1037.	39.6	29.8	.02510	.4910	11.4	5.22	1.40
5	33.40	.03740	37.3	1.74	988.	39.1	31.5	.02400	.3740	8.7	5.02	1.32
6	64.90	.06300	33.9	1.85	955.	44.7	29.6	.02290	.6110	14.7	4.22	1.26
7	52.80	.05240	33.7	1.85	823.	45.0	34.1	.01980	.5030	12.5	3.59	1.02
8	54.30	.04230	33.7	1.85	765.	46.0	36.5	.01950	.4080	10.3	3.38	.82
9	40.80	.02870	34.1	1.84	681.	44.3	41.7	.01660	.2780	7.2	3.04	.76
10	60.90	.04680	31.2	1.93	745.	49.1	34.9	.01580	.4440	11.5	3.02	.82
11	62.50	.03850	31.0	1.93	648.	49.4	39.9	.01590	.3620	9.8	2.60	.74
12	74.00	.03470	31.0	2.00	539.	56.4	41.8	.01390	.3170	9.4	1.90	.69
13	32.10	.03110	36.1	1.77	879.	40.6	37.8	.02130	.3170	7.4	1.30	.82
14	21.40	.02325	37.5	1.73	925.	38.8	36.8	.02210	.2840	5.5	4.82	1.25
15	23.30	.02780	37.7	1.73	1045.	38.5	30.1	.02340	.2760	6.4	5.32	1.42
16	15.80	.01928	37.3	1.74	1075.	39.1	28.9	.02510	.1930	4.5	5.48	1.50
17	8.80	.01500	37.1	1.75	1325.	39.4	20.3	.03780	.1560	3.3	7.65	2.36
18	48.60	.03350	38.7	1.86	1384.	45.5	40.8	.01650	.3330	8.4	2.97	.78
19	41.60	.02340	38.7	1.85	670.	45.0	42.0	.01630	.2740	7.1	2.97	.75
20	32.90	.02345	33.9	1.84	749.	44.7	37.7	.01630	.2450	6.2	3.35	.90

TABLE XII
SERIES NO. 10 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per gallon water per foot of lift, cubic feet.	Average volume of air in eduction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	H _d	r	H _{exp}	S	E	Q	V _{av}	V _{av}	G	H'
21	29.40	.02000	33.3	1.86	678.	45.7	41.3	.01640	.1920	5.0	2.94	.76
22	29.30	.01402	33.0	1.87	721.	45.1	38.7	.01360	.1840	3.4	3.38	.87
23	32.60	.01700	33.6	1.85	742.	45.2	37.7	.01800	.1580	4.2	3.38	.90
24	33.30	.01690	30.5	1.84	536.	50.2	47.4	.01380	.1580	4.9	3.13	.56
25	33.80	.02860	31.4	1.92	606.	48.8	43.2	.01360	.2700	6.9	3.13	.66
26	46.30	.02610	31.1	1.93	591.	49.5	43.8	.01460	.2470	6.8	2.39	.64
27	55.00	.02900	31.0	1.93	591.	49.5	48.7	.01470	.2730	7.5	2.40	.64
28	55.80	.02475	27.3	2.04	469.	55.5	48.4	.01210	.2250	7.0	1.69	.46
29	56.30	.02030	26.0	2.08	422.	57.6	51.3	.01110	.1830	6.0	1.46	.39
30	41.40	.01510	26.5	2.07	422.	56.9	52.4	.01000	.1890	4.5	1.50	.37
31	76.80	.02950	25.4	2.10	454.	58.6	46.6	.01190	.2660	8.5	1.56	.46
32	69.70	.02445	24.7	2.12	421.	59.7	48.9	.01140	.2180	7.5	1.41	.41
33	59.80	.02025	24.9	2.11	404.	59.4	51.3	.01090	.1800	6.1	1.35	.38
34	46.40	.01500	24.7	2.12	391.	59.7	52.7	.01080	.1340	4.6	1.35	.36
35	81.00	.02520	21.8	2.21	393.	64.5	46.2	.01140	.2210	7.8	7.22	.40
36	72.50	.02095	21.8	2.21	364.	64.5	49.9	.01060	.1820	6.7	7.13	.35
37	65.30	.01781	22.0	2.20	344.	64.2	52.4	.00985	.1550	5.9	7.07	.31
38	79.60		18.9	2.30		69.1						
39	21.40		32.9	1.87		46.4						
40	21.20	.01560	32.8	1.87	785.	46.5	37.2	.01820	.1500	3.8	3.19	.90

TABLE XII
SERIES NO. 10 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in reduction pipe, cubic feet per second.	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'
41	16.80	.01340	33.6	1.85	780	45.2	35.9	.01900	.1290	3.2	3.62	.98
42	11.20	.01156	33.6	1.85	1010	45.2	27.7	.01840	.1120	2.7	4.50	1.42
43	4.80	.00777	33.7	1.85	1525	45.1	18.5	.00884	.0747	1.7	6.99	2.45
44	33.20	.01765	30.6	1.94	561	50.1	45.5	.01330	.1650	4.7	2.23	.59
45	28.90	.01550	30.9	1.93	561	49.6	45.8	.01390	.1450	4.0	2.34	.59
46	27.00	.01585	31.4	1.90	600	48.8	43.6	.01580	.1480	4.1	2.46	.66
47	17.80	.01105	31.2	1.91	642	49.1	40.5	.01600	.1030	2.8	2.59	.75
48	11.90	.00894	30.9	1.91	780	49.6	33.1	.01450	.0828	2.1	3.12	1.02
49	11.40	.00713	27.3	2.02	702	55.4	32.4	.01840	.0638	1.7	2.51	.93
50	15.50	.00852	27.3	2.02	617	55.4	31.7	.01620	.0765	2.2	2.21	.76
51	24.60	.01063	26.9	2.04	492	56.2	45.5	.01280	.0955	2.9	1.74	.52
52	35.50	.01397	26.8	2.04	446	56.3	50.1	.01180	.1250	4.0	1.58	.43
53	34.80	.01190	24.8	2.10	405	59.6	51.1	.01100	.1050	3.6	1.35	.39
54	27.10	.00932	24.6	2.11	409	59.9	50.2	.01120	.0827	2.8	1.37	.40
55	14.30	.00718	25.4	2.08	586	58.6	36.1	.01593	.0545	1.9	2.03	.73
56	17.30	.00627	22.0	2.18	450	64.1	40.8	.01320	.0540	1.8	1.40	.52
57	26.40	.00812	22.0	2.18	383	64.1	48.0	.01125	.0702	2.5	1.19	.39
58	36.10	.01010	22.2	2.18	328	63.7	53.3	.01010	.0867	3.3	1.01	.23
59	44.60	.01161	21.5	2.20	328	65.0	54.8	.00970	.0999	3.9	1.01	.29
60	49.5	.01320	21.9	2.19	384	64.3	54.7	.00973	.1140	4.4	1.03	.29

TABLE XII
SERIES NO. 10 (Continued)

Run Number	2	3	4	5	6	7	8	9	10	11	12	13
	Discharge of water in gallons per	Quantity of air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expansion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in suction pipe, feet per second.	Average volume of air Volume of water	Loss of head per foot of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'
61	55.00	.01500	21.8	2.19	341.	64.4	53.2	.01103	.1290	4.9	1.05	.31
62	62.00	.01311	18.6	2.29	279.	69.7	55.5	.00912	.1100	4.8	.80	.24
63	69.80	.01560	19.0	2.27	293.	69.0	54.2	.00940	.1310	5.6	.84	.26
64	55.00	.01250	18.9	2.28	290.	69.2	54.2	.00940	.1080	4.4	.84	.26
65	45.30	.00985	18.8	2.28	284.	69.4	55.0	.00924	.0827	3.6	.82	.25
66	48.50	.01047	19.0	2.27	282.	69.0	56.1	.00911	.0883	3.8	.82	.24
67	38.30	.00808	18.1	2.30	280.	70.4	54.0	.00932	.0674	3.0	.79	.25
68	31.60	.00714	18.3	2.29	299.	70.2	51.1	.00989	.0595	2.5	.84	.29
69	91.40	.01685	15.3	2.39	257.	75.1	49.6	.00964	.1390	6.7	.68	.26
70	90.40	.01655	15.3	2.39	255.	76.1	50.1	.00959	.1360	6.6	.68	.24
71	77.10	.01902	14.9	2.40	236.	75.7	52.5	.00907	.1060	5.4	.62	.21
72	71.10	.01255	16.0	2.36	248.	73.9	53.8	.00907	.1060	5.2	.67	.22
73	61.80	.01004	15.1	2.39	256.	75.4	55.8	.00821	.0821	4.3	.59	.19
74	59.60	.00993	15.4	2.38	231.	74.8	57.4	.00869	.0812	4.2	.61	.20
75	50.90	.00857	15.8	2.37	225.	74.3	60.0	.00855	.0706	3.6	.62	.19
76	42.60	.00760	15.9	2.36	242.	74.0	56.9	.00837	.0619	3.1	.65	.21
77	71.60	.00851	12.9	2.51	176.	80.4	53.5	.00803	.0637	4.5	.44	.15
78	58.90	.01280	12.8	2.46	207.0	79.0	52.4	.00842	.0542	3.5	.43	.16
79	52.00	.00667	12.4	2.47	167.0	79.8	56.8	.00829	.0441	3.3	.47	.16
80	31.60	.00430	12.1	2.51	199.5	80.2	52.0	.00902	.0349	2.1	.60	.19

TABLE XII
SERIES NO. 10 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	p _q	r	H'''	S	E	Q	V _{av}	var	449V _{av} G	H'
81	40.60	.00537	12.4	2.49	192.5	79.8	55.5	.00846	.0437	2.6	.48	.17
82	14.20	.00275	12.4	2.49	282	79.8	37.8	.01220	.0224	1.1	.71	.36
83	14.40	.00275	12.5	2.49	279	79.7	38.6	.01210	.0224	1.1	.70	.34
84	24.40	.00368	12.8	2.49	219	80.0	48.2	.00980	.0297	1.6	.55	.23
85	24.40	.00463	15.4	2.41	267	75.0	49.7	.00982	.0384	1.8	.71	.27
86	30.00	.00550	15.6	2.40	255	74.5	52.6	.00941	.0457	2.2	.68	.24
87	10.30	.00296	16.4	2.41	404	75.0	32.9	.01490	.0246	.9	1.07	.53
88	16.70	.00385	15.5	2.40	322	74.8	41.3	.01190	.0318	1.4	.86	.38
89	16.80	.00473	17.7	2.33	392	71.1	38.8	.01320	.0399	1.5	1.10	.48
90	20.50	.00568	19.0	2.29	366	69.1	44.8	.01160	.0476	1.8	1.04	.41
91	25.90	.00280	9.2	2.60	164	85.1	48.2	.00942	.0223	1.6	.39	.17
92	35.70	.00340	9.2	2.60	144.5	85.1	54.6	.00830	.0270	2.1	.34	.13
93	41.40	.00414	9.5	2.59	152	84.6	53.7	.00844	.0329	2.4	.36	.14
94	59.90	.00571	9.5	2.59	144.5	84.6	56.5	.00803	.0455	3.5	.34	.13
95	63.90	.00583	9.5	2.59	138.8	85.1	56.9	.00768	.0465	3.7	.33	.12

TABLE XIII
 SERIES NO. 11
 Diameter of Education Pipe 2½ and 3 inches. Foot-piece C. Length of Education Pipe 42.96 and 19.08 feet. General Arrangement of Apparatus as in Figure 3.
 Average Temperature of Water 3.8°.

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of Expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air lift, cubic feet, per foot of lift.
G	W		ft		H	S	E	Q
1	7.85	.00183	11.2	2.56	350.	82.0	26.7	.01680
2	23.85	.00177	6.1	2.71	118.	90.1	43.3	.00974
3	74.50	.00265	1.1	2.87	60.	98.3	14.6	.02810
4	81.10	.00853	1.6	2.86	73.	97.5	17.7	.02510
5	59.60	.00353	4.6	2.79	97.	92.5	39.5	.01080
6	39.60	.00404	7.7	2.67	160.	87.6	40.1	.01065
7	26.65	.00350	11.0	2.57	198.	82.3	46.2	.00958
8	18.05	.00350	14.3	2.47	280.	77.1	42.6	.01090
9	13.60	.00352	15.4	2.43	367.	75.3	35.0	.01350
10	16.90	.00436	17.5	2.37	361.	71.9	40.4	.01200
11	25.60	.00422	13.4	2.49	241.	78.5	46.4	.00989
12	34.90	.00418	10.5	2.58	182.	83.1	48.0	.00911
13	48.50	.00416	7.3	2.68	135.	88.4	45.0	.00949
14	62.75	.00460	5.1	2.75	116.	91.8	36.8	.01125
15	54.70	.00449	2.1	2.84	136.	96.8	12.5	.03610
16	89.20	.00544	2.7	2.81	100.	96.8	22.0	.01840
17	77.50	.00545	4.2	2.76	118.	93.2	30.7	.01340
18	61.90	.00540	7.2	2.67	137.	88.5	43.5	.00979
19	45.80	.00537	10.7	2.57	179.	82.8	49.7	.00895
20	33.95	.00537	13.6	2.47	229.	78.1	49.7	.00982

TABLE XIII
SERIES NO. 11 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.
G	W	H _d	r	H _u	S	E	Q	
21	23.10	.00527	17.6	2.39	319	71.7	46.4	.01040
22	18.35	.00527	19.9	2.29	379	68.0	43.7	.01158
23	13.10	.00528	23.2	2.19	505	62.7	38.2	.01390
24	10.10	.00524	26.2	2.09	613	57.8	35.3	.01590
25	5.94	.00635	32.4	1.90	1100	47.8	24.6	.02650
26	8.60	.00625	29.7	1.99	800	52.1	31.0	.01960
27	12.40	.00618	26.3	2.09	587	57.6	37.4	.01520
28	17.60	.00628	23.3	2.18	445	62.5	43.7	.01230
29	24.60	.00680	19.3	2.31	342	69.0	47.1	.01060
30	31.65	.00637	16.3	2.40	281	73.9	48.7	.00991
31	39.70	.00621	13.3	2.48	227	78.5	49.0	.00947
32	48.75	.00625	10.9	2.56	193	82.5	46.8	.00949
33	64.25	.00626	7.7	2.66	152	87.6	42.2	.01020
34	79.15	.00607	5.1	2.74	124	91.8	34.5	.01210
35	90.50	.00610	3.1	2.80	110	95.0	23.0	.01770
36	2.70	.00867	36.9	1.76	2910	40.5	10.0	.06970
37	13.85	.00846	30.7	1.95	651	50.5	39.4	.01590
38	21.30	.00850	25.8	2.10	474	56.8	45.3	.01240
39	25.40	.00850	23.5	2.17	415	62.1	47.3	.01190
40	34.55	.00847	20.1	2.28	322	67.7	52.0	.00978

TABLE XIII
SERIES NO. 11 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.
G	W	H _g	r	H	S	E	Q	
41	43.35	.00851	16.6	2.38	272.	73.3	50.7	.00951
42	51.75	.00849	13.9	2.47	237.	77.7	48.7	.00949
43	65.50	.00835	10.4	2.57	194.	83.3	44.3	.00988
44	78.70	.00839	7.5	2.66	166.	88.0	36.7	.01145
45	89.50	.00905	6.4	2.70	162.	90.0	32.7	.01275
46	90.10	.01311	8.9	2.62	224.	85.6	33.2	.01310
47	71.50	.01313	13.8	2.47	255.	77.0	43.6	.01060
48	61.2	.01318	16.7	2.38	238.	72.7	48.6	.01040
49	50.25	.01317	20.1	2.28	344.	67.7	48.7	.01050
50	37.50	.01312	24.4	2.14	427.	60.7	47.7	.01150
51	35.75	.01415	26.8	2.07	461.	56.8	48.4	.01182
52	29.10	.01413	30.0	1.95	529.	51.6	47.3	.01295
53	22.55	.01416	32.5	1.80	643.	47.7	42.2	.01550
54	15.60	.01416	35.3	1.78	841.	41.5	35.9	.02000
55	12.80	.01650	29.5	1.69	996.	36.4	35.4	.03510
56	19.10	.01648	36.3	2.09	1015.	41.6	32.9	.01900
57	26.20	.01648	32.6	1.89	642.	47.3	42.3	.01550
58	34.20	.01642	30.0	1.97	522.	51.6	47.9	.01280
59	42.75	.01622	25.0	2.09	448.	58.1	48.4	.01170
60	47.50	.01515	23.6	2.17	419.	62.0	47.0	.01160

TABLE XIII
SERIES NO. 11 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.
	G	W	H_d	r	H''	S	E	Q
61	56.70	.01617	20.4	2.27	373.	67.2	45.5	.01120
62	70.50	.01613	15.5	2.42	323.	75.0	39.9	.01187
63	79.70	.01615	13.7	2.44	288.	72.7	39.7	.01180
64	90.00	.01602	10.8	2.56	267.	82.7	33.2	.01330
65	94.90	.01983	11.4	2.54	312.	81.8	30.3	.01475
66	84.60	.01999	14.2	2.46	339.	77.2	34.8	.01337
67	78.80	.01999	16.1	2.40	354.	74.1	37.8	.01270
68	64.65	.02000	20.4	2.27	404.	67.2	42.1	.01220
69	54.70	.02002	23.6	2.17	452.	62.0	43.5	.01240
70	45.70	.01995	26.9	2.07	507.	56.8	44.3	.01300
71	35.85	.01995	26.9	1.97	507.	56.8	44.3	.01300
72	28.75	.01986	33.2	1.98	752.	46.5	36.8	.01670
73	22.10	.01982	36.3	1.78	830.	41.5	36.5	.01980
74	14.85	.01982	39.3	1.69	1123.	36.8	29.1	.02720
75	7.24	.01978	42.2	1.61	2080.	32.1	16.9	.05180
76	96.00	.01582	10.1	2.57	249.	83.7	33.8	.01292
77	79.40	.01580	14.7	2.44	283.	76.7	36.7	.01090
78	66.30	.01570	18.6	2.32	318.	70.0	48.6	.01020
79	59.00	.01568	20.9	2.24	343.	66.3	50.7	.01020
80	46.60	.01565	24.2	2.14	409.	61.1	49.2	.01115

TABLE XIII
SERIES NO. 11 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per foot of lift, cubic feet.
G	W	Hd	r	H"	S	E	Q	
81	38.65	.01547	27.7	2.04	455.	55.4	50.8	.01155
82	31.00	.01560	30.5	1.95	538.	50.7	47.4	.01820
83	24.00	.01560	33.3	1.87	650.	46.4	42.3	.01565
84	18.75	.01540	36.3	1.78	735.	41.5	40.0	.01815
85	10.15	.01555	40.3	1.66	1238.	35.0	27.1	.03050
86	9.83	.02670	42.5	1.62	2110.	31.5	16.8	.05130
87	19.10	.02665	39.4	1.69	1170.	34.6	23.0	.02850
88	30.00	.02640	35.5	1.80	827.	39.8	35.7	.01980
89	38.00	.02625	32.3	1.90	768.	43.0	38.0	.01710
90	48.10	.02615	29.3	1.99	599.	52.8	40.7	.01480
91	55.30	.02500	26.9	2.06	543.	56.7	41.2	.01400
92	65.70	.02590	23.3	2.17	488.	62.5	39.7	.01360
93	76.70	.02590	20.6	2.27	444.	67.6	38.5	.01320
94	87.70	.02580	16.9	2.36	400.	72.8	35.2	.01400
95	96.00	.02560	14.0	2.45	382.	72.5	27.9	.01530
96	14.30	.03160	42.1	1.60	1510.	32.2	23.2	.04210
97	23.10	.03115	38.8	1.70	1140.	37.6	38.3	.02520
98	32.90	.03285	35.7	1.79	931.	42.4	31.9	.02240
99	45.00	.03280	31.9	1.91	751.	48.6	35.4	.01830
100	52.25	.03260	29.7	1.97	678.	52.1	36.4	.01680

TABLE XIII

SERIES NO. 11 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	W Quantity of air sup- plied, pounds per second.	H_d Delivery head, feet.	r Ratio of expansion of air.	H''' Energy of expan- sion of air used per gallon of water pumped, foot pounds.	S Submergence, per cent.	E Efficiency of pump, per cent.	Q Volume free air per gallon water per foot of lift, cubic feet.
101	61.30	.03235	25.4	2.07	614.	57.5	35.7	.01610
102	71.50	.03210	23.4	2.16	552.	52.5	35.3	.01530
103	95.20	.03205	15.8	2.39	468.	54.5	28.1	.01710
104	14.9	.03970	42.7	1.58	1965.	31.2	18.1	.05000
105	25.40	.03960	39.10	1.69	1310.	35.9	25.0	.03200
106	36.20	.03940	35.7	1.79	1015.	42.5	29.3	.02440
107	48.60	.03925	32.1	1.90	827.	42.2	27.3	.02020
108	66.10	.03865	32.10	2.07	691.	47.2	32.3	.01600
109	74.00	.03825	22.7	2.36	646.	51.3	29.2	.01860
110	93.80	.03875	17.6	2.54	560.	71.2	26.2	.01880

TABLE XIV

SERIES NO. 12

Diameter of Education Pipe 2, 2½, and 3 inches. Foot-pieces C, Length of Education Pipe 20.38, 21.62 and 19.08 feet. General Arrangement of Apparatus as in Figure 3. Average Temperature of Water 3.5°C.

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of air pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.
G	W	h _d	r	H	S	E	Q	
1	87.1	.0043	23.5	2.16	606.	62.8	32.4	01714
2	94.5	.0043	21.5	2.90	771.	102.2	41.2	01282
3	16.2	.0054	21.5	2.22	430.	66.0	19.1	00800
4	6.4	.0049	22.2	2.19	970.	64.3	59.5	00870
5	19.2	.0046	21.8	2.20	305.	64.8	49.2	01100
6	15.9	.0048	22.3	2.19	381.	64.2	57.1	01000
7	20.2	.0056	22.3	2.20	347.	65.0	34.7	01580
8	15.1	.0044	21.7	2.20	371.	64.7	55.9	00770
9	6.51	.0027	22.0	2.20	529.	60.0	45.8	02000
10	16.8	.0051	24.9	2.11	363.	60.0	40.5	01381
11	13.25	.0049	24.9	2.11	442.	60.0	43.7	01125
12	1.01	.0043	24.9	2.03	513.	56.2	41.1	01413
13	11.63	.0054	27.5	2.03	526.	70.0	54.6	02550
14	13.10	.0050	27.4	2.04	437.	70.2	54.6	00916
15	10.01	.0049	27.3	2.04	554.	70.0	50.8	00984
16	26.8	.0057	18.7	2.30	286.	74.9	55.7	00838
17	22.2	.0047	18.6	2.30	282.	74.9	55.7	00833
18	16.9	.0039	18.7	2.30	306.	74.9	55.7	00833
19	33.6	.0054	15.6	2.39	226.	74.9	55.7	00833
20	23.9	.0049	15.6	2.39	226.	74.9	55.7	00833

TABLE XIV
SERIES NO. 12 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used, per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.
	G	W	P _d	r	H'''	S	E	Q
21	18.9	.0032	15.2	2.40	239.	75.5	53.2	.00700
22	42.5	.0035	12.5	2.48	187.	80.0	55.2	.00826
23	38.8	.0047	12.2	2.49	175.	80.3	53.8	.00788
24	28.1	.0035	12.2	2.49	178.	80.3	57.2	.00806
25	18.5	.0024	12.2	2.49	171.	80.3	53.6	.00863
26	53.1	.0054	9.2	2.58	153.	85.1	50.2	.00876
27	46.9	.0044	9.2	2.58	138.	85.2	55.7	.00793
28	37.4	.0034	9.2	2.58	139.	84.7	57.0	.00773
29	26.6	.0025	9.2	2.58	134.	85.2	57.5	.00737
30	45.2	.0303	31.1	1.94	723.	50.0	37.3	.01750
31	39.0	.0211	31.4	1.93	572.	49.6	45.8	.01380
32	38.2	.0329	34.3	1.85	845.	44.9	33.8	.02010
33	38.2	.0259	34.5	1.84	766.	44.5	37.6	.01830
34	27.4	.0147	33.9	1.86	532.	45.4	53.2	.01260
35	29.0	.0316	37.0	1.76	995.	40.5	31.4	.01260
36	19.9	.0135	37.5	1.74	602.	39.8	51.8	.01440
37	22.6	.0327	40.8	1.66	170.	35.4	28.6	.02890
38	13.5	.0158	39.0	1.70	852.	37.4	38.1	.02070
39	16.9	.0345	43.1	1.57	156.	30.6	23.1	.04030
40	53.7	.0334	27.2	2.06	692.	56.4	32.3	.01770

TABLE XIV
SERIES NO. 12 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of wa- ter pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per foot of lift, cubic feet.
	G	W	h _d	r	H'''	S	E	Q
41	42.8	.0144	27.1	2.07	390.	56.5	58.0 ^a	.00994
42	59.3	.0311	25.5	2.12	630.	59.0		.01650
43	55.4	.0313	25.1	2.07	656.	59.8	31.9	.01810
44	46.1	.0199	24.44	2.14	525.	60.6	38.9	.01415
45	66.3	.0319	21.8	2.21	607.	64.9	30.0	.01770
46	49.1	.0139	21.8	2.21	357.	64.9	51.0	.01035
47	78.3	.0219	17.7	2.33	378.	72.5	38.9	.01270
48	63.2	.0169	17.7	2.33	360.	72.5	40.8	.01210
49	82.6	.0334	15.7	2.40	564.	74.7	23.2	.02060
50	63.2	.0148	15.3	2.41	328.	75.4	38.9	.01220
51	88.8	.0104	13.3	2.46	168.	78.7	65.9	.00698
52	79.6	.0143	11.7	2.51	264.	81.0	37.1	.01270
53	53.3	.0317	42.3	1.59	162.	32.0	21.8	.04140
54	95.6	.0555	10.7	2.55	859.	82.7	10.4	.04330
55	95.0	.0582	11.7	2.52	904.	81.1	10.8	.04170
56	80.0	.0500	16.0	2.39	867.	74.2	15.9	.03120
57	74.7	.0518	18.4	2.31	927.	70.5	16.6	.03030
58	67.2	.0512	21.3	2.22	969.	65.8	18.4	.02860
59	58.4	.0646	24.7	2.12	325.	60.3	15.5	.03540
60	53.0	.0693	27.4	2.03	477.	56.0	15.4	.03820

TABLE XIV
SERIES NO. 12 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used, per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, percent.	Volume free air per foot of lift, cubic feet.
	G	W	h _d	r	H"	S	E	Q
61	48.7	0285	27.4	2.03	1732	56.0	12.1	0174
62	42.7	0700	31.4	1.91	1595	48.5	13.4	0418
63	34.4	0305	32.3	1.88	892	45.2	20.2	0219
64	29.6	0291	33.8	1.83	945	46.4	20.2	0231
65	29.4	0314	34.6	1.81	1050	44.4	27.3	0247
66								
67	24.2	0326	36.9	1.74	1180	40.6	25.9	0292
68	17.7	0355	40.2	1.64	1650	37.8	20.2	0414
69	94.2	0700	43.0	1.56	1800	37.0	12.7	0730
70	81.0	0660	9.2	2.60	1070	85.2	7.1	0607
72	86.1	0609	15.2	2.39	1045	75.5	12.2	0395
73	68.1	0778	9.2	2.60	310	85.2	24.8	0176
74	53.9	0075	12.0	2.52	178	85.2	43.7	0108
75	66.9	0128	12.7	2.50	205	80.6	48.9	0093
76	44.1	0029	15.3	2.42	278	79.7	37.8	0120
77	59.3	0118	15.0	2.42	255	75.4	50.2	00950
78	56.0	0101	14.7	2.43	281	76.8	44.5	01055
79	37.1	0078	18.2	2.33	257	76.3	47.9	00978
80	43.9	0099	18.3	2.33	302	70.8	50.7	00941
								00979

TABLE XIV
SERIES NO. 12 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.
	G	W	H _d	r	H ¹	S	E	Q
81	51.4	.0121	18.3	2.33	317.	70.8	48.3	.01028
82	29.1	.0076	21.6	2.21	332.	66.8	54.2	.00956
83	38.0	.0103	21.4	2.23	347.	65.6	51.6	.01010
84	43.9	.0132	22.0	2.21	381.	64.6	48.3	.01092
85	27.6	.0076	24.6	2.13	382.	60.6	53.9	.00953
86	30.5	.0102	24.8	2.12	401.	60.1	51.7	.00938
87	43.7	.0122	24.8	2.12	355.	60.1	51.8	.00962
88	32.2	.0115	25.0	2.12	417.	59.7	50.1	.01119
89	17.3	.0070	27.3	2.05	465.	56.2	48.7	.01190
91	24.8	.0098	27.4	2.04	451.	56.0	50.5	.01150
92	29.6	.0119	27.8	2.05	451.	56.2	49.3	.01180
93	36.3	.0148	27.4	2.04	457.	55.3	48.9	.01990
94	11.8	.0715	31.2	1.93	682.	49.8	39.5	.01820
95	12.8	.0108	31.0	1.93	579.	50.0	48.0	.01400
96	26.1	.0144	31.5	1.92	573.	49.3	45.7	.01400
97	18.8	.0133	34.1	1.84	686.	45.2	45.4	.01650
98	19.8	.0124	33.8	1.85	890.	45.1	31.7	.01560
99	12.4	.0173	37.2	1.75	886.	40.3	35.0	.02140
100	20.5	.0163	34.1	1.84	774.	46.3	42.2	.01860
103	19.8	.0280	38.7	1.65	1140.	37.8	28.3	.0293

TABLE XIV
SERIES NO. 12 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, percent.	Volume free air per foot of lift, cubic feet.
	G	W	H _d	r	H _m	S	E	Q
104	39.1	.0131	24.0	2.09	334.	61.3	50.8	.0112
105	17.9	.0051	21.3	2.17	354.	65.7	50.3	.0107
106	90.5	.0161	7.6	2.68	281.	87.7	22.5	.0186
107	90.6	.0193	6.9	2.63	339.	88.9	17.5	.0247
108	71.0	.0059	6.2	2.69	152.	90.0	33.9	.0121
109	82.7	.0112	6.6	2.68	214.	89.3	26.9	.0164
110	90.4	.0158	6.9	2.67	275.	88.9	21.0	.0196
111	70.3	.0076	2.7	2.79	177.	95.9	12.9	.0314
112	89.5	.0109	4.1	2.75	196.	93.4	17.4	.0227
113	33.3	.0023	6.1	2.69	111.	90.1	46.0	.00897
114	39.4	.0034	6.6	2.68	137.	89.3	40.4	.01055
115	56.9	.0053	6.6	2.68	147.	89.3	37.7	.0122
116	33.2	.0053	3.1	2.78	118.	95.0	22.1	.01860
117	85.2	.0032	1.1	2.87	103.	99.7	35200	
119	69.1	.0045	3.0	2.78	106.	95.1	23.8	.01750
120	53.2	.0045	6.6	2.68	133.	89.2	41.8	.01020
121	56.9	.0034	3.0	2.68	93.	95.1	26.9	.01530
122	50.4	.0023	2.6	2.80	113.	93.7	23.5	.01450
123	22.4	.0022	10.6	2.83	163.	88.2	38.2	.00637
127	21.3	.0034	12.8	2.52	241.	73.3	44.4	.01025

TABLE XIV
SERIES NO. 12 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used, per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per foot of lift, cubic feet.
	G	W	h _d	r	H	s	E	Q
128	18	.0032	15.8	2.43	411.	73.0	32.5	.01455
129	18	.0025	13.7	2.49	365.	77.2	31.3	.02670
130	12.8	.0030	14.1	2.48	394.	77.2	31.3	.01350
131	10.8	.0031	10.7	2.58	297.	82.2	30.3	.01450
132	9.9	.0023	14.7	2.46	433.	76.3	25.0	.01870
133	29.0	.0023	5.7	2.74	128.	90.8	37.3	.01100
134	37.1	.0020	3.6	2.80	83.	94.2	33.3	.01200
135	25.9	.0028	9.2	2.63	169.	81.2	34.9	.00948
136	22.5	.0041	14.2	2.48	259.	77.1	45.9	.01050
137	19.9	.0022	9.7	2.59	168.	84.4	48.3	.00914
138	31.7	.0034	9.9	2.59	165.	84.1	50.2	.00875
139	40.1	.0041	9.2	2.61	156.	85.1	42.5	.00884
140	48.0	.0048	8.4	2.63	154.	86.5	45.8	.00951
141	53.5	.0056	8.5	2.63	163.	86.4	43.8	.00992
142	18.8	.0020	10.3	2.56	163.	83.5	53.1	.00841
143	35.5	.0031	9.2	2.59	132.	85.3	58.7	.00752
144	45.1	.0045	9.7	2.57	149.	84.4	54.7	.00812
145	54.9	.0058	9.6	2.58	156.	84.5	50.5	.00833
146	21.4	.0022	11.9	2.52	157.	82.2	59.0	.00721
147	38.4	.0039	10.3	2.55	152.	83.5	56.9	.00753

TABLE XIV
SERIES NO. 12 (Continued)

1	2	3	4	5	6	7	8	9
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.
G	W	h _d	r	H'''	S	E	Q	
148	55.5	.0064	10.2	2.55	173	83.5	49.3	.00911
149	56.5	.0026	14	2.43	234	77.5	50.6	.00950
150	29.3	.0041	13.5	2.45	201	78.5	56.5	.00832

TABLE XV
 SERIES NO. 13
 Diameter of Education Pipe 1 inch. Foot-piece C. Length of Education Pipe 62.43 feet. General Arrangement of Apparatus as in Figure 3.
 Average Temperature of Water 12.5° C.

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon of lift, cubic feet.	Average volume of air in education pipe, cubic feet per second.	Average velocity of mixture in edu- cation pipe, feet per second.	Average volume of air 449V _{av} G	Loss of head per ft. of pipe, feet of water.
1	5.05	.00399	34.2	1.86	791.0	45.3	36.0	.01850	.0386	8.3	3.40	.95
2	5.01	.00240	30.7	1.96	517.0	50.9	49.5	.01250	.0225	5.6	2.00	.38
3	4.94	.00187	28.0	2.04	433.0	55.2	53.8	.01085	.0172	4.7	1.60	.38
4	6.54	.00183	22.1	2.22	355.0	64.6	51.8	.01060	.0158	5.1	1.10	.32
5	7.45	.00183	18.7	2.33	332.0	70.1	46.8	.01050	.0156	5.4	.94	.33
6	7.45	.00244	22.3	2.22	418.0	64.4	44.3	.01180	.0215	6.4	1.29	.44
7	7.47	.00258	24.9	2.14	418.0	60.1	49.7	.01105	.0230	6.6	1.33	.39
8	6.47	.00500	30.2	1.98	1031.0	51.6	34.4	.02510	.0561	11.7	3.98	1.45
9	5.24	.00598	33.2	1.89	1160.0	46.8	23.9	.02750	.0574	11.5	4.82	1.66
10	3.71	.00599	37.9	1.75	1414.0	39.4	22.3	.03360	.0576	11.0	6.84	2.09
11	1.45	.00595	42.6	1.60	1832.0	31.8	19.4	.04580	.0626	11.3	11.50	2.80
12	1.28	.00595	46.4	1.99	2945.0	25.8	13.1	.08030	.0651	11.3	22.80	4.88
13	3.03	.00453	41.1	1.64	1175.0	33.6	29.4	.02900	.0481	9.2	7.13	1.58
14	7.54	.00438	26.5	2.12	698.0	59.2	30.5	.01830	.0393	9.4	2.34	.91
15	6.52	.00438	29.1	2.01	751.0	53.5	32.3	.01850	.0405	9.2	2.79	.95
16	5.41	.00438	33.1	1.89	825.0	47.0	33.4	.01960	.0420	9.1	3.48	1.03
17	3.80	.00435	37.8	2.32	1000.0	39.6	31.5	.02370	.0436	8.7	5.01	1.30
18	8.98	.00435	18.9	2.32	631.0	69.9	24.1	.02060	.0370	9.5	1.85	.68
19	9.94	.00373	17.8	2.36	513.0	71.6	28.8	.01690	.0316	9.0	1.43	.68
20	7.57	.00370	24.8	2.14	595.0	60.3	34.8	.01580	.0330	8.3	1.93	.73

TABLE XV
SERIES NO. 13 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of Air supplied, pounds	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air 449V _{av} G	Loss of head per ft. of pipe, feet of water.
	G	W	P _q	F	H'''	S	E	Q	V _{av}	V _{av}	G	H'
21	11.03	.00305	13.0	2.50	404.0	79.3	26.7	.01710	.0249	8.3	1.01	.55
22	12.1	.00306	11.2	2.55	680.0	82.1	13.8	.03230	.0458	12.2	1.65	1.07
23	5.46	.00345	32.7	1.90	1023.0	47.6	26.6	.02440	.0520	10.7	4.27	1.40
24	2.19	.00374	43.4	1.58	1913.0	30.5	18.9	.04840	.0608	10.9	12.41	2.95
25	5.36	.00373	32.9	1.89	1090.0	47.3	25.1	.02600	.0548	11.1	4.59	1.53
26	8.59	.00595	23.1	2.19	870.0	63.0	22.1	.02410	.0524	11.9	2.74	1.25
27	5.82	.00331	20.0	2.29	743.0	68.0	22.4	.02250	.0465	11.3	2.17	1.07
28	12.85	.00572	12.0	2.53	637.0	80.7	14.6	.03080	.0465	12.3	1.69	1.08
29	4.23	.00310	35.1	1.80	1347.0	40.5	22.3	.03180	.0617	11.9	6.51	1.97
30	3.64	.00548	38.3	1.73	1352.0	38.6	22.7	.03140	.0565	10.8	6.98	1.97
31	3.80	.00290	36.7	1.76	707.0	41.3	43.3	.01685	.0295	6.3	3.48	0.77
32	2.93	.00213	37.6	1.76	655.0	39.7	48.0	.01545	.0218	4.7	3.35	0.65
33	3.69	.00305	37.7	1.75	744.0	39.7	42.2	.01800	.0313	6.6	3.81	0.82
34	4.09	.00447	37.0	1.72	951.0	40.8	32.4	.02360	.0433	7.1	4.76	1.21
35	3.39	.00704	36.4	1.74	1604.0	41.8	18.9	.03990	.0675	12.7	7.77	1.65
36	3.39	.00859	37.7	1.75	2275.0	39.6	13.8	.05390	.0881	15.9	11.65	3.69
37	4.13	.00542	36.8	1.78	1209.0	41.0	26.1	.02860	.0551	10.7	5.98	1.70
38	3.13	.01978	37.1	1.77	5780.0	40.5	5.3	.03170	.2015	34.8	28.90	10.14
39	12.91	.00552	10.4	2.58	648.0	83.4	13.3	.03310	.0456	12.4	1.58	1.02
40	10.39	.00341	16.2	2.41	461.0	74.0	29.3	.01680	.0293	8.7	1.26	0.60

TABLE XV
SERIES NO. 13 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume of free air per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air 449V _{av} G.	Volume of water of pipe, feet of water.
	G	W	H _d	r	H _{av}	S	E	Q	V _{av}	V _{av}		H'
41	9.38	.00244	16.4	2.40	364.0	73.7	37.5	.01270	.0211	7.0	1.01	0.58
42	8.98	.00305	20.1	2.29	450.0	67.8	37.1	.01350	.0271	7.8	1.35	0.52
43	8.23	.00357	23.6	2.18	541.0	62.2	36.6	.01550	.0324	8.5	1.77	0.64
44	6.15	.00224	26.1	2.08	425.0	58.2	51.3	.01120	.0204	5.7	1.49	0.39
45	3.74	.00455	38.1	1.74	1078.0	33.9	29.5	.02560	.0469	9.2	5.63	1.43
46	2.22	.00355	43.9	1.56	1140.0	29.7	32.1	.02920	.0387	7.3	7.80	1.47
47	16.17	.00263	9.0	2.61	267.0	85.4	28.2	.01440	.0209	9.5	.58	.34
48	9.91	.00279	9.8	2.59	457.0	84.4	17.9	.02290	.0224	7.4	1.01	.68
49	11.96	.00272	10.5	2.57	366.0	83.2	23.9	.01730	.0220	8.1	.83	.51
51	11.55	.00224	10.4	2.57	312.2	83.4	27.7	.01490	.0180	7.3	.70	.41
52	11.49	.00209	10.1	2.58	287.3	83.8	29.3	.01450	.0168	7.1	.66	.38
53	11.01	.00174	9.2	2.61	259.0	85.4	29.5	.01380	.0140	8.1	.57	.33
54	10.88	.00173	9.2	2.62	261.0	85.4	29.4	.01380	.0139	6.4	.57	.33
55	10.03	.00178	11.8	2.53	281.8	81.1	35.0	.01210	.0149	6.3	.65	.33
56	9.75	.00169	11.6	2.54	275.0	81.3	35.2	.01190	.0138	5.9	.64	.32
57	8.88	.00145	11.6	2.54	259.4	81.3	37.2	.01130	.0119	4.4	.80	.30
58	8.68	.00205	17.1	2.37	347.5	72.6	41.0	.01100	.0172	6.1	.89	.37
59	7.87	.00146	16.1	2.40	276.0	74.2	48.6	.00925	.0122	5.0	.69	.25
60	7.43	.00150	17.3	2.36	296.5	72.3	48.7	.00937	.0125	4.9	.76	.27

TABLE XV
SERIES NO. 13 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in edu- ction pipe, feet per second.	Average volume of air 449V _{av}	Loss of head per ft. of pipe, feet of water.
	G	W	h _d	r	H'''	S	E	Q	V _{av}	V _{av}	G	H'
61	7.16	.00125	16.0	2.70	282.0	74.4	50.8	.00375	.0103	4.4	.65	.23
62	6.08	.00133	16.4	2.39	251.4	73.7	54.3	.00323	.0086	3.7	.63	.21
63	5.38	.00139	17.1	2.22	210.0	72.6	52.1	.00354	.0084	3.4	.70	.23
64	5.33	.00131	20.4	2.27	346.0	67.4	49.2	.00370	.0113	3.9	.93	.32
65	4.17	.00111	20.4	2.21	372.0	67.4	45.6	.01040	.0095	3.1	1.03	.37
66	3.16	.00097	24.5	2.15	399.0	70.8	51.2	.01000	.0086	2.6	1.22	.36
67	2.18	.00093	28.5	2.02	511.0	51.3	46.5	.01190	.0085	2.7	1.76	.30
68	2.17	.00114	33.8	1.87	541.0	43.8	56.3	.01230	.0112	2.7	2.31	.31
69	2.17	.00143	37.6	1.75	629.0	38.8	49.8	.01600	.0141	2.2	2.93	.33
70	2.13	.00209	40.2	1.61	781.5	32.4	42.8	.01910	.0219	4.5	4.52	.78
71	1.98	.00237	43.6	1.56	856.0	30.2	42.5	.02200	.0251	4.9	5.68	1.05
72	4.08	.00156	30.1	1.97	445.0	51.3	56.4	.01040	.0146	3.9	1.61	1.35
73	1.13	.00249	48.5	1.42	1275.0	22.3	31.7	.03630	.0278	5.2	11.00	1.62
74	1.15	.00205	47.3	1.45	1113.0	24.1	35.4	.03020	.0227	4.2	8.87	1.37
75	1.13	.00177	46.1	1.49	1047.0	26.2	36.7	.02720	.0194	3.7	7.71	1.26
76	1.15	.00141	44.1	1.55	905.0	29.4	40.6	.02230	.0151	2.9	5.88	1.02
77	1.16	.00117	40.8	1.65	835.0	34.6	40.8	.01970	.0121	2.4	4.68	.96
78	1.13	.00111	39.1	1.70	866.0	37.4	37.6	.02010	.0128	2.6	5.09	1.04
79	2.59	.00269	42.3	1.60	845.0	32.2	41.8	.01960	.0282	5.7	4.90	.89

TABLE XV
SERIES NO. 13 (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
Run Number	Discharge of water in gallons/min.	Quantity of air sup- plied, pounds per second.	Delivery head, feet.	Ratio of expansion of air.	Energy of expan- sion of air used per gallon of water pumped, foot pounds.	Submergence, per cent.	Efficiency of pump, per cent.	Volume free air per gallon water per foot of lift, cubic feet.	Average volume of air in suction pipe, cubic feet per second.	Average velocity of mixture in pipe, feet per second.	Average volume of air Volume of water	Loss of head per ft. of pipe, feet of water.
	G	W	h_d	r	H	S	E	Q	V_{av}	V_{av}	$449V_{av}$	H'
80	2.50	.00186	39.6	1.68	795.0	36.5	41.6	.0149	.01920	4.1	3.44	.61
81	2.41	.00155	37.0	1.77	608.0	40.6	50.8	.01380	.0154	3.5	2.87	.57
82	2.38	.00122	34.3	1.84	521.0	45.0	55.0	.01190	.0118	2.9	2.23	.45
83	3.33	.00262	38.5	1.73	716.0	38.2	44.8	.01640	.0268	5.7	3.61	.76
84	3.42	.00186	34.8	1.84	549.0	44.2	52.9	.01250	.0183	4.3	2.40	.50
85	3.34	.00145	31.5	1.94	481.0	49.5	54.7	.01100	.0140	3.6	1.88	.42
86	4.55	.00284	37.1	1.86	645.0	45.7	44.2	.01450	.0278	6.3	2.74	.69
87	4.56	.00207	31.3	1.94	501.0	49.7	52.1	.01160	.0197	5.0	1.94	.46
88	4.45	.00155	37.4	2.06	418.0	56.1	54.6	.01010	.0143	4.0	1.43	.37
89	4.52	.00131	24.8	2.14	367.0	60.3	55.7	.00935	.0118	3.7	1.18	.31
90	1.07	.00220	39.4	1.39	1182.0	20.9	34.8	.03420	.0251	4.6	10.58	1.40
91	1.05	.00172	46.9	1.47	1060.0	24.9	36.9	.02780	.0191	3.6	8.23	1.28
92	1.70	.00415	47.0	1.47	1570.0	24.7	25.0	.04150	.0463	8.3	12.23	1.94
93	1.66	.00294	46.9	1.47	1140.0	25.0	34.3	.03640	.0326	6.1	8.85	1.45
94	1.65	.00175	43.5	1.57	798.0	30.3	45.4	.01940	.0187	3.7	5.08	.84
95	2.81	.00393	43.2	1.59	1080.0	30.9	33.3	.02300	.0420	8.0	6.72	1.37
96	2.78	.00346	42.6	1.60	982.0	31.8	36.1	.02330	.0365	7.1	5.90	1.30
97	2.73	.00225	40.4	1.66	701.0	35.2	48.1	.01630	.0233	4.9	3.84	.69
98	2.53	.00351	40.1	1.68	855.0	35.7	39.1	.01980	.0364	7.4	4.64	.99
99	3.58	.00288	38.8	1.71	721.0	37.8	45.0	.01660	.0307	6.4	3.87	.77

TABLE XVI

Source of Test Data	Length of Education Pipe in Feet.	Diameter of Education Pipe in inches.	Lift in feet.	Submergence in Feet.	Per Cent Submergence	Gallons of Water per Minute.	Cubic Feet of Free Air per Foot of Lift.	Ratio (Average Volume of Free Air to Volume of Water)	Average velocity of the mixture.	Loss in Head per foot of Pipe.	Efficiency of Pipe in Percent.
Mining and Metallurgy No. 158, Sec. 35 by S. F. Shaw "Tiro General Mine Air Lift."	370	6	181	190	51	340	.00365	1.87	10.1	.445	57.6
	370	6	192	178	48	392	.00385	1.92	12.8	.472	54.0
	430	6	214	216	50	450	.00313	1.60	12.8	.342	62.7
	495	6	300	195	39	307	.00507	3.07	16.2	.895	41.3
	575	6	354	221	38	255	.00472	3.57	15.3	.885	41.3
	615	6	396	219	36	270	.00464	4.27	15.6	.896	42.9
San Fernando Air Lift, 5" Spiral Riveted Education Pipe											
Mining and Metallurgy No. 158, Sec. 35 by S. F. Shaw "Tiro General Mine Air Lift."	453	5	239	214	47	278	.00476	2.88	17.6	.715	41.7
	494	5	280	214	43	237	.00572	4.03	19.5	1.031	34.7
	532	5	340	212	38	194	.00556	4.82	18.5	1.092	35.7
	572	5	372	200	35	166	.00553	6.37	20.1	1.420	30.9
	572	5	406	166	29	96	.00666	8.03	14.1	1.566	33.6
	575	5	234	350	60	120	.00843				50.0
Harris' Compressed Air	Composed of										
	328	3						95	12.4	.754	
Iverson Pumping by Compressed Air	247	4						4.58	19.0	1.500	
	320	3 1/2	112	208	65	120	.00534	1.466	12.2	.572	37.0

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BULLETIN OF THE UNIVERSITY OF WISCONSIN

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**COMPARATIVE TESTS OF BUTTON HEAD AND
COUNTERSUNK RIVETED JOINTS**

By

J. B. KOMMERS

Associate Professor of Mechanics

The University of Wisconsin
ENGINEERING EXPERIMENT STATION

Researches in Applied Mechanics
EDWARD R. MAURER
Professor of Mechanics

MADISON
1925

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COMPARATIVE TESTS OF BUTTON HEAD AND COUNTERSUNK RIVETED JOINTS

INTRODUCTION

Inception of Present Tests.—A number of years ago F. E. Schmitt, associate editor of the *Engineering News-Record*, suggested that the strength of countersunk rivets would be a desirable subject for investigation.

During the school year 1922-23 Frank L. Bumer and Ora C. Ralibitt, civil engineering seniors at the University of Wisconsin, carried on a series of tests on button head and countersunk riveted joints as a project for a senior thesis. Tensile tests were made on a series of fifty-four joints.

During the summer of 1923 a second series of ninety-six joints was tested (some in tension and some in bending) by the writer; and during the summer of 1924 a third series of eighteen joints was tested in tension by Carl L. Neumeister and Rufus S. Phillips, instructors in the Mechanics Department at the University.

Outline of Tests.—The first series of tests was confined to joints fastened together by a single rivet for the lap joints, and by one rivet on each side for the single covered butt joints. The lap joints included three different plate thicknesses, and five different rivet diameters.

The second series of tests was limited to one diameter of rivet and one thickness of plate, but included lap joints and single covered and double covered butt joints. Lap joints were made with two and with three rivets; single and double covered butt joints with one, two, and three rivets on each side.

The third series was a continuation of the second series and consisted of double covered butt joints with two and with three rivets on each side.

Since the investigation was concerned primarily with the comparison in strength of button head and countersunk rivets, the joints were designed so that failure would occur by shearing of the rivets. In a few cases, however, failure occurred by tension in the plates.

For each series of tests auxiliary tension tests were made on the plate material, and tension and shear tests on the rivet material.

Acknowledgement.—Thank are due the Wisconsin Bridge and Iron Company of North Milwaukee and the Worden-Allen Company of Milwaukee for defraying the larger part of the expense of the first and second series of test specimens. The Wisconsin Bridge and Iron Company not only fabricated the test specimens for all three series of tests, but also defrayed the larger part of the expense of the third series of tests.

Credit is due Frank L. Bumer and Ora C. Rabbitt for their conscientious work in carrying out the first series of tests, and to Carl L. Neumeister and Rufus S. Phillips for carrying out the third series of tests.

Thanks are due Professor M. O. Withey of the University of Wisconsin, for his helpful suggestions in the planning and carrying out of the tests.

PREVIOUS TESTS AND PRESENT SPECIFICATIONS

R. Fleming, of the American Bridge Company, discusses countersunk rivets in an article in *Engineering News* for September 14, 1916. This article was the outcome of a research of the literature on the strength of countersunk rivets. In the following discussion this article will be freely referred to, but in each case the original sources of information have been examined, and detailed information regarding tests will be given where such information seems desirable.

Fleming says that the usual American practice for countersinking rivets in bridge and building work is to make the depth of the head one-half the diameter of the shank, and the slope 30° from the axis of the shank. The practice is different in shipbuilding. The present (1925) regulations for shipbuilding of the English "Lloyd's Register of Shipping" require that, where rivet holes are countersunk, the countersinking is to extend through the whole thickness of the plate where the thickness of the plate does not exceed 0.50 in. Above this thickness the countersinking is to extend through nine-tenths the thickness of the plate. The angle of countersink for plates whose thickness does not exceed 0.50 in. is to be about 60° , and for plates exceeding this thickness the angle is to be about 45° .

At the present time (1925) the Bureau of Construction and Repair of the United States Navy Department has the following rule:

In plates and shapes of thickness corresponding to a nominal weight of $12\frac{1}{2}$ lb. or less, the depth of countersink may be the full thickness of the material; in plates and shapes of greater thickness the depth of countersink shall be approximately $\frac{1}{8}$ in. less than the thickness of the material.

The slope varies according to the diameter of the rivet.

The use of countersunk rivets is permitted wherever flush surfaces are required or where the heads of rivets may require calking for water tightness. No distinction is made between the allowable unit shearing stress for countersunk rivets and other types of rivets.

Fleming refers to a paper on "Experimental Inquiry Into the Strength of Wrought-Iron Plates and Their Riveted Joints as Applied to Shipbuilding and Vessels Exposed to Severe Strains," by William Fairbairn in the *Philosophical Transactions of the Royal Society of London* for 1850. Fairbairn says:

The system of countersinking the rivets is only used when smooth surfaces are required; under other circumstances their introduction would not be desirable as they do not add to the strength of the joint but to a certain extent reduce it. This reduction is not observable in the experiments, but the simple fact of sinking the head of the rivet into the plate and cutting out a greater portion of metal must of necessity lessen its strength, and render it weaker than the plain joint with raised heads. This must appear evident from the fact of the sectional area of the plate being diminished, and the consequent reduction of the heads of the rivets, which in this state are less able to sustain the effects of an oblique or transverse strain.

It is, however, satisfactory to observe that countersinking the heads of the rivets does not seriously injure the joint in its powers of resistance to direct tensile force; but the rivets are liable to start when exposed to collisions or a strong impinging force, such as the sides of ships are frequently doomed to encounter.

On referring to experiments (Table XI), the same results as to strength are obtained with the countersunk rivets as those with rounded heads; they are rather under the mean of the former experiments; but not more than is easily accounted for by the reduced section of the countersunk plates.

It should be pointed out in connection with Fairbairn's statement regarding the reduction in tensile strength of the plate due to the

removal of metal, that tensile tests¹ of plates with reamed or drilled holes have shown about 11% higher ultimate unit stress than solid plates, and that plates with punched holes have shown about the same strength as solid plates. While, therefore, countersinking decreases the net area in tension, this decrease is accompanied by an improvement, due to the removal of metal which has been damaged in the process of punching the rivet hole.

Fairbairn tested only two joints with countersunk rivets. One joint was a single covered butt joint, countersunk on the cover side and button heads on the plate side, with three rivets in a row across the joint. The rivets were $\frac{3}{8}$ in. in diameter and the plates were $\frac{3}{8}$ in. wide and 0.22 in. thick. This joint failed in tension across the rivet holes at a load of 16,351 lbs.

A second joint was the same as the first but had five rivets arranged in two rows. This joint failed by tearing across the three rivet holes at a load of 23,707 lbs.

A third joint was the same as the first but was made with button head rivets. This joint failed by shearing the rivets at a load of 16,351 lbs.

Fairbairn says that double covered butt joints are seldom used in shipbuilding, but he advises their use whenever it is possible.

Fleming's article mentions next a paper on "The Strength of Plates and Rivets Used in Shipbuilding," by J. G. Wildish, in the Transactions of the *Institution of Naval Architects*, 1885.

Wildish first tested a series of eight single covered butt joints using iron rivets. In four of the joints there were four $\frac{3}{4}$ in. rivets on each side, two joints being riveted with pan heads and countersunk points, and two joints riveted with pan heads and snap points. It should be stated here that a pan head is so named because its shape is like an inverted pan. The "point" of the rivet is the end which is upset when the rivet is driven. The "snap" point is like the ordinary button head.

All of these joints failed in tension. The $\frac{3}{4}$ in. riveted joints gave an average ultimate strength of 28.77 tons per sq. in. for the countersunk points, and 25.05 tons per sq. in. for the snap points. The $\frac{7}{8}$ in. riveted joints gave an average ultimate strength of 29.03 tons per sq. in. for the countersunk points, and 24.75 tons per sq. in. for the snap points.

¹Proc. Amer. Eng. and Main of Way Assoc., 1905

Wildish next tested a series of joints with steel rivets, using pan heads with snap points, pan heads with countersunk points, and countersunk heads and points. These specimens were made with $\frac{3}{4}$ in., $\frac{7}{8}$ in., and 1 in. rivets in single covered butt joints with four rivets on each side, two in a row.

For the joints with pan heads and countersunk points there were four tests made for each size of rivet.

For the joints with countersunk heads and points four tests were made with the $\frac{7}{8}$ in. rivets, and two tests each with the $\frac{3}{4}$ in. and 1 in. rivets. The $\frac{3}{4}$ in. and $\frac{7}{8}$ in. rivets were all drawn through the holes, while the 1 in. rivets sheared.

For the joints with pan heads and snap points two tests were made on each size of rivet.

The following results were obtained:

	Ultimate Load in tons.		
	$\frac{3}{4}$ in. rivet	$\frac{7}{8}$ in. rivet	1 in. rivet
Pan heads, countersunk points	11.5	15.25	20.25
Countersunk heads and points	11.0	13.6	20.9
Pan heads, snap points	11.0	14.75	19.0

Wildish says:

Half of these tests had the countersinking stopped $\frac{1}{8}$ in. short of the full thickness of the plates, and in the other half the countersinking extended right through; but the results were on the whole as good one way as the other, and there was but little variation in them individually.

These results indicate that for the 1 in. rivets with countersunk heads and points in which the rivets were not drawn through the holes, the strength is greater than for the other two types of riveting. For the other two sizes of rivet the strength of the rivets with pan heads and countersunk points is greater than for pan heads and snap points.

Wildish also made some tests on rivets with hexagonal heads with screw points, and countersunk heads with screw points. Those with the countersunk heads were stronger by about 16%.

He also made some tests on hammered countersunk rivets, some with countersunk points and some with snap points. The ones with countersunk points were about 6% stronger.

He describes also some tests made on friction resistance of riveted joints. These joints were made with a center plate and two outside plates. The center plate was slotted in front of the

rivets so that the rivets would not be sheared when the load was applied. Four tests were also made with $\frac{3}{4}$ in. plates and 1 in. rivets, and four tests with $\frac{1}{2}$ in. plates and $\frac{3}{4}$ in. rivets, there being in each case three rivets across the joint.

The results were as follows:

	Frictional Stress per Rivet, tons.	
	$\frac{3}{4}$ in. plates 1 in. rivets	$\frac{1}{2}$ in. plates $\frac{3}{4}$ in. rivets
Snap head and point	6.4	4.72
Pan head and conical point	7.37	4.52
Pan head and countersunk point	8.55	6.25
Countersunk head and point	9.04	4.95

It will be noted that both the pan head with countersunk point and the countersunk head and point gave higher results than the other types of rivets.

It is of interest to note the attitude of ship constructors toward the use of countersunk rivets. Wildish makes the statement: "The pan headed and countersunk pointed rivet is, as is well known, commonly used in the outside plating, upper deck, and other important parts of a ship's structure."

W. H. White, in the discussion upon the above paper by Wildish and another by J. T. Milton stated:

It was my duty, having had to do with these experiments throughout, to draft the instructions for the treatment of mild steel on which the Admiralty have worked, and I believe are still working; and since I have been outside the Admiralty I have again looked very carefully and independently into the matter, and it is my belief that the practice of a countersunk point with the countersinking going right through the thickness of the plate, is for steel work, in all important parts of the structure the most certain and trustworthy method of fastening. I should like also to say this—it really means very little additional expense.

N. Barnaby, director of naval construction, in a paper on "The Use of Steel in Naval Construction," in the proceedings of *The Iron and Steel Institute*, 1879, refers to some tests on $\frac{1}{2}$ in. plates, butt jointed, and treble riveted. The results were as follows:

Breaking Strength of Plates in tons per sq. in.			
		Annealed before riveting	Unannealed before riveting
Countersunk points (Not countersunk through)	Open hearth steel	30.6	29.84
	Converter steel	30.2	21.8
	Best-best iron	—	19.9
Snap points	Open hearth steel	28.6	21.2
	Converter steel	26.9	15.25
	Best-best iron	—	18.3

It will be noted that the countersunk points gave the better results in every case. If the higher strength for the countersunk points is largely due to the fact that countersinking removes most of the metal which has been injured in punching the rivet holes, then annealing the plates should show a smaller difference between the two types of rivets. The experiments seem to bear out this statement.

Barnaby says:

Snap riveting is only to be used for the internal work on transverse bulkheads, floors, framing, and other subordinate parts of the structure; but on stringers, deck plating, and other parts subjected to considerable tensile strain, countersunk riveting is to be used, and the holes treated similarly to those in the outside plating.

This quotation, Barnaby stated, was taken from instructions issued by the director of naval construction.

J. H. Biles in his book *The Design and Construction of Ships*, 1908, gives the results on some $\frac{1}{4}$ in. and $\frac{3}{8}$ in. rivets. Since his results are based on joints which were not exactly alike in the arrangement of the rivets, and since the $\frac{3}{8}$ in. rivets came from various firms, the conclusions from the tests are only of a qualitative kind. For $\frac{1}{4}$ in. countersunk rivets used with $\frac{1}{8}$ in. plates the ultimate shearing strength varied from 14.63 to 20.83 tons per sq. in.; and for the snap headed and pan headed rivets it varied from 18.28 to 18.64 tons per sq. in. For the $\frac{3}{8}$ in. countersunk rivets used with $\frac{1}{8}$ in. plates the ultimate shearing strength varied from 14.87 to 19 tons per sq. in.; and for the pan headed rivets varied from 17.27 to 18.73 tons per sq. in.

Fleming, in the résumé mentioned above, states that the majority of specifications for bridges and buildings give but little if any consideration to countersunk rivets.

The present specifications (1925) prepared by the State Engineer of New York require that: "In metal $\frac{3}{8}$ in. thick and over the depth of the countersink shall be omitted in calculating bearing area. In metal less than $\frac{3}{8}$ in. thick the rivets shall not be assumed to carry stress."

Waddell in his book on *Bridge Engineering* requires that:

Rivets with flat heads shall be preferred to countersunk rivets; the height or thickness of the flat head shall be three-eighths of an inch.

In important members rivets shall not be countersunk in plates of thickness less than one-half of the diameter of the rivet. Rivets with flattened heads shall be assumed to have only eight-tenths of the strength of rivets that have full heads.

The Pennsylvania Railroad Company in its *Standard Specifications for Steel Bridges* requires that: "Rivets with countersunk heads shall be assumed to have three-fourths the value of corresponding rivets with full heads."

The present specifications (1925) of the American Railway Engineering Association for steel railway bridges require the following unit stresses:

Shear in power driven rivets and pins, 12,000 lb. per sq. in.

Bearing on power-driven rivets, pins, outstanding legs of stiffener angles, and other steel parts in contact, 24,000 lb. per sq. in.

The above-mentioned values for shear and bearing shall be reduced 25 per cent for countersunk rivets, hand-driven rivets, floor-connection rivets, and turned bolts.

A special committee of the American Society of Civil Engineers on *Specifications for Design and Construction of Steel Highway Bridge Superstructure*, presented their final report on February 4, 1924. They recommended: "Bearing on countersunk rivets: one-half the depth of countersink shall be omitted in calculated bearing area and, in metal less than $\frac{3}{8}$ in. thick, countersunk rivets shall not be assumed to carry stress."

Other specifications might be quoted, but the above will give some idea of the attitude of engineers of bridges and buildings toward countersunk rivets.

Fleming recommends the following:

Rivets with countersunk heads shall be assumed to have three-fourths the value of corresponding rivets with full heads, but no value shall be allowed for countersunk rivets in plates of a thickness less than one-half the diameter of the rivet.

From these quotations it will be noted that engineers concerned with bridges and buildings believe that countersunk rivets are not as strong as button head rivets. The statements of the ship constructors, previously quoted, show that these men believe that countersunk rivets are stronger than button head rivets. While the test results given above show that there is not much experimental information available on the comparative strength of countersunk and button head rivets, yet the majority of the results seem to show that there is little basis for the prejudice of the engineer of bridges and buildings against countersunk rivets.

MATERIALS, TESTS, TESTING APPARATUS AND TEST SPECIMENS

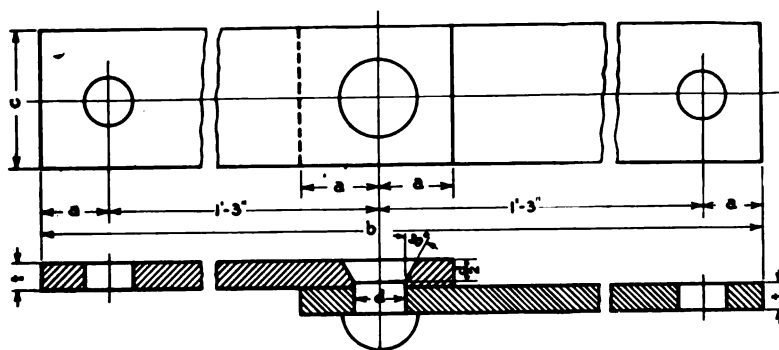
Test Specimens.—Figure 1 shows the types of joints tested in the first series of tests, and gives their principal nominal dimensions. Figures 2, 3, and 4 show the second series of joints, and Figure 5 shows the third series. Each joint was marked so that it could be identified.

The joints were made of open hearth structural steel, and press riveting was employed. The rivet holes in the plates were punched $\frac{1}{8}$ in. small and then reamed to size.

It will be noted that for each type of joint tested there were prepared two specimens, so that some judgment could be formed as to the uniformity of the results. In general the plan was to have six joints for each variable investigated, these consisting of two joints with button head rivets, two joints with rivets having one button head and one countersunk head, and two joints with rivets having two countersunk heads.

The above three types of joint would be described by the ship-builder as snap head and point, snap head and countersunk point, and countersunk head and point.

Tests and Apparatus.—Each tension test was made with an extensometer attached to the joint. The procedure followed was to apply an initial load of 1000 lb., take readings, apply a load increment, take readings, reduce the load to the initial value, take readings, apply double the first increment, and repeat as before until the load was somewhere near the expected ultimate strength of the joint. The extensometer was then removed and



Joint No.	Thick-ness of Plate	Size of Rivets	Dia. of Holes	Kind of Heads	a	b	c
A	$\frac{1}{4}$ "	$\frac{3}{8}$ "	$\frac{7}{16}$ "	Ctsk. 1	$1\frac{1}{4}$ "	$2'-8\frac{1}{2}$ "	$2\frac{1}{2}$ "
B	$\frac{1}{4}$ "	$\frac{3}{8}$ "	$\frac{7}{16}$ "	Ctsk. 2	$1\frac{1}{4}$ "	$2'-8\frac{1}{2}$ "	$2\frac{1}{2}$ "
C	$\frac{1}{4}$ "	$\frac{3}{8}$ "	$\frac{7}{16}$ "	B. H.	$1\frac{1}{4}$ "	$2'-8\frac{1}{2}$ "	$2\frac{1}{2}$ "
D	$\frac{1}{4}$ "	$\frac{1}{2}$ "	$\frac{9}{16}$ "	Ctsk. 1	$1\frac{1}{4}$ "	$2'-8\frac{1}{2}$ "	$2\frac{1}{2}$ "
E	$\frac{1}{4}$ "	$\frac{1}{2}$ "	$\frac{9}{16}$ "	Ctsk. 2	$1\frac{1}{4}$ "	$2'-8\frac{1}{2}$ "	$2\frac{1}{2}$ "
F	$\frac{1}{4}$ "	$\frac{1}{2}$ "	$\frac{9}{16}$ "	B. H.	$1\frac{1}{4}$ "	$2'-8\frac{1}{2}$ "	$2\frac{1}{2}$ "
G	$\frac{1}{2}$ "	$\frac{3}{8}$ "	$\frac{1}{16}$ "	Ctsk. 1	$1\frac{3}{8}$ "	$2'-8\frac{3}{4}$ "	$2\frac{3}{4}$ "
H	$\frac{1}{2}$ "	$\frac{3}{8}$ "	$\frac{1}{16}$ "	Ctsk. 2	$1\frac{3}{8}$ "	$2'-8\frac{3}{4}$ "	$2\frac{3}{4}$ "
K	$\frac{1}{2}$ "	$\frac{3}{8}$ "	$\frac{1}{16}$ "	B. H.	$1\frac{3}{8}$ "	$2'-8\frac{3}{4}$ "	$2\frac{3}{4}$ "
L	$\frac{1}{2}$ "	$\frac{3}{8}$ "	$\frac{1}{16}$ "	Ctsk. 1	$1\frac{3}{8}$ "	$2'-8\frac{3}{4}$ "	$2\frac{3}{4}$ "
M	$\frac{1}{2}$ "	$\frac{3}{8}$ "	$\frac{1}{16}$ "	Ctsk. 2	$1\frac{3}{8}$ "	$2'-8\frac{3}{4}$ "	$2\frac{3}{4}$ "
N	$\frac{1}{2}$ "	$\frac{3}{8}$ "	$\frac{1}{16}$ "	B. H.	$1\frac{3}{8}$ "	$2'-8\frac{3}{4}$ "	$2\frac{3}{4}$ "
O	$\frac{1}{2}$ "	$\frac{7}{8}$ "	$\frac{1}{16}$ "	Ctsk. 1	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
P	$\frac{1}{2}$ "	$\frac{7}{8}$ "	$\frac{1}{16}$ "	Ctsk. 2	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
R	$\frac{1}{2}$ "	$\frac{7}{8}$ "	$\frac{1}{16}$ "	B. H.	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
S	$\frac{3}{4}$ "	$\frac{3}{4}$ "	$\frac{1}{16}$ "	Ctsk. 1	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
T	$\frac{3}{4}$ "	$\frac{3}{4}$ "	$\frac{1}{16}$ "	Ctsk. 2	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
U	$\frac{3}{4}$ "	$\frac{3}{4}$ "	$\frac{1}{16}$ "	B. H.	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
V	$\frac{3}{4}$ "	$\frac{7}{8}$ "	$\frac{1}{16}$ "	Ctsk. 1	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
W	$\frac{3}{4}$ "	$\frac{7}{8}$ "	$\frac{1}{16}$ "	Ctsk. 2	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
X	$\frac{3}{4}$ "	$\frac{7}{8}$ "	$\frac{1}{16}$ "	B. H.	$1\frac{3}{8}$ "	$2'-9\frac{1}{4}$ "	$3\frac{1}{4}$ "
Y	$\frac{3}{4}$ "	1"	$\frac{1}{16}$ "	Ctsk. 1	2"	$2'-10"$	$3\frac{3}{4}$ "
Z	$\frac{3}{4}$ "	1"	$\frac{1}{16}$ "	Ctsk. 2	2"	$2'-10"$	$3\frac{3}{4}$ "
Aa	$\frac{3}{4}$ "	1"	$\frac{1}{16}$ "	B. H.	2"	$2'-10"$	$3\frac{3}{4}$ "

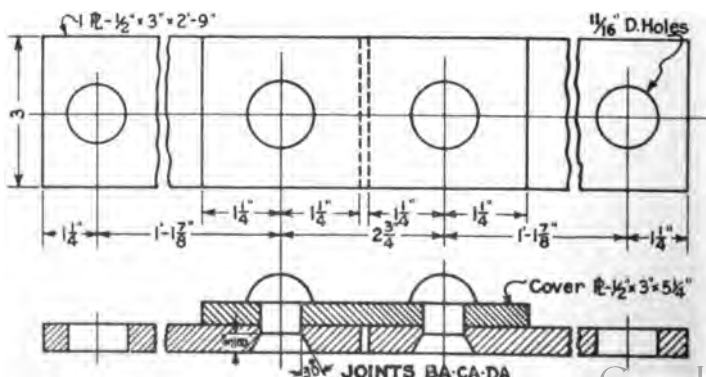


Figure 1—Test Specimens for First Series

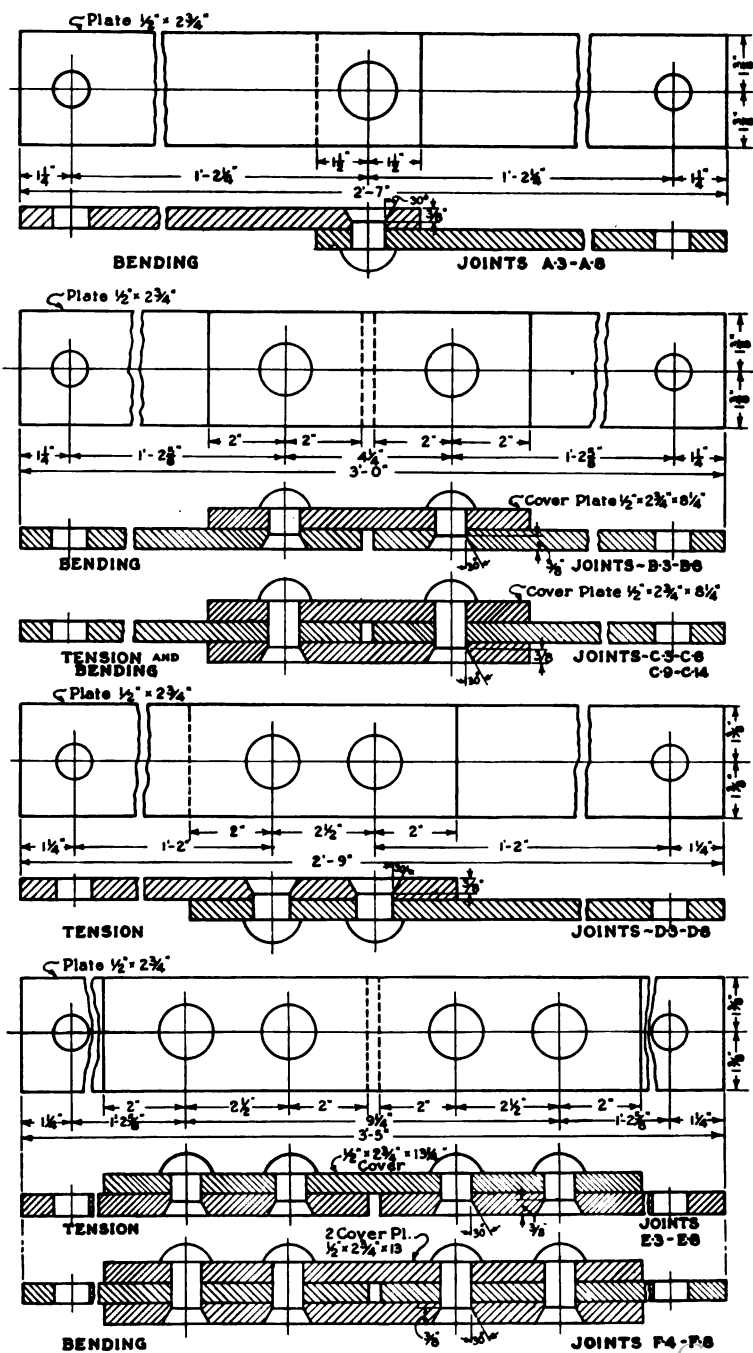


Figure 2—Test Specimens for Second Series

the joint tested to failure. This procedure enabled the slipping in the joint to be determined.

The bend tests which were made on some of the joints of the second series were for the purpose of determining whether there

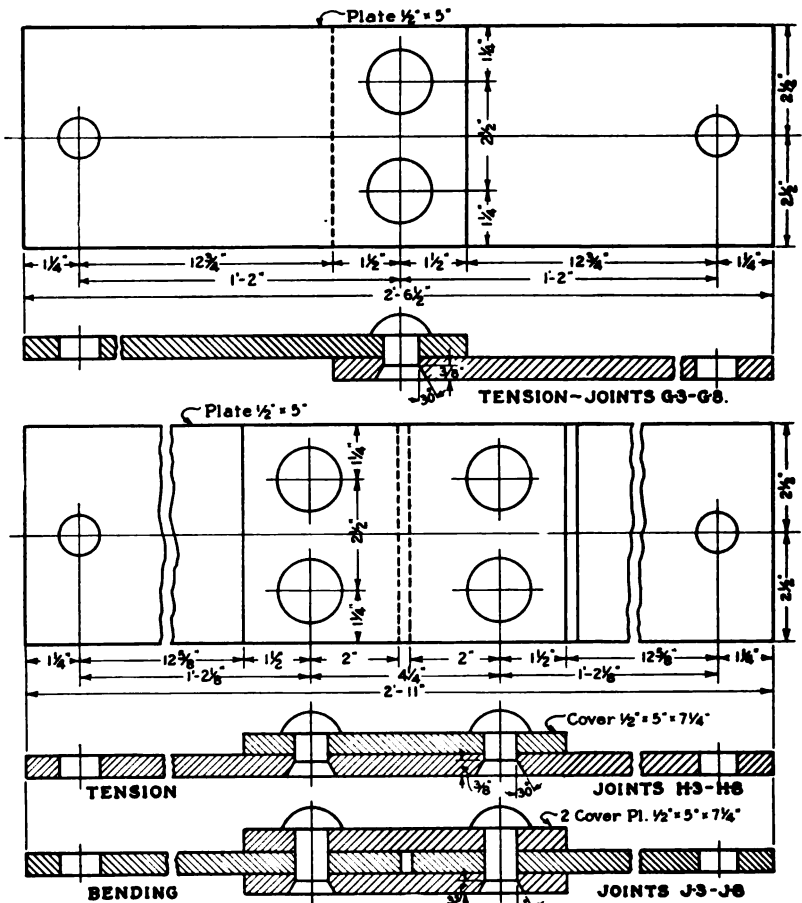


Figure 3—Test Specimens for Second Series

was any difference in the action of button head and countersunk riveted joints when subjected to lateral forces at right angles to the flat side of the joint. In these tests the joint was laid flat on supports and a load applied at the center of the span length. Deflections of the joint at the center of the span were determined by an Ames dial reading to 0.001 in.

Auxiliary tension tests on the plate and rivet material were made according to the standard specification of the American Society for Testing Materials. The percentage of elongation in

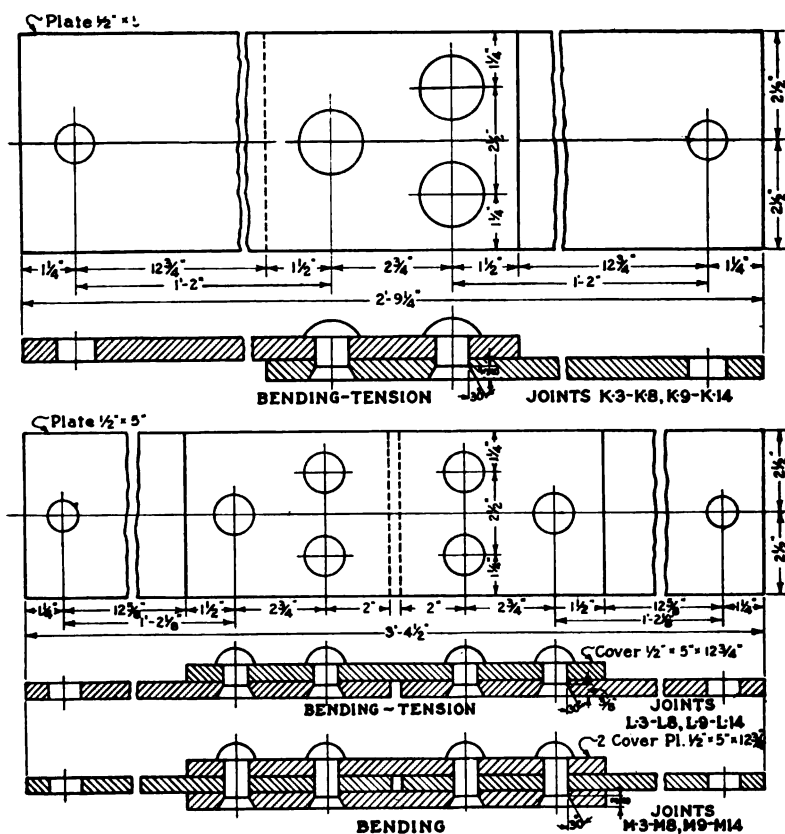


Figure 4—Test Specimens for Second Series

2 in. was determined, and also in 8 in. when the specimen was long enough.

The speed of the testing machine up to the yield point was either 0.03 or 0.06 in. per minute, and from the yield point up to the ultimate 0.55 in. per minute or less.

The auxiliary shear tests on the rivet material were made on specimens of rectangular cross-section cut from the center of the rivet material. The specimens were flat, about 3 in. long, and were tested in double shear. These tests were made merely as

check tests of the material. It was thought that the shear strength of the full size rivets would be very definitely determined by the tests of the joints themselves.

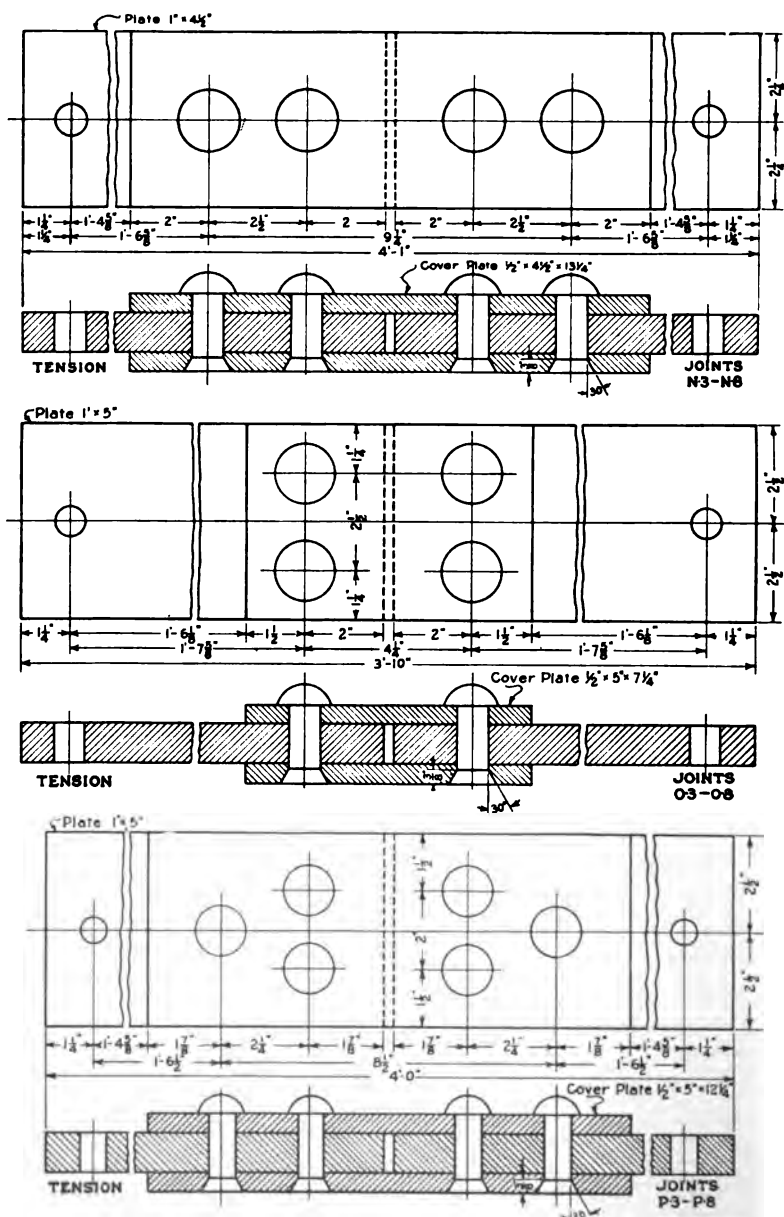


Figure 5—Test Specimens for Third Series

All of the tests were made in a 100,000 lb. Riehle universal testing machine, except those for the third series of joints, which were tested in a 200,000 lb. Riehle universal machine.

It will be noted that each riveted joint was provided with a hole

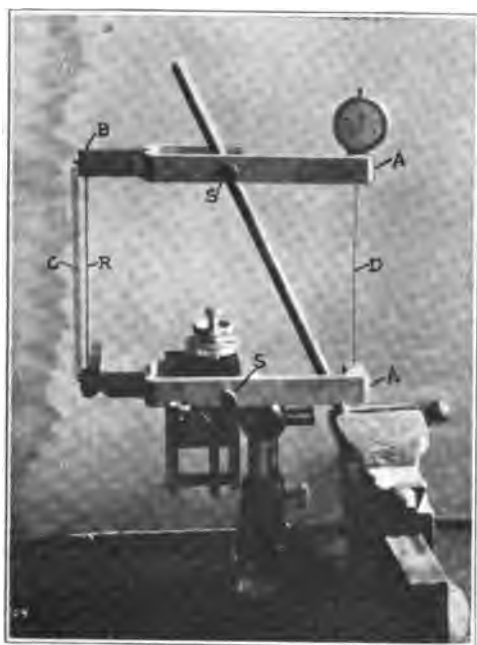


Figure 6 Extensometer in Position to be Calibrated

at each end. During the test, pins slipped through these holes rested against the wedge grips of the testing machine, and thus made it impossible for the specimens to slip in the grips.

The extensometer used in measuring the slip and deformation in the joints is shown in Figure 6. The apparatus consists of two clamps, A, which are fastened to the specimen by means of set screws, S. Fastened securely to the lower clamp is a rod, R, which is pointed at the upper end and rests in a conical socket in and adjustable screw, B. During the test the spring, C, holds the upper clamp against the distance rod, R. The pull rod, D, is fastened to the lower clamp by screws, and at its upper end is screwed into the plunger of an Ames dial which reads to 0.0001 in. Since the screws, S, are midway between the rods R and D,

the dial reading is equal to twice the deformation of the specimen. A gauge bar which holds the clamps, A, the proper distance apart is used in attaching the extensometer to the specimen. This bar is removed after the extensometer has been fastened and adjusted. The correction factor for the extensometer readings was



Figure 7--Extensometer Attached to Riveted Joint

determined by a calibration of the apparatus using a filar micrometer screw. The extensometer readings are correct to about five-tenths of one percent. Figure 6 shows the extensometer in position to be calibrated, while Figure 7 shows the extensometer attached to one of the riveted joints.

In the first series of tests a piece of cross-section paper was glued across the plate edges of the joint near the seam. The paper was cut between the plates of the joint before the joint was tested, and the amount of slip was read directly from the paper. It was found that this method was unsatisfactory because it was not delicate enough. The extensometer method, on the other hand, was found to be very satisfactory. The extensometer

proved to be particularly rugged and reliable, and read correctly even after suffering severe shocks due to the sudden slipping of the plates with respect to each other.

JOINT NO. L3
SINGLE COVERED BUTT—THREE RIVETS

Load	Dial Reading	Slip Inches	Deformation Inches	Unit Shear Stress	Corrected Slip In.	Corrected Deformation Inches
1000	0.3084	0	0	643	0	0
6000	0.3034		0.0050	3880		0.00254
1000	0.3081	0.0003			0.00015	
11000	0.2978		0.0106	7080		0.00539
1000	0.3071	0.0013			0.00066	
16000	0.2900		0.0184	10300		0.00936
1000	0.3046	0.0038			0.00193	
21000	0.2792		0.0292	13500		0.0148
1000	0.2993	0.0091			0.00462	
26000	0.2641		0.0443	16700		0.0225
1000	0.2903	0.0181			0.00921	
31000	0.2472		0.0612	19940		0.0311
1000	0.2794	0.0290			0.0147	
36000	0.2297		0.0787	23200		0.0400
1000	0.2677	0.0407			0.0207	
41000	0.2100		0.0984	26400		0.0500
1000	0.2530	0.0554			0.0282	
46000	0.1748		0.1336	29800		0.0679
1000	0.2232	0.0852			0.0433	
51000	0.0840		0.2244	32800		0.1142
1000	0.1431	0.1653			0.0841	
*1000	0.3087		0.3197			
56000	0.1543			36000		0.1626
1000	0.2232	0.2508			0.1275	
76000	Ultimate			48800		

Note—*New initial setting. At ultimate load the joint failed by shearing the rivets.
Gauge length = 14 in.

Figure 8—Sample Data Sheet of Tensile Test

TEST DATA

Figure 8 shows a sample data sheet such as was prepared during the testing of each joint.

Figure 9 shows sample curves of slip and total deformation of a

joint subjected to tension, and the deflection of a joint subjected to bending. In order that the slip and deformation (terms explained later) might be determined accurately at working unit stresses, slip curves and deformation curves were plotted for all the joints to a much larger scale than that shown in the figure, but the curves were plotted only up to about 12,000 or 15,000 lb. per sq. in. shearing unit stress on the rivets.

Tables No. 1, 2, and 3 give the results of the auxiliary tensile

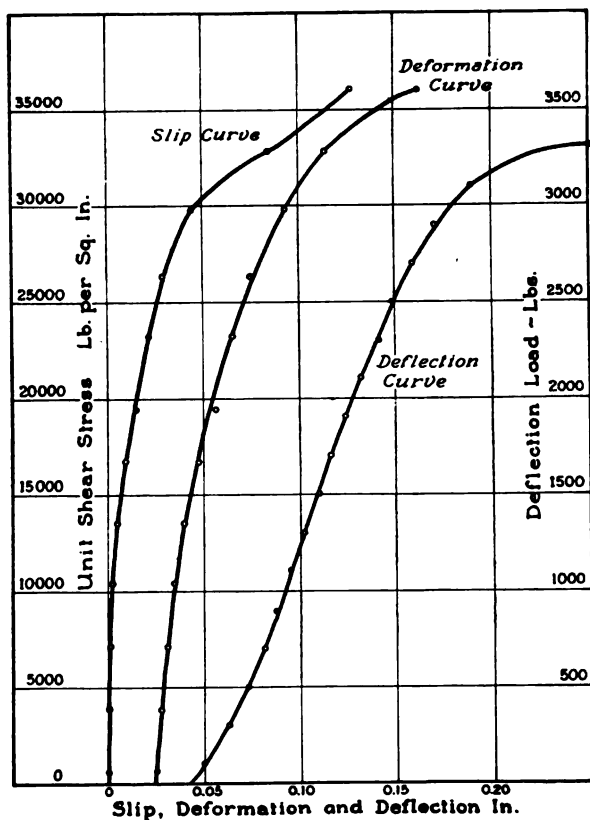


Figure 9—Sample Slip, Deformation, and Deflection Curves

and shear tests made on the material of which the joints were fabricated. The tables are for the first, second, and third series of tests, respectively.

Table No. 4 gives the results for the lap joints of the first

TABLE NO. 1
FIRST SERIES
TENSION TESTS
OF RIVET MATERIAL **SHEAR TESTS OF**
RIVET MATERIAL

1	2	3	4	5	1	2
Nominal Dia. in.	Yield Point Lbs. per Sq. in.	Ultimate Strength Lbs. per Sq. in.	% Elongation in 2 inches	% Reduction of Area	Nominal Dia. in.	Ultimate Strength Lbs. per Sq. in.
$\frac{3}{8}$	37800	59300	41	64.7	$\frac{3}{8}$	50100
$\frac{3}{8}$	39800	59300	40	64.7	$\frac{3}{8}$	49500
Ave	38000	59300	40.5	64.7	Ave	49800
$\frac{1}{2}$	32300	47000	50	71.5	$\frac{1}{2}$	39700
$\frac{1}{2}$	31200	47000	50	70.0	$\frac{1}{2}$	40800
Ave	31800	47000	50	70.8	Ave	40300
$\frac{5}{8}$	34400	52600	55	69.4	$\frac{5}{8}$	41800
$\frac{5}{8}$	34400	52300	55	70.2	$\frac{5}{8}$	40400
Ave	34400	52500	55	69.8	Ave	41100
$\frac{3}{4}$	31600	49000	60	69.1	$\frac{3}{4}$	41900
$\frac{3}{4}$	31300	48800	60	70.7	$\frac{3}{4}$	42600
Ave	31500	48900	60	69.9	Ave	42300
$\frac{7}{8}$	31100	48700	65	74.9	$\frac{7}{8}$	40600
$\frac{7}{8}$	31100	48700	70	74.9	$\frac{7}{8}$	40800
Ave	31100	48700	67.5	74.9	Ave	40700
1	32400	51300	55	58.2	1	42400
1	32100	51200	65	60.2	1	43400
Ave	32300	51300	60	59.2	Ave	42900

TENSION TESTS OF PLATE MATERIAL

1	2	3	4	5	6
Nominal Thickness in.	Yield Point Lbs. per Sq. in.	Ultimate Strength Lbs. per Sq. in.	% Elongation in 2 inches	% Elongation in 8 inches	% Reduction of Area
$\frac{1}{4}$	40900	64300	45	26.9	60.8
$\frac{1}{4}$	39300	65100	45	26.4	54.5
Ave	40100	64700	45	26.7	57.7
$\frac{1}{2}$	40400	52800	55	29.4	56.7
$\frac{1}{2}$	38100	51500	57	30.0	60.6
Ave	39300	52200	56	29.7	58.7
$\frac{3}{4}$	32900	67800	56	29.4	50.3
$\frac{3}{4}$	32700	68800	50	28.2	54.4
Ave	32800	68300	53	28.8	52.4

series of tests. These joints were all made with a single rivet, but the plate thickness and rivet diameter were varied. With $\frac{1}{4}$ in. plates there were used $\frac{3}{8}$ in. and $\frac{1}{2}$ in. rivets; with $\frac{1}{2}$ in. plates, $\frac{5}{8}$ in., $\frac{3}{4}$ in., and $\frac{7}{8}$ in. rivets; and with $\frac{3}{4}$ in. plates, $\frac{3}{4}$ in., $\frac{7}{8}$ in., and 1 in. rivets.

An explanation of the columns of Table No. 4 will help to make all similar tables clear. Practically all the joints failed by

TABLE NO. 2
SECOND SERIES
TENSION TESTS OF RIVET MATERIAL **SHEAR TESTS OF RIVET MATERIAL**

1	2	3	4	5	1	2
Nominal Dia. in.	Yield Point Lbs. per Sq. in.	Ultimate Strength Lbs. per Sq. in.	% Elongation in 2 inches	% Reduction of Area	Nominal Dia. in.	Ultimate Strength Lbs. per Sq. in.
$\frac{3}{8}$	36200	49900	56.0	68.7	$\frac{3}{8}$	41200
$\frac{3}{4}$	32800	49500	54.0	71.9	$\frac{3}{4}$	41400
$\frac{1}{2}$	32000	49100	54.0	71.8	$\frac{5}{8}$	41200
$\frac{1}{4}$	33800	51800	52.5	70.5	$\frac{3}{4}$	41100
Ave	33700	50100	54.1	70.7	Ave	41300

TENSION TESTS OF PLATE MATERIAL

1	2	3	4	5	6
Nominal Thickness in.	Yield Point Lbs. per Sq. in.	Ultimate Strength Lbs. per Sq. in.	% Elongation in 2 inches	% Elongation in 8 inches	% Reduction of Area
$\frac{1}{2}$	37800	59500	47.5	30.0	56.9
$\frac{1}{2}$	38300	59500	52.5	28.2	54.0
$\frac{1}{2}$	38100	66900	36.5	21.9	47.6
$\frac{1}{2}$	34300	66200	46.5	26.3	57.3
Ave	37200	62000	46.3	26.6	54.0

shearing the rivets, and Column 6 gives the ultimate shearing unit stress based on the area of the rivet hole, which the rivet was assumed to fill. Column 7 gives the maximum bearing stress existing in the joint at the ultimate load. The bearing area for button head rivets was taken as the diameter of the rivet holes times the thickness of the plate. For countersunk rivets there was added to the above area the amount of the small triangular portions due to the countersinking. (See Figure 1). Column 8

gives the maximum tensile unit stress existing in the joint at the ultimate load. For button head rivets the tensile area was taken as the width of the plate minus the diameter of the rivet holes, times the thickness of the plate. For countersunk rivets, the net area in tension was further reduced by the amount of the triangular portions due to the countersinking. (See Figure 1). When

TABLE NO. 3
THIRD SERIES
TENSION TESTS OF PLATE MATERIAL **SHEAR TESTS OF RIVET MATERIAL**

1	2	3	4	5	6	1	2
Nominal Thickness in.	Yield Point Lbs. per Sq. in.	Ultimate Strength Lbs. per Sq. in.	% Elongation in 2 inches	% Elongation in 8 inches	% Reduction of Area	Nominal Dia. in.	Ultimate Strength Lbs. per Sq. in.
$\frac{1}{2}$	33800	56700	47.5	25.6	59.5	$\frac{5}{8}$	41400
$\frac{1}{2}$	33900	57400	48.5	26.4	59.4	$\frac{5}{8}$	41300
$\frac{1}{2}$	35600	58100	46.5	26.8	57.4	$\frac{5}{8}$	39900
Ave	34400	57400	47.5	26.3	58.8	Ave	40900
1	33500	57300	59.5	31.9	63.0	$\frac{3}{4}$	37900
1	34300	60500	59.5	31.6	61.0	$\frac{3}{4}$	39400
1	35500	61000	60.0	31.3	60.9	$\frac{3}{4}$	38400
Ave	34400	59600	59.7	31.6	61.6	Ave	38600

TENSION TESTS OF RIVET MATERIAL

1	2	3	4	5	6
Nominal Dia. in.	Yield Point Lbs. per Sq. in.	Ultimate Strength Lbs. per Sq. in.	% Elongation in 2 inches	% Elongation in 8 inches	% Reduction of Area
$\frac{5}{8}$	44300	63400	45.0	29.2	67.5
$\frac{5}{8}$	44700	64000	48.0	30.2	68.0
$\frac{5}{8}$	45400	64800	45.5	28.6	65.6
$\frac{5}{8}$	45100	64700	44.5	27.5	66.9
Ave	44800	64200	45.8	28.9	67.0
$\frac{3}{4}$	48700	77000	43.0	25.6	58.2
$\frac{3}{4}$	50000	79100	36.5	23.1	42.7
$\frac{3}{4}$	50200	77900	40.5	24.4	54.7
$\frac{3}{4}$	50400	79300	42.0	25.0	56.1
Ave	49800	78300	40.5	24.5	52.9

TABLE NO. 4
LAP JOINTS SINGLE RIVETS

1	2	3	4	5	6	7	8	9	10	11	12	13
Test Spec. No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Unit Stress at Ultimate Load - Lbs. per Sq. In.			Slip at Shear Stress of 9000 Lbs. per Sq. In.	Deformation at Shear Stress 9000 Lbs. per Sq. In.	Slip at Shear Stress in Column 11	Deformation at Shear Stress in Col. 12	Shear Stress Lbs. per Sq. In. for Column 11 & 12
					Shearing	Bearing	Tension					
A 1	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	Csk.	51600	70800	17210	0.0001	0.0021	0.0016	0.0090	
A 2	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	1	53200	73000	17300	0.0014	0.0027	0.0052	0.0127	
Ave					52400	71900	17300	0.0008	0.0024	0.0034	0.0108	38000
B 1	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	Csk.	48600	56200	15700	0.0000	0.0001	0.0304	0.0385	
B 2	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	2	59900	69400	19050	0.0002	0.0015	0.0027	0.0083	
Ave					54300	62800	17400	0.0001	0.0008	0.0166	0.0234	38000
C 1	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	B.H.	52200	71500	15700	0.0003	0.0013	0.0030	0.0088	
C 2	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{1}{4}$	B.H.	48200	66300	14300	0.0001	0.0040	0.0132	0.0230	
Ave					50200	68900	15000	0.0002	0.0027	0.0081	0.0159	38000
D 1	$\frac{1}{2}$	$\frac{9}{8}$	$\frac{1}{4}$	Csk.	59900	106000	36800	0.0014	0.0038	0.0107	0.0241	
D 2	$\frac{1}{2}$	$\frac{9}{8}$	$\frac{1}{4}$	1	57700	102000	34600	0.0080	0.0103	0.0256	0.0390	
Ave					58800	104000	35700	0.0047	0.0071	0.0182	0.0315	40000
E 1	$\frac{1}{2}$	$\frac{9}{8}$	$\frac{1}{4}$	Csk.	53300	75000	30400	0.0005	0.0034	0.0133	0.0268	
E 2	$\frac{1}{2}$	$\frac{9}{8}$	$\frac{1}{4}$	2	58500	82400	33800	0.0006	0.0033	0.0106	0.0239	
Ave					55900	78700	32100	0.0006	0.0034	0.0119	0.0253	40000
F 1	$\frac{1}{2}$	$\frac{9}{8}$	$\frac{1}{4}$	B.H.	52600	93300	29200	0.0009	0.0025	0.0166	0.0312	
F 2	$\frac{1}{2}$	$\frac{9}{8}$	$\frac{1}{4}$	B.H.	53100	93800	28800	0.0006	0.0023	0.0176	0.0321	
Ave					52900	93600	29000	0.0002	0.0024	0.0171	0.0317	40000
G 1	$\frac{5}{8}$	$\frac{11}{8}$	$\frac{1}{2}$	Csk.	54300	59800	21700	0.0059	0.0076	0.0267	0.0440	
G 2	$\frac{5}{8}$	$\frac{11}{8}$	$\frac{1}{2}$	1	50900	56100	20700	0.0016	0.0022	0.0223	0.0297	
Ave					52600	58000	21200	0.0038	0.0049	0.0245	0.0369	38000
H 1	$\frac{5}{8}$	$\frac{11}{8}$	$\frac{1}{2}$	Csk.	55200	52000	23000	0.0056	0.0083	0.0224	0.0362	
H 2	$\frac{5}{8}$	$\frac{11}{8}$	$\frac{1}{2}$	2	55200	52100	21700	0.0039	0.0062	0.0192	0.0341	
Ave					55200	52100	22400	0.0047	0.0072	0.0208	0.0351	38000
K 1	$\frac{5}{8}$	$\frac{11}{8}$	$\frac{1}{2}$	B.H.	56300	63400	21800	0.0001	0.0017	0.0155	0.0270	
K 2	$\frac{5}{8}$	$\frac{11}{8}$	$\frac{1}{2}$	B.H.	56600	62400	21500	0.0003	0.0024	0.0229	0.0318	
Ave					56500	62900	21700	0.0002	0.0021	0.0192	0.0294	38000
L 1	$\frac{3}{4}$	$\frac{13}{8}$	$\frac{1}{2}$	Csk.	47900	63700	26700	0.0026	0.0046	0.0349	0.0517	
L 2	$\frac{3}{4}$	$\frac{13}{8}$	$\frac{1}{2}$	1	48600	63300	25700	0.0010	0.0034	0.0340	0.0497	
Ave					48300	63500	26200	0.0018	0.0040	0.0345	0.0507	36000
M 1	$\frac{3}{4}$	$\frac{13}{8}$	$\frac{1}{2}$	Csk.	61100	66100	34500	0.0045	0.0084	0.0196	0.0359	
M 2	$\frac{3}{4}$	$\frac{13}{8}$	$\frac{1}{2}$	2	62100	67200	34600	0.0047	0.0060	0.0157	0.0315	
Ave					61600	66700	34600	0.0046	0.0072	0.0177	0.0337	36000
N 1	$\frac{3}{4}$	$\frac{13}{8}$	$\frac{1}{2}$	B.H.	48900	63800	24000	0.0005	0.0035	0.0225	0.0364	
N 1	$\frac{3}{4}$	$\frac{13}{8}$	$\frac{1}{2}$	B.H.	51100	66600	26200	0.0004	0.0026	0.0170	0.0317	
Ave					50000	65200	25100	0.0005	0.0030	0.0197	0.0341	36000

Note: Gauge length - 8 in.

TABLE NO. 4 (cont)
LAP JOINTS SINGLE RIVETS

1	2	3	4	5	6			7	8	9	10	11	12	13
Test Spec. No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate in Rivet Heads	Kind of Rivet	Unit Stress at Ultimate Load - Lbs. per Sq. In.			Slip at Shear Stress of 9000 Lbs. per Sq. In.	Deformation at Shear Stress 9000 Lbs. per Sq. In.	Slip at Shear Stress in Column 11	Deformation at Shear Stress in Col. 11	Shear Stress Lbs. per Sq. In. for Column 11 & 12		
					Shearing	Bearing	Tension							
O 1	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{1}{2}$	Csk	53000	79700	33700	0.0047	0.0087	0.0189	0.0340	30000		
O 2	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{1}{2}$	I	49300	75800	31700	0.0035	0.0063	0.0269	0.0432			
Ave					51200	77800	32700	0.0041	0.0075	0.0229	0.0386			
P 1	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{1}{2}$	Csk	55300	68200	36800	0.0079	0.0120	0.0202	0.0390	30000		
P 2	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{1}{2}$	2	55700	69500	35100	0.0022	0.0073	0.0190	0.0324			
Ave					55500	68900	36000	0.0050	0.0096	0.0196	0.0357			
R 1	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{1}{2}$	B.H.	46200	69400	26600	0.0002	0.0030	0.0126	0.0247	30000		
R 2	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{1}{2}$	B.H.	45200	67900	25500	0.0009	0.0038	0.0364	0.0533			
Ave					45700	68700	26100	0.0005	0.0034	0.0245	0.0390			
S 1	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{3}{4}$	Csk	48200	38500	16800	0.0002	0.0016	0.0250	0.0432	32000		
S 2	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{3}{4}$	I	45300	37100	15000	0.0046	0.0067	0.0401	0.0525			
Ave					46800	37800	15900	0.0024	0.0042	0.0325	0.0478			
T 1	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{3}{4}$	Csk	44300	31800	14400	0.0008	0.0024	0.0269	0.0370	32000		
T 2	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{3}{4}$	2	60100	43200	19700	0.0010	0.0029	0.0049	0.0160			
Ave					52200	37500	17100	0.0009	0.0026	0.0159	0.0265			
U 1	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{3}{4}$	B.H.	49000	39100	14600	0.0014	0.0024	0.0289	0.0394	32000		
U 2	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{3}{4}$	B.H.	44900	35900	13800	0.0009	0.0016	0.0205	0.0293			
Ave					47000	37500	14200	0.0012	0.0020	0.0247	0.0343			
V 1	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	Csk	49200	45300	18300	0.0009	0.0030	0.0083	0.0179	27000		
V 2	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	I	47800	44000	17200	0.0018	0.0035	0.0104	0.0185			
Ave					48500	44700	17700	0.0013	0.0033	0.0093	0.0182			
W 1	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	Csk	52000	41700	19800	0.0024	0.0044	0.0079	0.0165	27000		
W 2	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	2	51100	41400	20600	0.0053	0.0080	0.0124	0.0229			
Ave					51600	41600	20200	0.0038	0.0062	0.0102	0.0197			
X 1	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	B.H.	47400	44100	16600	0.0000	0.0011	0.0068	0.0167	27000		
X 2	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{3}{4}$	B.H.	46900	43700	17000	0.0006	0.0015	0.0130	0.0222			
Ave					47100	43900	16800	0.0003	0.0013	0.0099	0.0195			
Y 1	I	$1\frac{1}{16}$	$\frac{3}{4}$	Csk	58100	61400	24900	0.0008	0.0026	0.0132	0.0249	30000		
Y 2	I	$1\frac{1}{16}$	$\frac{3}{4}$	I	61200	64800	26800	0.0018	0.0043	0.0118	0.0241			
Ave					59700	63100	25800	0.0013	0.0035	0.0125	0.0245			
Z 1	I	$1\frac{1}{16}$	$\frac{3}{4}$	Csk	61700	56300	28000	0.0034	0.0086	0.0105	0.0264	30000		
Z 2	I	$1\frac{1}{16}$	$\frac{3}{4}$	2	52100	47500	21900	0.0025	0.0075	0.0115	0.0277			
Ave					56900	51900	25000	0.0030	0.0080	0.0110	0.0270			
As 1	I	$1\frac{1}{16}$	$\frac{3}{4}$	B.H.	47900	50000	19700	0.0007	0.0025	0.0196	0.0321	30000		
As 2	I	$1\frac{1}{16}$	$\frac{3}{4}$	B.H.	44000	46500	17000	0.0001	0.0017	0.0250	0.0372			
Ave					46000	48300	18400	0.0004	0.0021	0.0223	0.0347			

Note: Gauge length = 8 in.

the plates in a joint varied in thickness the smallest area in bearing and in tension was used to obtain the maximum tension and bearing stresses.

In order to compare the stiffness or resistance to deformation imparted to the joints by the various types of riveting it was decided that the actual deformation at a working unit stress and the actual deformation at a unit stress near the ultimate would be the best basis of comparison. This decision was based on the fact that the curves are often not straight lines even at low stresses and it would be very difficult to decide what the average slope of the stress-deformation curve really is. A glance at the sample curves will also show that it would be very difficult to decide what the elastic limit or yield point of a joint really is, because neither of these is well defined.

It was decided, therefore, that no attempt would be made to discuss the joints on the basis of elastic limit or yield point. The unit stress chosen as a criterion of the action of the joints under working conditions was a unit shearing stress in the rivet of 9,000 lbs. per sq in. The stress chosen as a criterion of the action of the joints near the ultimate was a unit shearing stress as near the ultimate as could be obtained from the curves. This last unit stress varied slightly with different groups of joints, but was kept constant for any one group of six joints.

Columns 9 and 10 in Table No. 4, therefore, give the slip and deformation at a shearing unit stress of 9,000 lbs. per sq. in. in the rivet, and Columns 11 and 12 give the slip and deformation at a shearing unit stress near the ultimate. Column 13 gives the particular unit stress used in obtaining the data in Columns 11 and 12. The values listed in Columns 9, 10, 11, and 12 were in each case those which occurred in the gauge length used for the joint. This gauge length was always kept constant for each group of six joints.

Table No. 5 shows the results for lap joints of the second series. The joints with two rivets had the rivets arranged in two ways; in a line parallel to the load (called two rivets tandem), and in a line perpendicular to the load (called two rivets team). The joints with three rivets had the rivets arranged to form a triangle. All the joints of Table No. 5 were made with a plate thickness of $1\frac{1}{2}$ in. and a rivet diameter of $\frac{3}{4}$ in.

Table No. 6 gives the results for single covered butt joints.

TABLE NO. 5

LAP JOINTS—TWO RIVETS TANDEM

1	2	3	4	5	6	7	8	9	10	11	12	13
Test Spec. No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Unit Stress at Ultimate Load Lbs. per Sq In			Slip at Shear Stress of 9000 Lbs per Sq In.	Deformation at shear Stress 9000 lbs per sq. in.	Slip at Shear Stress in Column 11	Deformation at Shear Stress in Column 12	Shear Stress Lbs per Sq. In. for Column 11 & 12
					Shearing	Bearing	Tension					
D 5	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	Csk.	49600	62000	55400	0.0017	0.0098	0.0680	0.0936	
D 6	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	1	43500	56600	54400	0.0029	0.0110	0.1091	0.1384	
Ave.					46600	59300	54900	0.0023	0.0104	0.0885	0.1160	35000
D 7	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	Csk.	47900	50100	52400	0.0024	0.0105	0.0870	0.1138	
D 8	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	2	50200	52500	55700	0.0034	0.0115	0.0720	0.0978	
Ave.					49100	51300	54100	0.0029	0.0110	0.0795	0.1058	35000
D 3	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	B.H.	49700	63500	50100	0.0012	0.0070	0.0733	0.0972	
D 4	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	B.H.	47500	58300	47500	0.0006	0.0072	0.0820	0.1072	
Ave.					48600	60900	48800	0.0009	0.0071	0.0776	0.1022	35000
LAP JOINTS—TWO RIVETS TEAM												
G 5	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	Csk.	42000	53600	28400	0.0024	0.0074	0.0230	0.0385	
G 6	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	1	39100	49900	26600	0.0037	0.0090	0.0366	0.0529	
Ave.					40600	51800	27500	0.0030	0.0082	0.0298	0.0457	28000
G 7	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	Csk.	46100	49000	31300	0.0032	0.0092	0.0239	0.0409	
G 8	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	2	48300	51400	32700	0.0036	0.0087	0.0208	0.0362	
Ave.					47200	50200	32000	0.0034	0.0090	0.0223	0.0385	28000
G 3	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	B.H.	45100	57500	27500	0.0005	0.0034	0.0177	0.0307	
G 4	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	B.H.	45300	57900	27900	0.0003	0.0037	0.0121	0.0253	
Ave.					45200	57700	27700	0.0004	0.0035	0.0149	0.0280	28000
LAP JOINTS—THREE RIVETS												
K 5	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	Csk.	47300	60200	48100	0.0069	0.0121	0.0828	0.1032	
K 6	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	1	46400	59200	47300	0.0044	0.0098	0.0753	0.0956	
Ave.					46900	59700	47700	0.0057	0.0110	0.0790	0.0994	35000
K 7	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	Csk.	46400	49300	47300	0.0049	0.0106	0.0844	0.1078	
K 8	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	2	46300	49200	47300	0.0045	0.0106	0.0836	0.1032	
Ave.					46400	49300	47300	0.0047	0.0106	0.0840	0.1055	35000
K 3	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	B.H.	52200	66600	48200	0.0007	0.0047	0.0330	0.0484	
K 4	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	B.H.	46700	59700	43100	0.0006	0.0045	0.0540	0.0750	
Ave.					49500	63200	45700	0.0007	0.0046	0.0435	0.0617	35000

Note: Joints "D" gauge length = 14 in.
 Joints "G" and "K" gauge length = 8 in.
 * Joint failed in tension at rivet holes

Joints Ca, Ba, and Da were from the first series, and the remaining ones were from the second series. For these joints, also, the plate thickness was $\frac{1}{2}$ in., and the rivet diameter was $\frac{3}{4}$ in.

Table No. 7 gives the results for the double covered butt joints. Joints C were from the second series, and joints N, O, and P from the third series. For joints C the main plate and cover plates were each $\frac{1}{2}$ in. thick; while for joints N and O the main plate

TABLE NO. 6
SINGLE COVERED BUTT JOINTS - SINGLE RIVET

1	2	3	4	5	6	7	8	9	10	11	12	13
Test Spec. No.	Nominal Size of Rivet, in.	Size of Rivet Holes, in.	Nominal Thickness of Plate, in.	Kind of Rivet Heads	Unit Stress at Ultimate Load Lbs. per Sq. in.			Slip at Shear Stress of 9000 Lbs. per Sq. in.	Deformation at shear stress 9000 lbs per Sq. in.	Slip at Shear stress in Column 11	Deformation at Shear Stress in Column 12	Shear Stress Lbs. per Sq. in. for Column 13
					Shearing	Bearing	Tension					
Ba1	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk.	47200	62800	26100	0.0053	0.0091	0.0285	0.0494	
Ba2	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	1	47200	61500	25000	0.0009	0.0078	0.0274	0.0506	
Ave.					47200	62200	25600	0.0031	0.0084	0.0280	0.0500	28000
Ca1	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk.	55900	61500	33200	0.0008	0.0095	0.0108	0.0352	
Ca2	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	2	54700	60300	31700	0.0063	0.0171	0.0347	0.0652	
Ave.					55300	60900	32400	0.0035	0.0133	0.0227	0.0502	28000
Da1	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	47700	63400	25500	0.0008	0.0043	0.0230	0.0400	
Da2	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	46800	62400	23300	0.0004	0.0039	0.0235	0.0370	
Ave.					47300	62900	24400	0.0006	0.0041	0.0233	0.0385	28000
SINGLE COVERED BUTT JOINT - TWO RIVETS TANDEM												
E5	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk.	44700	56900	51300	0.0035	0.0150	0.1972	0.2294	
E6	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	1	43500	55500	48900	0.0027	0.0156	0.1850	0.2154	
Ave.					44100	56200	50100	0.0031	0.0153	0.1911	0.2224	35000
E7	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk.	46300	50200	53400	0.0072	0.0204	0.1961	0.2297	
E8	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	2	45200	48000	50200	0.0087	0.0212	0.2122	0.2335	
Ave.					45800	49100	51800	0.0080	0.0208	0.2041	0.2316	35000
E3	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	46600	59600	47500	0.0029	0.0135	0.1642	0.1932	
E4	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	45700	59500	48800	0.0012	0.0097	0.1530	0.1838	
Ave.					46200	59600	48200	0.0020	0.0116	0.1586	0.1885	35000
SINGLE COVERED BUTT JOINTS - TWO RIVETS TEAM												
H5	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk.	39900	50900	27100	0.0034	0.0115	0.1804	0.2232	
H6	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	1	41500	52900	27600	0.0041	0.0121	0.1207	0.1677	
Ave.					40700	51900	27400	0.0037	0.0118	0.1505	0.1954	35000
H7	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk.	47400	50400	32200	0.0042	0.0126	0.0970	0.1285	
H8	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	2	46400	49300	31400	0.0090	0.0181	0.0913	0.1259	
Ave.					46900	49900	31800	0.0066	0.0153	0.0941	0.1272	35000
H3	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	46500	59400	28600	0.0007	0.0056	0.0530	0.0805	
H4	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	44300	57600	27600	0.0005	0.0058	0.0800	0.1105	
Ave.					45400	58500	28100	0.0006	0.0057	0.0665	0.0955	35000
SINGLE COVERED BUTT JOINTS - THREE RIVETS												
L5	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk.	45300	57900	41700	0.0045	0.0154	0.1140	0.1475	
L6	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	1	41600	53100	38100	0.0052	0.0173	0.1050	0.1429	
Ave.					43500	55500	39900	0.0048	0.0164	0.1095	0.1452	32000
L7	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk.	48700	51700	49400	0.0064	0.0207	0.1300	0.1603	
L8	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	2	45800	49500	47400	0.0104	0.0256	0.1190	0.1541	
Ave.					47300	50600	48400	0.0084	0.0232	0.1245	0.1572	32000
L3	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	48800	62300	45100	0.0013	0.0080	0.0740	0.1005	
L4	$\frac{3}{8}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	46000	58700	42300	0.0023	0.0115	0.0651	0.0928	
Ave.					47400	60500	43700	0.0018	0.0097	0.0695	0.0966	32000

Note: Joint "Ca" failed by the rivet head being pulled through the rivet hole.
 Joint failed in tension across rivet holes.
 Joints "C" and "H" gauge length = 8 in.
 Joints "E" and "L" gauge length = 14 in.

TABLE NO. 7
DOUBLE COVERED BUTT JOINTS - SINGLE RIVET

1	2	3	4	5	6	7	8	9	10	11	12	13
Test Spec. No.	Nominal Size of Rivet In.	Size of Rivet holes In.	Nominal thickness of Plate In.	Kind of Rivet Heads	Unit Stress at Ultimate Load. Lbs. per Sq. In.			Slip at Shear Stress of 9000 lbs per Sq. In.	Deformation at shear stress of 9000 lbs per Sq. In.	Slip at Shear Stress in Column 11	Deformation at Shear Stress in Col 12	Shear Stress in lb. per sq. in. for Column 13
					Shearing	Bearing	Tension					
C11	3/4	13/16	1/2	Csk	47700	122200	50800	0.0042	0.0074	0.0580	0.0790	35000
C12	3/4	13/16	1/2	1	46600	119000	50600	0.0016	0.0040	0.0721	0.0927	
Ave					47200	120600	50700	0.0029	0.0057	0.0650	0.0858	
C13	3/4	13/16	1/2	Csk	55200	141000	59300	0.0001	0.0034	0.0040	0.0212	35000
C14	3/4	13/16	1/2	2	47200	120500	49500	0.0002	0.0036	0.0110	0.0293	
Ave					51200	130800	54400	0.0002	0.0035	0.0075	0.0252	
C9	3/4	13/16	1/2	B.H.	42700	106900	44800	0.0007	0.0032	0.1191	0.1272	35000
C10	3/4	13/16	1/2	B.H.	40200	102700	43300	0.0007	0.0031	0.1500	0.1742	
Ave					41500	104800	44100	0.0007	0.0032	0.1345	0.1507	
DOUBLE COVERED BUTT JOINTS - TWO RIVET TANDEM												
N5	3/4	13/16	1	Csk	42200	53400	24600	0.0029	0.0046	0.0468	0.0642	29000
N6	3/4	13/16	1	1	42600	54000	25000	0.0046	0.0072	0.0302	0.0450	
Ave					42400	53700	24800	0.0038	0.0059	0.0385	0.0546	
N7	3/4	13/16	1	Csk	43800	56000	26500	0.0052	0.0168	0.0382	0.0700	29000
N8	3/4	13/16	1	2	42900	54600	24700	0.0035	0.0132	0.0602	0.0800	
Ave					43400	55300	25600	0.0044	0.0150	0.0492	0.0750	
N3	3/4	13/16	1	B.H.	40400	52600	22800	0.0001	0.0023	0.0320	0.0480	29000
N4	3/4	13/16	1	B.H.	39500	51800	22900	0.0004	0.0027	0.0550	0.0840	
Ave					40000	52200	22900	0.0003	0.0025	0.0435	0.0660	
DOUBLE COVERED BUTT JOINTS - TWO RIVET TEAM												
O5	3/4	13/16	1	Csk	40400	51900	26000	0.0037	0.0051	0.0435	0.0423	29000
O6	3/4	13/16	1	1	48100	61100	30300	0.0049	0.0073	0.0236	0.0340	
Ave					44300	56500	26200	0.0043	0.0062	0.0335	0.0382	
O7	3/4	13/16	1	Csk	42900	54500	29400	0.0055	0.0092	0.0280	0.0456	29000
O8	3/4	13/16	1	2	39900	51900	26900	0.0030	0.0068	0.0375	0.0530	
Ave					41400	53200	28200	0.0043	0.0080	0.0328	0.0493	
O3	3/4	13/16	1	B.H.	38300	49100	24000	0.0013	0.0028	0.0790	0.0947	29000
O4	3/4	13/16	1	B.H.	40200	51100	24800	0.0005	0.0024	0.0732	0.0902	
Ave					39300	50100	24400	0.0009	0.0026	0.0761	0.0925	
DOUBLE COVERED BUTT JOINTS - THREE RIVETS												
P5	5/8	1 1/16	1	Csk	40300	42900	25400	0.0004	0.0025	0.0378	0.0512	29000
P6	5/8	1 1/16	1	1	42200	45000	27100	0.0010	0.0030	0.0372	0.0505	
Ave					41300	44000	26300	0.0007	0.0028	0.0375	0.0509	
P7	5/8	1 1/16	1	Csk	42000	44800	28100	0.0077	0.0109	0.0568	0.0726	29000
P8	5/8	1 1/16	1	2	37800	40200	24800	0.0126	0.0205	0.1118	0.1335	
Ave					39900	42500	26500	0.0102	0.0157	0.0843	0.1031	
P3	5/8	1 1/16	1	B.H.	40800	43800	24800	0.0007	0.0026	0.0580	0.0566	29000
P4	5/8	1 1/16	1	B.H.	40800	44500	25300	0.0000	0.0017	0.0522	0.0620	
Ave					40800	44200	25100	0.0004	0.0022	0.0551	0.0593	

Note: Joints "C" and "O" gauge length = 14 in.
 Joints "N" and "P" gauge length = 15 in.

TABLE NO. 8
BENDING TESTS
LAP JOINT - SINGLE RIVET

1	2	3	4	5	6	7
Test Spec. No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 2000 Lbs.	Total Deflection at 5500 Lbs.
A 5	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.038	0.250
A 6	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.033	0.423
Ave					0.036	0.337
A 7	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.037	0.416
A 8	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.035	0.252
Ave					0.036	0.334
A 3	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.036	0.248
A 4	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.035	0.302
Ave					0.036	0.275

LAP JOINT - THREE RIVETS

1	2	3	4	5	6	7
Test Spec. No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 5000 Lbs.	Total Deflection at 15000 Lbs.
K 11	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.033	0.287
K 12	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.031	0.246
Ave					0.032	0.267
K 13	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.032	0.224
K 14	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.032	0.322
Ave					0.032	0.273
K 9	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.023	0.116
K 10	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.024	0.252
Ave					0.024	0.184

Note: Joints "A" span length = 8 in.
 Joints "K" span length = 8 in.

was 1 in. thick and the cover plates $\frac{1}{2}$ in. For joints C, N, and O the rivets were $\frac{3}{4}$ in. in diameter. For joints P the main plate was 1 in. thick, the cover plates $\frac{1}{2}$ in., and the rivet diameter $\frac{5}{8}$ in.

Tables No. 8, 9, and 10 give the results obtained from bending tests on lap joints, single covered butt joints, and double covered butt joints from the second series. The specimens were loaded up to a point which caused serious permanent deflection, and the

TABLE NO. 9
BENDING TESTS

SINGLE COVERED BUTT JOINTS—SINGLE RIVET

1	2	3	4	5	6	7
Test Spec. No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 1000 Lbs.	Total Deflection at 3000 Lbs.
B 5	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.028	0.123
B 6	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.025	0.121
Ave					0.027	0.122
B 7	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.034	0.110
B 8	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.030	0.192
Ave					0.032	0.151
B 3	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.025	0.082
B 4	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.022	0.154
Ave					0.024	0.118

Note: Joints "B" span length = 10 in.

SINGLE COVERED BUTT JOINTS—THREE RIVETS

1	2	3	4	5	6	7
Test Spec. No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 1000 Lbs.	Total Deflection at 3000 Lbs.
L 11	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.060	0.174
L 12	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.058	0.197
Ave					0.059	0.186
L 13	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.063	0.247
L 14	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.063	0.181
Ave					0.063	0.214
L 9	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.049	0.147
L 10	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.045	0.140
Ave					0.047	0.144

Note: Joints "L" span length = 15 in

loads and corresponding maximum deflections were recorded throughout the test.

To obtain some idea of the relative stiffness of these joints in bending, the deflection was determined at a load which was on the straight portion of the load-deflection curve and which was considered to represent working conditions within the elastic limit of the material. Stiffness under conditions near the ultimate was determined from deflections at a much higher load, well beyond the "knee" of the curve, which had produced serious permanent deformation in the joint. These deflections are recorded in the tables.

TABLE NO.10
BENDING TESTS
DOUBLE COVERED BUTT JOINTS.

SINGLE RIVET							TWO RIVET TEAM						
1	2	3	4	5	6	7	1	2	3	4	5	6	7
Test Spec No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 4000 Lbs.	Total Deflection at 13000 Lbs.	Test Spec No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 6000 Lbs.	Total Deflection at 20000 Lbs.
C5	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.037	0.230	J5	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.023	0.157
C6	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.029	0.257	J6	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.027	0.205
Ave					0.033	0.244	Ave					0.025	0.181
C7	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.042	0.201	J7	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.051	0.390
C8	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.026	0.172	J8	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.037	0.340
Ave					0.034	0.187	Ave					0.044	0.365
C3	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.025	0.200	J3	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.024	0.217
C4	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.033	0.297	J4	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.025	0.218
Ave					0.029	0.249	Ave					0.025	0.218

THREE RIVETS							THREE RIVETS						
1	2	3	4	5	6	7	1	2	3	4	5	6	7
Test Spec No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 8000 Lbs.	Total Deflection at 22000 Lbs.	Test Spec No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 5000 Lbs.	Total Deflection at 16000 Lbs.
M5	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.038	0.175	M11	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.049	0.220
M6	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.042	0.171	M12	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.043	0.232
Ave					0.040	0.173	Ave					0.046	0.226
M7	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.090	0.442	M13	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.059	0.292
M8	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.068	0.315	M14	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.097	0.490
Ave					0.079	0.379	Ave					0.078	0.391
M3	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.033	0.163	M9	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.034	0.192
M4	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.048	0.321	M10	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.034	0.256
Ave					0.041	0.242	Ave					0.034	0.224

TWO RIVET TANDEM						
1	2	3	4	5	6	7
Test Spec No.	Nominal Size of Rivet In.	Size of Rivet Holes In.	Nominal Thickness of Plate In.	Kind of Rivet Heads	Total Deflection at 4000 Lbs.	Total Deflection at 11000 Lbs.
F5	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.045	0.230
F6	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	1	0.044	0.205
Ave					0.045	0.218
F7	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	Csk	0.104	0.443
F8	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	2	0.088	0.471
Ave					0.096	0.457
F4	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	B.H.	0.049	0.362

DISCUSSION OF RESULTS

LAP JOINTS—SINGLE RIVETS

Effect of Type of Riveting on Ultimate Shearing Strength of Rivets.—In Table No. 4 are eight different sets of six lap joints each, from which the ultimate unit shearing strength of countersunk and button head rivets may be compared. In five out of the eight sets, both the rivets with one countersunk head and those with two countersunk heads gave higher average ultimate strengths than the button head rivets. In two out of eight sets the rivets with two countersunk heads were stronger than the button heads, but the rivets with one countersunk head were slightly weaker. In only one set out of eight were the button head rivets stronger than both types of countersunk rivets.

Taking the average ultimate shearing unit stress for all the joints the results were 49,400 lbs. per sq. in. for button head rivets, 52,300 for rivets with one countersunk head and 55,400 for rivets with two countersunk heads. In other words the rivets with one countersunk head showed 6% greater strength, and those with two countersunk heads 12% greater strength than button head rivets.

Effect of Diameter of Rivet on Ultimate Shearing Strength of Rivets.—In Table No. 4 there were two sizes of rivets, $\frac{3}{8}$ in. and $\frac{1}{2}$ in., used with $\frac{1}{4}$ in. plates. The $\frac{1}{2}$ in. rivets in all but one joint showed a higher unit shearing strength than the corresponding $\frac{3}{8}$ in. rivets. The average for the joints was 55,900 lb. per sq. in. for the $\frac{1}{2}$ in. rivets, and 52,300 for the $\frac{3}{8}$ in. rivets, which shows an advantage of 7% for the $\frac{1}{2}$ in. rivets.

There were $\frac{5}{8}$ in., $\frac{3}{4}$ in., and $\frac{7}{8}$ in. rivets used with the $\frac{1}{2}$ in. plates. With these rivets the individual results were not always consistent so that only the average result is of interest. The average ultimate shearing unit stress for $\frac{5}{8}$ in., $\frac{3}{4}$ in., and $\frac{7}{8}$ in. rivets was 54,800, 53,300, and 50,800 lbs. per sq in., respectively. These results are not so greatly different from each other, and agree fairly well with the tensile and shear results on the rivet material given in Table No. 1. The results indicate that the larger rivets may be expected to have lower ultimate strengths.

There were $\frac{3}{4}$ in., $\frac{7}{8}$ in., and 1 in. rivets used with the $\frac{3}{4}$ in. plates. Here again the individual results were not consistent.

The average ultimate shearing unit stress for $\frac{3}{4}$ in., $\frac{7}{8}$ in., and 1 in. rivets was 48,700, 49,100, and 54,200 lbs. per sq. in., respectively. The results for $\frac{3}{4}$ in. and $\frac{7}{8}$ in. rivets are in reverse order in these plates compared with the results of similar rivets in the $\frac{1}{2}$ in. plates. The tensile and shear results in Table No. 1 on the rivet material show very little difference between these sizes.

Taking the average of all the results for each size of rivet the values are 52,300, 55,900, 54,800, 51,000, 49,900, and 54,200 lbs. per sq. in. for the $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, and 1 in. sizes. These results are not consistent, and the results in Table No. 1 on the rivet material are not consistent either. However, it should be noted that these ultimate strengths do not differ greatly from each other. It seems evident from the above that the diameter of the rivet has little effect on the ultimate unit shearing strength which may be expected from a joint.

It should be mentioned here that the ultimate unit shearing strengths recorded for the joints are considerably higher than the results obtained from the small shear specimens of the rivet material as given in Table No. 1. In this table the average ultimate shearing unit stress is 42,900 lbs. per sq. in., and the average ultimate unit tensile stress is 51,300 lbs. per sq. in. The average ultimate shearing unit stress of the rivets in the joints as recorded in Table No. 4 is 52,300 lbs. per sq. in. These results indicate that in general the ultimate shearing unit strength of the rivets in a joint will be about equal to the ultimate tensile unit strength of the rivet material.

Effect of Thickness of Plate on Ultimate Shearing Strength of Rivets.—With the $\frac{3}{4}$ in. rivets in Table No. 4 there were used both $\frac{1}{2}$ in. and $\frac{3}{4}$ in. plates. The average ultimate shearing unit stress with the $\frac{1}{2}$ in. plates was 53,300 lbs. per sq. in., and with the $\frac{3}{4}$ in. plates it was 48,700 lbs. per sq. in.

With the $\frac{7}{8}$ in. rivets both $\frac{1}{2}$ in. and $\frac{3}{4}$ in. plates were used. The average ultimate shearing unit stress with the $\frac{1}{2}$ in. plates was 50,800 lbs. per sq. in., and with the $\frac{3}{4}$ in. plates it was 49,100 lbs. per sq. in.

The above results are not numerous enough to justify the drawing of conclusions, but the indications are that the thicker plate gives a somewhat lower ultimate shearing strength of rivet.

Bearing and Tensile Stress.—Table No. 4 records the maximum bearing and also tensile unit stress that existed in the joint

at the ultimate load. While the unit stress was fairly low for most of the joints, it is seen that in some cases the bearing stresses were over 100,000 lbs. per sq. in. These results indicate that the common practice of allowing heavy bearing stresses in comparison with shearing and tensile stresses is justified. Care must, however, be exercised to see that in such cases the joint is strong enough to resist tension in the net section, and also shear in the plate back of the rivet.

Effect of Type of Riveting on Slip and Deformation.—In order to show the relative stiffness and tightness of the three types of riveting there are recorded in Columns 8 and 10 of Table No. 4 the slip and deformation at a unit shearing stress in the rivets of 9,000 lbs. per sq. in., which was taken as representative of working conditions. In Columns 11 and 12 are recorded the slip and the deformation at a unit shearing stress in the rivets which was somewhere near the ultimate strength of the joints.

The slip was found by going back to the initial load of 1,000 lbs. after every increment of load and finding the difference in the extensometer reading at this load and the very first load of 1,000 lbs. which was applied to the joint. The slip therefore represents the sum of any frictional slip between the plates plus any permanent set which occurred in the steel. The permanent set in the steel would be negligible at working loads but might be quite large near the ultimate load.

The slip at a shearing stress of 9,000 lbs. per sq. in. will be discussed first. In six sets of joints out of eight the button head riveted joints showed much less slip than the countersunk. In the other two sets only the joints countersunk at one rivet head showed less slip than those with button heads. On the average the joints countersunk at one rivet head showed 5.7 times, and those countersunk at both heads showed 6.5 times as much slip as the joints with button heads.

In the same way the deformation at a shearing stress of 9,000 lbs. per sq. in. was less for button head joints in seven out of eight sets of joints. On the average the joints countersunk at one rivet head showed 1.9 times, and those countersunk at both heads showed 2.4 times as much deformation as the joints with button head rivets.

In this connection it is interesting to note the ratio which the slip bore to the deformation in the three types of joints. For joints

countersunk at one rivet head this ratio was 55%, for those countersunk at both heads it was 50%, and for the button head it was 18%. It is seen, therefore, that the deformation of the countersunk joints is greater than that for button head, and that at least half of the deformation of the countersunk joints is due to slip.

Since a shearing unit stress of 9,000 lbs. per sq. in. is less than one-fifth of the lowest ultimate shearing unit stress recorded in Table No. 4, and since experiments have shown that the frictional resistance in riveted joints is often equivalent to a unit shearing stress in the rivets of 7,000 to 10,000 lbs. per sq. in.,¹ it seems likely that the difference in the behavior of the different types of joints was due to a difference in the squeezing force of the rivets, producing greater or less friction between the plates. However, the squeezing force of a rivet would be no greater than that of its weaker head, and for that reason a rivet with one countersunk head would have no more squeezing force than one with two countersunk heads. The results on slip seem to bear this out because the two types of countersinking gave very similar results, indicating that most of this slip was frictional slip.

In order to determine whether the gripping power of rivets and therefore the frictional resistance between the plates is directly proportional to the area of the rivets, a comparison may be made between the slip for different sizes of rivets. With the $\frac{1}{2}$ in. plates there were used $\frac{5}{8}$ in., $\frac{3}{4}$ in., and $\frac{7}{8}$ in. rivets. The average slip for these joints at 9,000 lbs. per sq. in. shearing stress was 0.0029, 0.0023, and 0.0032 in., respectively. With $\frac{3}{4}$ in. plates there were used $\frac{3}{4}$ in., $\frac{7}{8}$ in., and 1 in. rivets. The average slip for these joints at 9,000 lbs. per sq. in. was 0.0015, 0.0018, and 0.0016 in., respectively. These results are not exactly consistent but the fact that the values in each set are so much alike indicates that the frictional resistance is directly proportional to the area of the rivets.

When the slip is subtracted from the deformation, the remainder may be called elastic deformation. The elastic behavior of the two types of countersunk joints is not the same, for the elastic deformation of the joints countersunk at both rivet heads is about 1.3 times as great as that of those countersunk at one rivet head. The average elastic deformation of joints countersunk at one rivet

¹See p. 117 of *Modern Framed Structures*, Part III, Johnson, Bryan, and Turneaure.

head is about the same as the elastic deformation of button head joints. These results seem to indicate that after the slip has taken place the joints countersunk at one rivet head and the button head joints behave about the same, but that the joints countersunk at both rivet heads deform about 30% more than the others.

It should be stated here that all of these slips and deformations are very small, the average for the joints countersunk at both rivet heads being only 0.0028 in. slip, and 0.0056 in. deformation, in a gauge length of 8 in.

The slip and deformation for shearing stresses somewhere near the ultimate strength of the joint will be discussed next. Two sets of joints out of eight showed less slip in both types of countersunk joints than in button head, seven out of eight showed less slip in one or the other countersunk joint than in button head, and only one set showed less slip in the button head than in either of the other two. However, when the results are averaged the slip is practically the same for all types of joint, showing 8% more slip for joints countersunk at one rivet head than for button head, and 15% less for joints countersunk at both rivet heads than for button head.

It should be recalled here that this so-called "slip" for stresses near the ultimate must necessarily include both frictional slip and the permanent set throughout the joint.

When the deformation near the ultimate is averaged for all the joints the deformation is practically the same for all types of riveting, those countersunk at one rivet head showing 9% more deformation than the button head, and the countersunk at both rivet heads showing 5% less deformation.

It is of interest to determine the ratio that the slip bears to the deformation. For the joints countersunk at both rivet heads this ratio is 55%, for those countersunk at one head it is 61%, and for button head it is 61%. The ratios are practically the same for the three types of joint, and they show that over half of the deformation near the ultimate is due to frictional slip and permanent set.

It is interesting to note that the absolute value of the slip plus permanent set near the ultimate is not very large, being only 0.0283 in. in a gauge length of 8 in. for the joints countersunk at both rivet heads.

Summarizing the results of Table No. 4 it may be said that

neither the type of riveting, the diameter of the rivet, nor the thickness of the plate had much effect on the ultimate shearing unit stress of the rivets. At 9,000 lbs. per sq. in. stress in the rivets the countersunk joints showed about 6 times as much slip and 2 times as much deformation as the button head joints. At stresses near the ultimate the slip plus permanent set and also the deformation were practically the same in all types of joints.

LAP JOINTS—TWO AND THREE RIVET

Effect of Type of Riveting on Ultimate Shearing Strength of Rivet.—Table No. 5 gives the results obtained on the lap joints of the second series which were connected with two and with three rivets. For joints with two rivets tandem the ultimate unit shearing strength of the rivets countersunk at one head was 4% less than for button heads, and for rivets countersunk at both heads was 1% greater than for button heads.

For joints with two rivets team the ultimate shearing strength of the rivets countersunk at one head was 10% less than for button heads, and for rivets countersunk at both heads was 4% greater than for button heads.

For joints with three rivets the ultimate shearing strength of the rivets countersunk at one head was 5% less than for button heads, and for rivets countersunk at both heads was 6% less than for button heads.

Taking the average for all the joints the ultimate unit shearing strength of the rivets countersunk at one head was 44,700 lbs. per sq. in., for those countersunk at both heads was 47,600 lbs. per sq. in., and for button heads was 47,800 lbs. per sq. in. The results for button heads and countersunk at both heads are practically identical, but those countersunk at one head show 7% less strength than the button heads.

Table No. 2 shows that the average shearing strength of the rivet material was 41,300 lbs. per sq. in., and the average tensile strength was 50,100 lbs. per sq. in. The shearing strength obtained from the rivets in the joints lies between these values and nearer the larger one.

Since the previous discussion shows that there is not much difference between the various types of rivets, all the rivets will be averaged for each of the three types of joint—two rivets tandem, two rivets team, and three rivets. These average ultimate strengths

are 48,100, 44,300, and 47,600 lbs. per sq. in., respectively. This would seem to show that the rivets in the two rivet team joint are a little weaker than the other two. Comparing these results with the average of all $\frac{3}{4}$ in. rivets in Table No. 4, which is 51,000 lbs. per sq. in., it is seen that the single rivets gave somewhat higher shearing unit ultimate strengths than the two rivet and three rivet joints.

Bearing and Tensile Stress.—Table No. 5 also gives the ultimate bearing and tensile unit stresses existing in the joints at the ultimate load. All the bearing stresses are comparatively low, but the average tensile unit stress in the two rivet tandem joints is 52,600 lbs. per sq. in., and in the three rivet joints it is 46,900 per sq. in. These values are respectively 85% and 76% of the average ultimate tensile strength of the plate material given in Table No. 2.

Joint No. K7 failed in tension at a maximum unit tensile stress which was less than the maximum unit tensile stress in some of the joints which failed by shearing the rivets. Since the maximum shearing stress for this joint was a little higher than the companion joint K8, and little different from the shearing ultimates for the other joints, no distinction has been made between this joint and the other joints in discussing the results in Table No. 5.

Effect of Type of Riveting on Slip and Deformation.—The slip of the joints in Table No. 5 at a working unit stress of 9,000 lbs. per sq. in. will be discussed next. It will be noted that for each set of joints the two types of countersunk joint have about the same slip, and in each set this is very much more than the slip for button head joints. The average for all the joints shows a slip for those countersunk at one rivet head of 0.0037 in., for those countersunk at both rivet heads 0.0037 in., and for the button head 0.0007 in. Both types of countersunk joint, therefore, show about 5.5 times as much slip as the button head.

It is interesting to note that here as in single rivet lap joints the two types of countersunk joint give identical results, indicating that this slip is probably mostly friction slip which is dependent on the gripping power of the rivets.

For deformation at 9,000 lbs. per sq. in. the results show that for each set of joints the deformation for the two types of coun-

tersunk joint is much the same, and always greater than for button head joints. The average for all the joints shows a deformation for those countersunk at one rivet head of 0.0099 in., for those countersunk at both rivet heads of 0.0102, in., and for the button head 0.0051 in. Both types of countersunk joint, therefore, show about 2 times as much deformation as the button head.

The ratio of slip to deformation at 9,000 lbs. per sq. in. are respectively 37%, 36%, and 13% for joints with rivets countersunk at one head, countersunk at both heads, and button head rivets.

The elastic deformation, found by subtracting the slip from the deformation, for joints countersunk at one rivet head is 0.0062 in., for those countersunk at both rivet heads 0.0065 in., and for button heads 0.0044 in. The countersunk joints, therefore, show an average elastic deformation about 1.4 times as great as the button head joints.

Considering next the slip that occurred at stresses near the ultimate strength of the rivets, the results show that this is least for joints with button head rivets and much the same for the two types of countersunk joint. The average result for all joints is 0.0639 in. for the countersunk and 0.0453 in. for the button head, or 1.4 times as much slip for the former as for the latter.

The average deformation for the joints countersunk at one rivet head is 0.0870 in., for those countersunk at both rivet heads 0.0833 in., and for button heads 0.0640 in. The average for the countersunk joints is therefore 1.3 times that for the button head.

The ratios of the slip to the deformation are 76%, 74%, and 71%, respectively, for joints countersunk at one rivet head, countersunk at both heads, and button head joints.

Referring next to the three types of joint—two rivet tandem, two rivet team, and three rivets—it is found that the average slip for the two rivets tandem at 9,000 lbs. per sq. in. is 2.9 times as great for the countersunk joints as for the button head. For two rivets team the slip for the countersunk joints is about 8 times as great as for button head, and for three rivets it is about 7.4 times as great.

In the same way for deformation at 9,000 lbs. per sq. in. the results show 1.5, 2.5, and 2.4 times as much, respectively, for countersunk as for button heads, for the three types of joint. For slip at stresses near the ultimate the results show 1.1, 1.8, and

1.9, respectively, for the three types. For deformation at stresses near the ultimate the results show 1.1, 1.5, and 1.7 for the three types.

These results indicate that in all cases the stiffness of the countersunk joint is most nearly equal to the button head for the two rivet tandem joint. For the two rivet team and the three rivet joints the lack of stiffness of the countersunk joint is more pronounced.

Since joints G and K of Table No. 5, and joints L, M, and N of Table No. 4 all had the same gauge length, thickness of plate, and diameter of rivet, a comparison of stiffness may be made between joints with one, two, and three rivets. For slip at 9,000 lbs. per sq. in. shearing stress the results show an average value of 0.0023, 0.0023, and 0.0037 in. for one rivet, two rivet, and three rivet joints respectively. For deformation at 9,000 lbs. per sq. in. the results are 0.0047, 0.0069, and 0.0087 in., respectively. In general these results show that the single rivet has the most stiffness and the three rivet joint the least stiffness.

Summarizing the results of Table No. 5 it may be said that the type of riveting does not seem to affect the ultimate shearing unit strength of the rivets. At 9,000 lbs. per sq. in. shearing stress the slip is about 5.5 times as much for countersunk as for button head joints, and the deformation is about 2 times as much. At stresses near the ultimate the slip plus permanent set is about 1.4 times as great for countersunk as for button head joints, and the deformation is about 1.3 times as great.

Summary on Lap Joints.—The results on the lap joints in Tables No. 4 and 5 may be summarized as follows:

1. The average ultimate shearing strength of countersunk rivets was somewhat greater than that of button head rivets.
2. The diameter of the rivet seemed to have little effect on the ultimate unit shearing strength of the rivet.
3. The thickness of the plate seemed to have comparatively little effect on the ultimate unit shearing strength of the rivet.
4. The bearing stress in some cases was 1.8 times the ultimate shearing unit stress of the rivet.
5. The slip at working loads was about 6 times as much for each type of countersunk joint as for button head joints.
6. The deformation at working loads was about 2 times as much for each type of countersunk joint as for button head joints.

7. The ratio of slip to deformation at working loads was about 45% for countersunk joints, and 15% for button head joints.

8. The elastic deformation at working loads was about 25% to 45% greater for countersunk joints than for button head joints.

9. The slip plus permanent set at loads near the ultimate was about 15% to 25% greater for countersunk joints than for button head joints.

10. The deformation at loads near the ultimate was about 15% to 25% greater for countersunk joints than for button head joints.

11. The ratio of slip plus permanent set to deformation, for loads near the ultimate, was practically the same for all types of riveted joints and averaged 66%.

SINGLE COVERED BUTT JOINTS

Effect of Type of Riveting on Ultimate Shearing Strength of Rivet.—In Table No. 6 are given the results obtained on single covered butt joints using one rivet, two rivets tandem, two rivets team, and three rivets on each side of the joint. The one rivet joints are from the first series, while the others are from the second series of tests. The average ultimate shearing unit stress for all joints is 43,900 lbs. per sq. in. for rivets countersunk at one head, 48,800 for those countersunk at both heads, and 46,600 for button head rivets.

Examined on the basis of number of rivets in a joint the results show that the joint with a single rivet is somewhat stronger per unit area than the other three.

It will be noted that joint Ca 1 failed by pulling the rivet head through the rivet hole, and joints L 7, L 4, and H 8, failed by tension in the plates. Since in two of these cases the ultimate unit shearing strength of the rivet was actually higher than the companion test piece, and in the other two cases only slightly lower, no distinction has been made between these joints and the others in discussing the results.

Bearing and Tensile Stress.—The maximum bearing and tensile unit stresses at the ultimate load are also given in Table No. 6. The bearing stresses are not high, but the tensile stresses in the two rivet tandem joints have an average value which is

about 81% of the ultimate tensile strength of the plates as shown in Table No. 2.

Effect of Type of Riveting on Slip and Deformation.—The slip at a unit shearing stress of 9,000 lbs. per sq. in. for all joints in Table No. 6 is greatest for the countersunk and least for the button head. The average for all the joints shows 0.0037 in. for those countersunk at one rivet head, 0.0066 in. for those countersunk at both rivet heads, and 0.0013 in. for button head. In other words joints countersunk at one rivet head had 2.9 times, and those countersunk at both rivet heads had 5.3 times as much slip as the button head joints.

These results indicate that the two types of countersunk joint do not act the same, which is different from that which was found to be the case for lap joints.

The average deformation at 9,000 lbs. per sq. in. for joints countersunk at one rivet head is 1.7 times, and for joints countersunk at both rivet heads it is 2.3 times as much as for button head joints.

The ratios of slip to deformation are 28% and 36% for the countersunk joints, and 16% for the button head.

The elastic deformations are 1.4 and 1.8 times as much, respectively, for countersunk joints as for button head.

The slip at stresses near the ultimate for joints countersunk at one rivet head is 1.5, and for those countersunk at both rivet heads it is 1.4 times as much as for button head joints. The deformations near the ultimate are 1.5 and 1.4 times as much, respectively, as for button head joints.

The ratio of slip to deformation at loads near the ultimate is about the same for all types of riveting and averages about 78%.

The average slips at a shearing unit stress of 9,000 lbs. per sq. in. for single rivet, two rivet tandem, two rivet team, and three rivet joints are respectively 5.5, 2.8, 8.6, and 3.7 times as much for the countersunk joints as for the button head. The deformations at 9,000 lbs. per sq. in. are 2.6, 1.6, 2.4, and 2.0, respectively. These results do not vary consistently and no general conclusions can be drawn as to the effect of number of rivets in a joint.

Summary on Single Covered Butt Joints.—The results on the single covered butt joints in Table No. 6 may be summarized as follows:

1. The average ultimate unit shearing strength of countersunk rivets was practically equal to that of button head rivets.
2. The slip at working loads was about 3 times as much for joints countersunk at one rivet head, and 5 times as much for joints countersunk at both rivet heads, as it was for button head joints.
3. The deformation at working loads was from 1.7 to 2.3 times as much for the two types of countersunk joints as it was for button head joints.
4. The ratio of slip to deformation at working loads was from 28% to 36% for countersunk joints, and 16% for button head joints.
5. The elastic deformation at working loads was about 40% to 80% greater for countersunk joints than for button head joints.
6. The slip plus permanent set at loads near the ultimate was about 1.5 times as much for countersunk joints as for button head joints.
7. The deformation at loads near the ultimate was about 1.5 times as much for countersunk joints as for button head joints.
8. The ratio of slip plus permanent set to deformation, at loads near the ultimate, was about the same for all types of joint and averaged 78%.

DOUBLE COVERED BUTT JOINTS

Effect of Type of Riveting on Ultimate Shearing Strength of Rivets.—In Table No. 7 are given the results obtained on double covered butt joints. The results on the joints with single rivets on each side are from the second series, while the remaining results are from the third series.

The average of all the ultimate unit shearing stresses for rivets countersunk at one head is 43,800 lbs. per sq. in., for those countersunk at both heads 44,000 lbs. per sq. in., and for button heads 40,400 lbs. per sq. in. The difference in these results is in favor of the countersunk rivets.

Examined on the basis of type of joint the results show no great difference between two rivet tandem, two rivet team, and three rivet joints, but the single rivet joints have a strength considerably higher than that of the others.

Bearing and Tensile Stresses.—The maximum bearing stresses in the single rivet joints are large, which tends to justify

the common use of high bearing stresses. The tensile stresses in the single rivet joint are also quite high, being about 80% of the average tensile strength of the plate material shown in Table No. 2.

Ultimate Shearing Strength in Lap and Butt Joints.—At this point it may be of interest to compare the ultimate shearing strength of rivets in lap joints and in single and double covered butt joints.

Taking the $\frac{3}{4}$ in. size of rivet for all lap joints tested (30 joints), the average ultimate stress is 45,800, 51,300, and 48,100 lbs. per sq. in. for rivets countersunk at one head, countersunk at both heads, and button head rivets, respectively. In the same way for all the single covered butt joints, (24 joints), the results are 43,900, 48,800, and 46,600 lbs. per sq. in., respectively. For all the double covered butt joints, (24 joints), the results are 43,800, 44,000, and 40,400 lbs. per sq. in., respectively. It will be noticed first of all that in each of these three sets of results the rivets countersunk at both heads have the highest ultimate unit shearing strength. This strength is highest for the lap joints, with single covered butt joints next, and double covered butt joints last. The same order of strength holds for rivets countersunk at one head and for button heads. When the average of all types of riveting is calculated for lap joints, single covered butt joints, and double covered butt joints, the results are 48,400, 46,400, and 42,700 lbs. per sq. in., respectively. This indicates that the average ultimate unit shearing strength for single covered butt joints, of the type tested, may be expected to be about 96% of the strength found in lap joints, and that the rivets in double covered butt joints may be expected to show about 88% of the strength found in lap joints. Or, if the single shear values for lap joints and for single covered butt joints are averaged, the result shows that the double shear of rivets may be expected to be about 90% of the strength in single shear. In other words the rivet in double shear will develop about 1.8 times the strength of the rivet in single shear.

Effect of Type of Riveting on Slip and Deformation.—The results on slip at 9,000 lbs. per sq. in. shearing stress for the joints in Table No. 7 are not altogether consistent, but in general the button head rivets show the least slip. Averaging all the results the slip is 0.0029 in., 0.0048 in., and 0.0006 in., for joints countersunk at one rivet head, countersunk at both rivet heads, and

button head joints, respectively. That is, the slip was 4.8 times as much for joints countersunk at one rivet head and 8 times as much for those countersunk at both rivet heads as it was for button heads. These results indicate that the two types of countersunk joint do not act the same, which is different from that which was found to be the case in lap joints.

In the same way the deformation at 9,000 lbs. per sq. in. averaged 0.0052 in., 0.0106 in., and 0.0026 in., for joints countersunk at one rivet head, countersunk at both rivet heads, and button head joints, respectively. Or the deformation was 2 times as much for joints countersunk at one rivet head and 4.1 times as much for those countersunk at both rivet heads, as for button head joints.

The average slips at a shearing stress of 9,000 lbs. per sq. in. for single rivet, two rivet tandem, two rivet team, and three rivet joints are respectively 2.3, 13.7, 4.8, and 13.7 as much for countersunk rivets as for button head rivets. For deformation at 9,000 lbs. per sq. in. the results are 1.4, 4.2, 2.7, and 4.2 respectively. These results would tend to indicate that the stiffest joint is obtained when all the rivets used are in one row across the joint.

The ratios of slip to deformation are 57%, 45%, and 23%, for joints countersunk at one rivet head, countersunk at both rivet heads, and button head joints, respectively.

The elastic deformation is 1.1 times as much for joints countersunk at one rivet head, and 2.8 times as much for those countersunk at both rivet heads, as for button head joints.

Taking next the slip and deformation at stresses near the ultimate the average result is about the same for the two types of countersunk joint. The button head joints show 1.8 as much slip and 1.5 times as much deformation as those countersunk.

The ratio of slip plus permanent set to deformation is 76%, 69%, and 84% for joints countersunk at one rivet head, countersunk at both rivet heads, and button head joints, respectively.

Summary on Double Covered Butt Joints.—The results on the double covered butt joints in Table No. 7 may be summarized as follows:

1. The ultimate shearing strength of countersunk rivets was somewhat greater than that of button head rivets.
2. The unit bearing stress in some cases was 2.6 times as great as the ultimate unit shearing stress of the rivet.

3. The slip at working loads was about 5 times as much for joints countersunk at one rivet head, and 8 times as much for joints countersunk at both rivet heads, as it was for button head joints.

4. The deformation at working loads was 2 times as much for joints countersunk at one rivet head and 4 times as much for joints countersunk at both rivet heads, as it was for button head joints.

5. The ratio of slip to deformation at working loads was 57% to 45% for countersunk joints and 23% for button head joints.

6. The elastic deformation at working loads was from 1.1 to 2.8 times as much for countersunk joints as for button head joints.

7. The slip plus permanent set at loads near the ultimate was 1.8 times as much for button head joints as for countersunk joints.

8. The deformation at loads near the ultimate was 1.5 times as much for button head joints as for countersunk joints.

9. The ratio of slip plus permanent set to deformation, at loads near the ultimate, was 76% to 69% for countersunk joints, and 84% for button head joints.

BENDING TESTS

Tables No. 8, 9, and 10 give the results obtained from the bending tests which were made to obtain some idea of the comparative strength and stiffness of countersunk and button head riveted joints when subjected to a lateral bending force.

Lap Joints.—Table No. 8 records the results on lap joints. The span length between supports was 8 in., which was large enough so that the supports came outside of the lapped plates. The plates were lapped 3 in. for the joints with single rivets, and $5\frac{3}{4}$ in. for the joints with three rivets.

For the joint with a single rivet the total deflection at a working load of 2,000 lbs. was the same for the two types of countersunk joint and for button head joints. For deflections near the ultimate, at 5,500 lbs., the two types of countersunk joint deflected about the same amount and about 1.2 times as much as the button head joints.

For the joints with three rivets the deflection at a working load of 5,000 lbs. is the same for the two types of countersunk joint, and about 1.3 times as great as for button head joints. At 15,000

lbs., near the ultimate, the deflection of the two types of countersunk joint is about the same, and is 1.5 times as great as for button head joints.

These two sets of results on lap joints indicate, therefore, that under working conditions the countersunk joint deflects about 1.2 times as much as button head, and at loads near the ultimate about 1.4 times as much.

The curves for these joints do not indicate that there is much difference between the joints in the maximum load that could be carried, so that the ultimate strengths of the joints are about the same for all types of riveting.

Single Covered Butt Joints.—Table No. 9 gives the results of bending tests for single covered butt joints, one set with a single rivet on each side, and one set with three rivets on each side. The former were tested with a span length between supports of 10 in., and the latter with a span length of 15 in. This was sufficient to bring the supports outside of the cover plate which was $8\frac{1}{4}$ in. long in the first case, and $12\frac{3}{4}$ in. in the second. One joint of each pair was tested with the cover plate on top, and one with the cover plate on the bottom.

For the joints with a single rivet on each side those countersunk at one rivet head, at a working load of 1,000 lbs., deflected 1.1 times as much, and those countersunk at both rivet heads deflected 1.3 times as much as the button head joints. In the same way at a load near the ultimate, 3,000 lbs., those countersunk at one rivet head deflected 1.03 times as much, and those countersunk at both rivet heads deflected 1.3 times as much as the button head joints.

For joints with three rivets on each side the deflections at a working load of 1,000 lbs., for joints countersunk at one rivet head, are 1.3 times as great, and for those countersunk at both rivet heads they are 1.3 times as great as for button heads. For a load near the ultimate, 3,000 lbs., the deflections are 1.3 and 1.5 times as great, respectively, as for button heads.

These results indicate that at working loads the deflections for joints countersunk at one rivet head are about 1.2 times as much, and for those countersunk at both rivet heads they are 1.3 times as much as for button head joints. At loads near the ultimate the deflections for joints countersunk at one rivet head are about

1.2 times, and for those countersunk at both rivet heads they are 1.4 times as much as for button head joints.

The deflection curves for these joints indicate that there is little difference between the strength of the joints as measured by maximum load.

Double Covered Butt Joints.—Table No. 10 gives the results of bending tests on five sets of double covered butt joints. For joints C, with one rivet on each side, the span between supports was 10 in. and the cover plates were $8\frac{1}{4}$ in. long. For joints F, with two rivets tandem on each side, the span was 15 in. and the cover plates were $13\frac{1}{4}$ in. long. For joints J, with two rivets team on each side, the span was 10 in. and the cover plates were $7\frac{1}{4}$ in. long. For joints M, with three rivets on each side, there were available four joints of each kind with cover plates $12\frac{3}{4}$ in. long. One set, joints M3 to M8, was tested with a span of $12\frac{3}{4}$ in., and the other set, M9 to M14, was tested with a span of 15 in. In the former, therefore, the supports rested on the cover plates, and in the latter the supports were outside of the cover plates.

For the joints with a single rivet on each side the deflections at a working load of 4,000 lbs. are 1.1 times as great for joints countersunk at one rivet head as for button head joints, and those countersunk at both rivet heads are 1.2 times as great. At a load of 13,000 lbs., near the ultimate, the deflections for joints countersunk at one rivet head are 0.98 times as great, and for those countersunk at both rivet heads they are 0.75 times as great as the button head joints.

For the joints with two rivets tandem on each side, deflections of those countersunk at one rivet head at a working load of 4,000 lbs. are 0.92 times as great, and of those countersunk at both rivet heads they are 2 times as great as for button head joints. At a load of 11,000 lbs., near the ultimate, the deflections for joints countersunk at one rivet head are 0.57 times as great, and for those countersunk at both rivet heads they are 1.2 times as great as for button head joints.

For the joints with two rivets team on each side the deflections at a working load of 6,000 lbs. are the same for joints countersunk at one rivet head as for button head, but those countersunk at both rivet heads show 1.8 times as much deflection as the but-

ton head joints. At a load of 20,000 lbs., near the ultimate, for the joints countersunk at one rivet head the deflections are 0.83 times as great, and for those countersunk at both rivet heads they are 1.7 times as great as for button head joints.

For joints M3 to M8 with three rivets on each side, the deflections at a working load of 8,000 lbs., for joints countersunk at one rivet head are 0.98 times as great, and for those countersunk at both rivet heads they are 1.9 times as great as for button head joints. At a load of 22,000 lbs., near the ultimate, the deflections for joints at one rivet head are 0.72 times, and for those countersunk at both rivet heads they are 1.6 times as great as for button head joints.

For joints M9 to M14 with three rivets on each side, deflections at a working load of 5,000 lbs., for joints countersunk at one rivet head, are 1.4 times, and for those countersunk at both rivet heads they are 2.3 times as great as for button head joints. At a load of 16,000 lbs., near the ultimate, the deflections for joints countersunk at one rivet head are 1.01 times, and for those countersunk at both rivet heads they are 1.7 times as great as for button head joints.

The results on these six sets of joints show, therefore, that at working loads the average deflections of the joints countersunk at one rivet head are 1.1 times, and of those countersunk at both rivet heads they are 1.8 times greater than for button head joints. For loads near the ultimate the deflections of joints countersunk at one rivet head are 0.80 times as great, and for those countersunk at both rivet heads they are 1.4 times as great as for button head joints. In general it may be said that the double covered butt joints countersunk at both rivet heads are less stiff than button head joints both at working and at ultimate loads. Joints countersunk at one rivet head have about the same stiffness as button head at working loads and are stiffer at ultimate loads.

The deflection curves for these joints do not indicate that there is much difference in the ultimate strength of the joints as measured by maximum load.

Summary on Bending Tests.—The results of the bending tests in Tables No. 8, 9, and 10 may be summarized as follows:

1. For lap joints the deflection at working loads for joints with countersunk rivets was about 20% more than for those with button head rivets.

2. For lap joints the deflection at loads near the ultimate was about 40% more for countersunk rivets than for button head rivets.

3. For single covered butt joints the deflection at working loads was about 25% more for countersunk rivets than for button head rivets.

4. For single covered butt joints the deflection at loads near the ultimate was about 30% more for countersunk rivets than for button head rivets.

5. For double covered butt joints the deflection at working loads was from 10% to 80% greater for countersunk rivets than for button head rivets.

6. For double covered butt joints the deflection at loads near the ultimate varied from 20% less to 40% more for countersunk rivets than for button head rivets.

7. The ultimate bending strength of joints was about the same for countersunk rivets and button head rivets. This was true for lap, single covered butt, and double covered butt joints.

GENERAL OBSERVATIONS

Out of a total of 72 joints with countersunk rivets, only one joint, Cal, failed by pulling the head of the rivet through the rivet hole. This joint was a single covered butt joint with one rivet on each side, and had rivets countersunk at both rivet heads. The failure was undoubtedly due to the fact that the joint was not rigidly riveted and the rivets did not fill completely the countersunk portion of the plate.

Out of a total of 108 joints tested in tension only four joints, H8, K7, L7, and L4, failed by tension in the plate. In all four cases the load at which failure occurred was so near the average ultimate for the other joints that no distinction was made between these joints and the others in discussing the results. In two of these four cases the load was higher than the load for the companion joint which failed by shearing the rivets. All of these failures were for joints with $\frac{3}{4}$ in. rivets. One joint was a lap joint countersunk at both rivet heads, two were single covered butt joints countersunk at both rivet heads and the fourth was a single covered butt joint with button head rivets. The maximum unit tensile stress in the joint with button head rivets was 42,300 lbs. per sq. in. and for the three joints with countersunk rivets it

was 31,400, 49,400, and 47,300 lbs. per sq. in. The average unit tensile strength of the plate material given in Table No. 2 is 62,000 lbs. per sq. in. This indicates that the plates in button head riveted joints developed only 68%, and the countersunk riveted joints only 51%, 80%, and 76% of the unit tensile strength of the solid plate. It is believed that this is due to the fact that some of the injury of the material resulting from the punching of the rivet holes may have remained even after the hole was reamed, and the further fact that at the ultimate load serious bending stresses must exist in the plates of all lap joints and single covered butt joints.

The bending of the plates which was present at the ultimate load is clearly shown in Figure 10 which shows a number of tensile and bending specimens. The joint marked A is part of a single covered butt joint, joint B part of a double covered butt joint, and joints C and D parts of a lap joint. In joint A the bending of the plates on either side of the rivet line is plainly seen, and is well brought out by the bent condition of the cover plate. This bending of the plates was a characteristic action in all the tests, and was evident even in the double covered butt joints.

The lap joints with $\frac{1}{4}$ in. plates were placed in the testing machine without any shims so that they were subjected to the maximum bending effect under tension load. All the other lap joints were shimmed at the top and bottom grips so that the center line of the joint would coincide with the center line of the testing machine, and so that bending of the plates would be reduced to a minimum.

At loads which could be considered working loads there was no noticeable bending such as has been described as existing at loads near the ultimate.

The joints of the second series were carefully examined for evidence of tension failure. Two of these joints showed tension cracks at the edges of the plate back of the rivet. One of these was a two rivet team lap joint with button heads, and the other was a two rivet team single covered butt joint with rivets countersunk at one rivet head. Six more joints showed tension cracks around the rivet holes, in one case $\frac{3}{8}$ in. long. Two of these joints were two rivet team lap joints with rivets countersunk at both rivet heads; one was a three rivet lap joint with button head rivets; three were three rivet lap joints, one with rivets counter-

sunk at one rivet head, and one with rivets countersunk at both rivet heads; and one was a three rivet single covered butt joint with rivets countersunk at both rivet heads.

The joints of the second series were also examined for elongation of rivet holes after failure. It was found that the joints generally showed evidences of rivet hole elongation, which in some cases amounted to a change in diameter from an initial value of 0.81 in. to a value of 0.93 in.

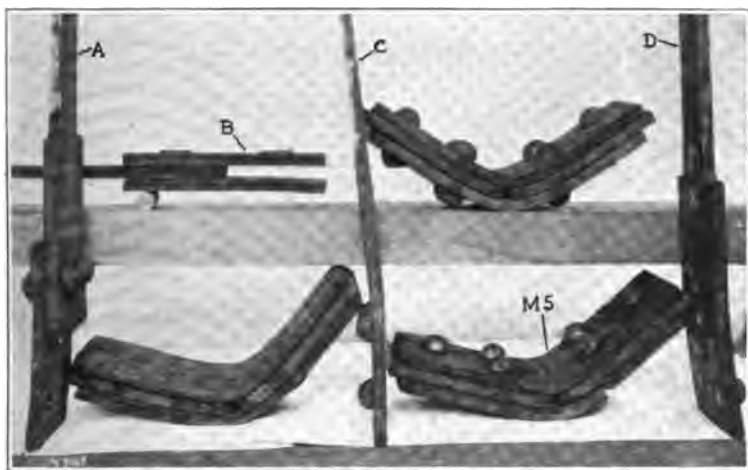


Figure 10—Samples of Tension and Bending Specimens after Testing

In connection with the bending tests, joints M3 to M8 were loaded in cross-bending up to very high loads; producing bends which varied from 57° to 89° from the original straight joint. In these severe bending tests none of the rivets failed. Some of the countersunk heads were tilted so that one edge projected beyond the plate while the other edge was pulled into the rivet hole. Four out of the six joints were cracked in tension due to this severe bending. Joint M5, countersunk at one rivet head, had the countersunk plate on the bottom. At a load of about 73,000 lbs., when the joint had bent through an angle of about 57° , the lower plate failed in tension across the two rivet holes nearest the seam. Joint M6, countersunk at one rivet head, had the countersunk plate at the top. At a load of about 72,000 lbs., when the joint had bent through an angle of about 89° , the lower plate failed in tension by cracking from the outside edge of the plate toward one of the

rivet holes. Joint M3, with button head rivets, cracked in tension on the under side at the outside edge of the plate when the load was about 72,000 lbs., and when the joint had bent through an angle of about 60° . Joint M4, with button head rivets, cracked in tension, on the under side from a rivet hole to the edge of the plate, when the load was about 83,000 lbs., and the joint had bent through an angle of about 69° .

Figure 10 shows joint M5 which failed in tension after being subjected to the severe bending mentioned above.

CONCLUSIONS

From the results on the tensile tests of the riveted joints the following conclusions may be drawn:

1. The average ultimate shearing strength of the countersunk joints was at least equal to that of the button head joints for lap, single covered butt, and double covered butt joints.
2. For lap joints the diameter of the rivet and the thickness of the plate seemed to have little effect on the ultimate shearing strength of the rivets.
3. The bearing stress in some of the lap joints was 1.8 times as great, and in some of the double covered butt joints 2.6 times as great as the ultimate unit shearing stress of the rivets.
4. The slip at working loads was from 3 to 6 times as great for joints with rivets countersunk at one head, and from 5 to 8 times as great for joints with rivets countersunk at both heads, as for those with button head rivets.
5. The deformation at working loads was from 1.7 to 2 times as great for joints with rivets countersunk at one head, and from 2 to 4 times as great for joints with rivets countersunk at both heads, as that for joints with button head rivets.
6. The ratio of slip to deformation at working loads varied from 28% to 57% for joints with countersunk rivets, and from 15% to 23% for joints with button head rivets.
7. The elastic deformation at working loads for joints with countersunk rivets varied from 35% less to 180% more than that for joints with button head rivets.
8. The slip plus permanent set at loads near the ultimate varied from 45% less to 50% more for joints with countersunk rivets than that for joints with button head rivets.

9. The deformation at loads near the ultimate varied from 33% less to 50% more for joints with countersunk rivets than that for joints with button head rivets.

10. The ratio of slip plus permanent set to deformation, for loads near the ultimate, was practically the same for all types of riveted joints and averaged about 74%.

From the results of the bending tests of the riveted joints the following conclusions may be drawn:

1. The deflection at working loads was from 10% to 80% more for joints with countersunk rivets than that for joints with button head rivets.

2. The deflection at loads near the ultimate varied from 20% less to 40% more for joints with countersunk rivets than that for joints with button head rivets.

From the results of all the tests, including tension and bending, the following general conclusions may be drawn:

1. For riveted joints in which strength is the primary consideration, either button head or countersunk rivets may be used.

2. For riveted joints in which great rigidity is desired, button head rivets should be used rather than countersunk.

3. Because joints with countersunk rivets showed high strength and very considerable deformation, the indication is that for joints which may be subjected to impact or collision, and in which a high capacity for the absorption of energy is desirable, countersunk rivets should be used rather than button head.

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Vol. IX, No. 6

6

THE JOULE-THOMSON EFFECT IN CARBON DIOXIDE

AN EXPERIMENTAL STUDY

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MADISON

1926

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EXPERIMENTAL STUDY OF THE JOULE-THOMSON EFFECT IN CARBON DIOXIDE

INTRODUCTION

When a student at the University of Wisconsin the writer became interested in certain possible applications of liquid air or other low boiling-point fluids that for feasibility depended upon the variation of their constant pressure specific heats, c_p , with pressure and temperature, or, in other words, upon the character of the variation of their specific energy-contents as functions of pressure and temperature. Being unable to obtain satisfactory information on the subject either by inquiry or by research in scientific literature, he sought to devise a practical way of determining, qualitatively at least, by experiment, the knowledge desired. The results of this preliminary investigation and a discussion of the experimental methods proposed are embodied in a report presented as a thesis for his B.S. degree, M.E. course, in June 1905, under the title, *The Specific Heats of Fluids as Functions of Pressure and Temperature*.

An award of a Scholarship in Engineering for the following year afforded the desired opportunity to undertake the experimental investigation, although enough fragmentary information had by then been acquired to determine the non-feasibility of the possible applications above mentioned. The writer's interest in the problem of pure science with a distinct engineering bearing led him, however, to devote most continuously the following several years to its solution. A report of these efforts and of the subsequent work on the data secured, with some other, and their interpretation, forms the subject matter of this bulletin.^o

It was at first planned to determine comparative values of mean constant-pressure specific heats of fluids at different pressures over comparable temperature ranges by observing the changes of temperature experienced by a fluid when circulated through a counter-current heat-interchanger, schemat-

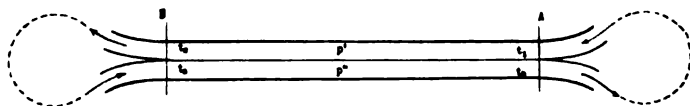


Fig. 1.

ically illustrated in Fig. 1. The fluid under investigation was to enter the interchanger at section A at a definite temperature t_1 ; pass through it from A to B at a substantially constant pressure p' ; arrive at section B with a temperature t_2 attained by interchange of heat (through the heat-conducting dividing wall of the interchanger) with the counter-flow of fluid. The latter was to enter at B with a temperature t_3 ; pass through the interchanger from B to A at a substantially constant pressure p'' ; exit at A with the temperature t_4 attained by the above heat interchange. Both entering and return flows were of course to be equal in amount, m units of mass per unit of time.

^oA much abbreviated account of this work and its important results, under the title "Experimental Study of the Joule-Thomson Effect in Carbon Dioxide", was published in the *Physical Review*, Ser. 2, Vol. 22, 1923.

For the ideal condition of adiabatic flow (in regard to external heat interchange) the heat gained or lost by unit flow from A to B would equal the heat lost or gained by unit flow from B to A, whence

$$m\bar{c}_p'(t_1 - t_2) = m\bar{c}_p''(t_4 - t_3) \quad (a)$$

or

$$\bar{c}_p' / \bar{c}_p'' = (t_4 - t_3) / (t_1 - t_2) \quad (b)$$

in which $\bar{c}_p'$ is the mean value of c_p over the temperature range t_1 to t_2 at the pressure p' , and $\bar{c}_p''$ is the mean value of c_p over the temperature range t_4 to t_3 at the pressure p'' . It is obvious from this that by varying the pressures and the temperature ranges a general qualitative comparison of mean constant-pressure specific heats could be secured by this method if external heat exchange could be avoided or properly allowed for.

In the course of these early studies attention was called to the possibility of utilizing observations of throttled or porous plug fluid flow as a means of determining the relative variation of c_p over the field of temperature and pressure. A "throttling curve" for a fluid is the locus of points on a pressure-temperature plane that represent the pressures and temperatures which a fluid attains as a result of throttling it from a fixed initial high-pressure condition to successively lower pressure conditions, provided that such expansive throttling be essentially adiabatic and that the kinetic energy of flow of the fluid be the same on either side of the throttling device, or preferably, negligible. This is a *constant enthalpy* process; that is, one in which the sum of the specific internal energy u of the fluid plus the product of its pressure p and its specific volume v , all expressed in agreeable units, is the same for all points which the curve represents. The enthalpy, $g (= u + pv)$, is commonly but less definitely referred to as the total energy or the total heat content of a fluid, and the throttling process as a constant total heat process.

A consideration of such throttling curves for steam from observations of various experimenters,* and particularly a study of a chart of constant enthalpy curves for air at pressures up to 130 atmospheres and temperatures from zero centigrade down to nearly its liquefaction points, which, with additions, is reproduced as Chart 0 from the computations of Witkowski¹ in 1899, led the writer to adopt this method as the experimental basis of his own determinations. The inherent advantages are many, chief among which, experimentally, is the fact that measurements of pressure and temperature only are needed since the effects are fundamentally independent of the quantity of material throttled. Furthermore an experimentally determined chart of isenthalpic curves, if sufficiently extensive, supplies a large amount of information regarding the thermal properties of the fluid represented.

A close study of the curves of Chart 0 discloses that any pair of isenthalpic curves are either convergent or divergent when viewed in the direction of increasing pressures, or they have a point of closest approach to each other and diverge therefrom in either direction. Hence, on constant-pressure lines, the temperature intervals intercepted by any two isenthalps are in general not alike (Fig. 2); the difference of enthalpy is the same however for all such intervals; it can be expressed as the product of the mean constant-pressure

* See References Nos. 10, 20, 21, 22, 23.

¹ Witkowski, A. W. *Zeitschr. f. kom. u. Aus. Gase*, 2, 161, 1899.

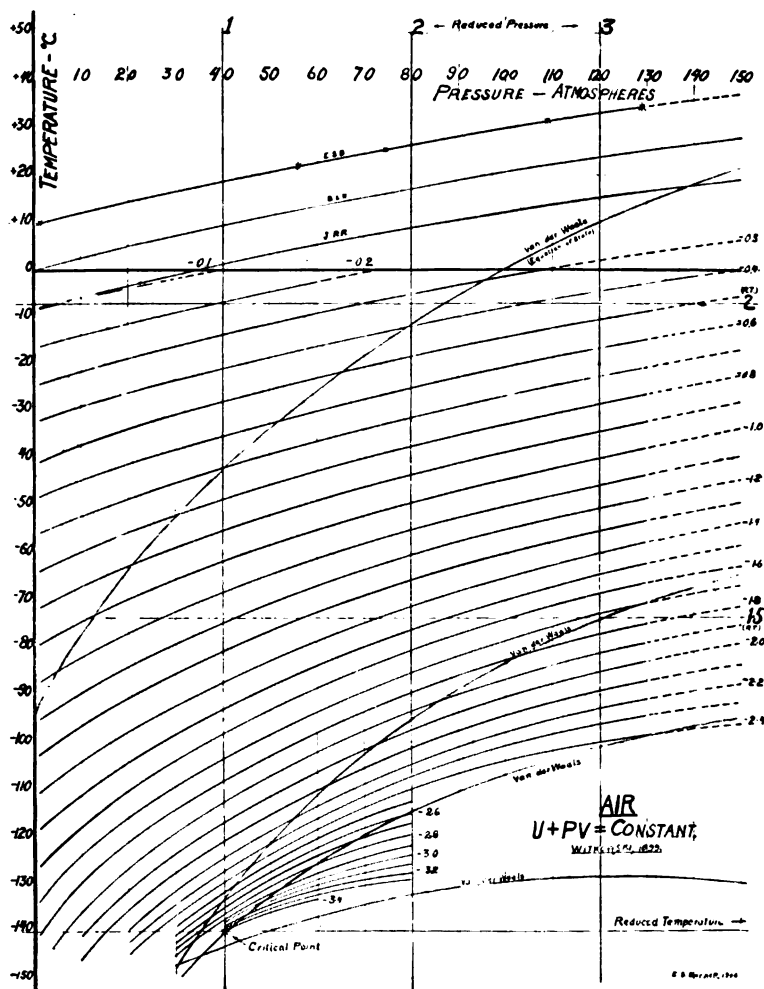


CHART 0

Throttling curves for air as computed by Witkowski; upper three, from experimental data by others; crossing curves, according to van der Waal's equation.

specific heat for any such interval, into that temperature difference, or algebraically by the expression,

$$\Delta g = \bar{c}_p \Delta t. \quad (c)$$

Since this expression is true for any and all pressures crossed by any two isenthalps we have by comparison,

$$\bar{c}_{p_1} : \bar{c}_{p_2} :: \Delta_2 t : \Delta_1 t \quad (d)$$

that is, the mean constant-pressure specific heats at two different pressures for the temperature intervals determined by the intersections of two isenthalps are inversely proportional to those temperature intervals. A chart of isenthalpic curves such as are presented in Charts 0, I, II, III, IX and XIII of this

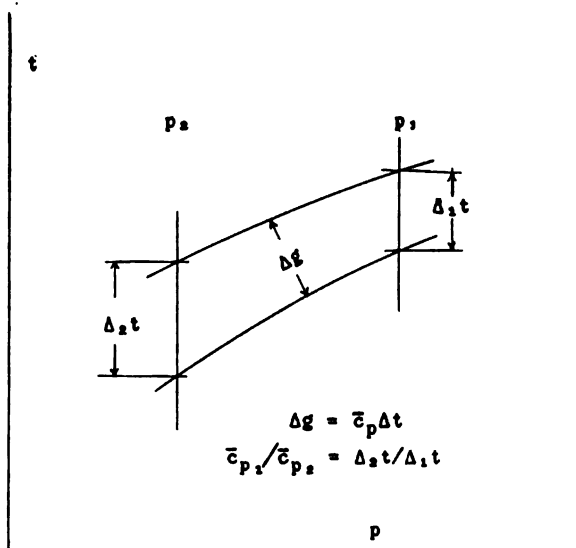


FIG. 2.

paper shows therefore at a glance the relative variation of mean constant-pressure specific heats over the field which it covers. The closer the curves converge, the greater are the relative specific heats. By considering the limiting values of the above ratios as the two isenthalps are taken closer and closer together, that is, as Δg approaches zero, an expression can be obtained showing the relation between the "instantaneous" or "point" specific heat c_p at any pressure p along any isenthalp, and the corresponding specific heat $c_{p'}$ at any other pressure p' along the same isenthalp.

If now the variation of c_p with temperature at any particular pressure be known or determined this relation can be combined with the one above to quantitatively "distribute" or evaluate the actual specific heats for all points of the field covered by the available isenthalpic curves. This of course is equivalent to determining the total heat or energy content (enthalpy) of the fluid for all such points, that content being measured from any particular isenthalp as a reference line.

Suppose that the chart of isenthalpic curves be sufficiently extensive to include both the liquid and the vapor phases of the fluid, as do several of the

charts of this paper; also that the temperature variation of the constant-pressure total heat content, that is enthalpy, of the fluid be known for a definite pressure from some low temperature of the liquid phase up to the boiling point at that pressure and on up into the superheat of the vapor phase, including the total heat of vaporization at the boiling point: then under these conditions it is possible to evaluate not only the enthalpy content for both phases of the fluid at all pressures and temperatures covered by the chart, but also to evaluate the total heat of vaporization at all points of the saturation curve thereon included if the latter curve be known; if it be not previously known the throttling experiments are sufficient to locate it as is explained in the text ahead.

Also, if the chart of isenthalpic curves extend beyond the critical point of the fluid as regards its pressure range, which some of those of this paper do (see Charts IX and XIII, p. 53 and 81) and the temperature variation of enthalpy be known over a sufficiently extended range of temperature at a pressure greater than the critical pressure of the fluid, then the information available will enable exact determination of: the saturation curve for the fluid, including the critical point; the total heat of vaporization along the saturation curve; the distribution of enthalpy over the whole field; and the variation of constant-pressure specific heat with pressure and temperature over the whole field of the chart.³

The present paper accounts the experimental development of porous plug apparatus for obtaining data for isenthalpic throttling curves of fluids; discusses the treatment of such data as was obtained for carbon dioxide, and their combination with similar data from other sources into isenthalpic curves; outlines the development of an empirical formulation of isenthalpic curves for both liquid and vapor phases of carbon dioxide including its critical point and saturation fields; compares these results with those secured by other investigators; presents in tabular and graphical forms various direct or associated properties of isenthalpic curves; reserves for later presentation the determination therefrom of the distribution of total energy content of carbon dioxide.

The experimental work leading up to and including that herein described was conducted by the writer at the University of Wisconsin under the joint auspices of the College of Engineering and the Department of Physics of the University during the years 1905-1910; it was financed by University funds, assisted toward the end of that period by grants from the Rumford Research Fund.

³ Davis, H. N., *Phys. Rev.* 2.5.359, 1915, has discussed the application of throttling curves to the determination of the properties of steam. [See also reference 31, p. 93]

GENERAL CONSIDERATIONS AND DEFINITION

1. If a fluid be caused to pass through a restricted passage such as a porous plug, Fig. 3, interposed in its line of flow, there will be established across the plug a difference of pressure sufficient to maintain that flow; there will in general occur changes in both density and temperature of the fluid which may also gain or lose energy in the form of heat exchanged with its environment during its passage through the plug.

2. When the conditions controlling the flow are such as to permit no exchange of energy in the form of heat between the fluid and its environment during the interval between the approach of the fluid to the plug and its recession therefrom, the flow is adiabatic. If the flow be adiabatic, or, what is equivalent thereto, if the net exchange of heat between the fluid and its environment during this interval be zero, then the conditions controlling the flow insofar as they effect a permanent change in the state of the fluid passing through the plug are defined by the statement that the total energy of the fluid receding from the plug is the same as that of the fluid approaching it.

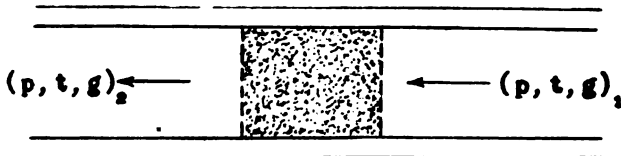


Fig. 3.

3. This statement may be expressed by the relation:

$$u_1'' + p_1''v_1'' + k_1'' = u_2'' + p_2''v_2'' + k_2'' \quad (1)$$

in which u_1'' is the internal or intrinsic energy of unit mass of the fluid approaching the plug, $p_1''v_1''$ is its external or extrinsic energy, and k_1'' is its kinetic energy due to its directed velocity of approach to the plug; u_2'' , $p_2''v_2''$, and k_2'' are the corresponding values of the respective properties of the fluid receding from the plug; p_1'' and p_2'' are the respective pressures of the fluid that would be measured by gauges carried with the fluid at its respective velocities of approach to or recession from the plug; v_1'' and v_2'' are the corresponding specific volumes of the fluid at the corresponding temperatures that would be measured by thermometers carried with the pressure gauges.

4. If the further condition controlling the flow be imposed such that the kinetic energy of the fluid due to its directed velocity of recession from the plug shall be the same as that due to its directed velocity of approach to the plug, then the conditions controlling the flow are expressed by the relation:

$$u_1' + p_1'v_1' + k_1' = u_2' + p_2'v_2' + (k_2' = k_1') \quad (2)$$

which reduces to

$$u_1' + p_1'v_1' = u_2' + p_2'v_2' \quad (3)$$

in which the terms have the significance corresponding to the specifications given in the preceding paragraph and the pressures and temperatures are determined in like manner.

5. Finally, if the conditions controlling the flow be still further restricted so that the greatest kinetic energy of the fluid due to either its directed velocity

of approach to or of recession from the plug shall be negligibly small, they are defined by the relation:

$$u_1 + p_1 v_1 = u_2 + p_2 v_2 \quad (4)$$

or,

$$u + pv = C, \text{ a constant} \quad (5)$$

in which the terms have the significance correspondingly specified above, except that the pressures and temperatures may be determined by stationary instruments properly located at entrance to and exit from the plug.

6. The function $(u + pv)$ is known as the *enthalpy** of the fluid; it is herein designated by the letter g . In what follows, unless otherwise stated, it will be assumed that the conditions governing the flow of fluid through a porous plug are such as may be defined by the statement:

$$\text{Enthalpy} = \text{Constant, or } g = u + pv = C \quad (6)$$

7. To repeat, for emphasis, it is understood that the conditions defined by this relation, (6), are that the flow of fluid through the plug is adiabatic or equivalent thereto; that the kinetic energies of the fluid due to its directed velocities of approach to and recession from the plug are negligibly small; and that all measurements of any attributes of the fluid undergoing porous plug flow are so made as to conform to these conditions.

8. It is further assumed in all the foregoing that all other possible factors that may effect changes in energy content or distribution, such as electrical, gravitational or other, are either absent or of inappreciable influence on a fluid undergoing isenthalpic transformation.

9. It is essential that these conditions be fully appreciated and adhered to in any considerations experimental or other dealing with isenthalpic properties of fluids; many of the results of experimental investigations of such properties of fluids heretofore published have been vitiated by failure to have satisfied these conditions.

10. Let (p, t, g) , Fig. 4, define the state of a fluid about to pass through a porous plug. Let Δp be the difference of pressure across the plug and Δt the accompanying temperature change that occurs as a result of the passage of the fluid through the plug under the conditions that the enthalpy shall remain constant. Then the locus of all the points $(p + \Delta p, t + \Delta t, g)$ will be the isenthalpic curve $g = C$.

11. The slope of a constant enthalpy curve on a pressure temperature diagram, that is, the limiting ratio of Δt to Δp as Δp approaches zero, is variously known as the Joule-Thomson or the Joule-Kelvin effect, after its original investigators,³ or simply the "porous plug effect." It is usually designated by μ and is here defined analytically by the relation:

$$\mu = (\partial t / \partial p)_g \quad (7)$$

12. For most gases at ordinary temperatures and moderate pressures porous plug "expansion" results in a cooling effect which for a given drop in pressure is greater the lower the temperature at which the expansion starts; it may disappear altogether, or a heating effect may occur, if the initial temperature

* The enthalpy of a fluid is sometimes called "potential", correctly, "thermodynamic potential," or "thermal head." Also, more frequently, but less definitely, it is referred to as the total heat or total energy content of the fluid. It has been variously designated by h , H , i , E , g , etc.

³ Thomson and Joule; *Phil. Trans.* 143, 357, 1853; 144, 321, 1854; 152, 579, 1862.

Joule's Sci. Papers, 2, 247.

Kelvin's Math. and Phys. Papers, 2, 387.

Harpers Sci. Memoirs, The Free Expansion of Gases, ed. by J. S. Ames.

be sufficiently high. It is the cooling effect at low temperatures however that has proven of most interest from the standpoint of practical application for it is because of this effect, coupled with counter-current heat interchange, that the liquefaction of air and other gases by throttled expansion has been effected.

13. The value of this function, $\mu = (\partial t / \partial p)_g$, as a criterion of many of the thermodynamic properties of fluids is well known and has frequently been discussed. Many scattering attempts have been made toward its experimental determination or toward its deduction from its thermodynamic relations to other properties of fluids. The reader is referred to texts on physics or thermo-

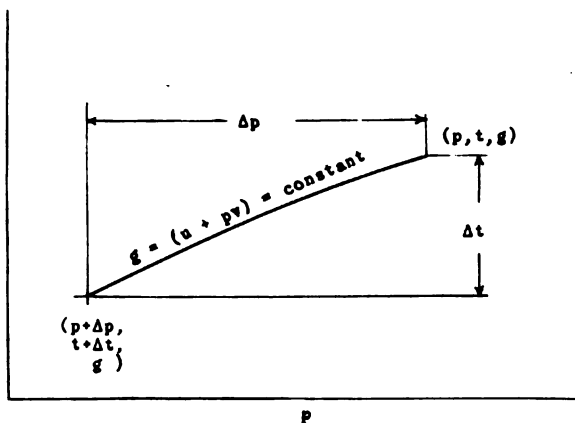


Fig. 4.

dynamics or to some of the papers hereinafter mentioned for considerations of these various relations since a general discussion of them is not a part of the purpose of this paper. The extent to which a knowledge of the thermodynamic properties of a fluid may be deduced from information concerning its porous plug effect as a function of pressure and temperature will be considered in a subsequent paper in which a quantitative determination of the thermal properties of carbon dioxide, based upon the experimental observations herein discussed and upon necessary data assembled from other sources, will be presented.

14. The present paper will be limited to an account of the experimental arrangements and procedure by which these observations were secured; to a discussion of their reduction and subsequent treatment for grouping into isenthalpic curves; to a discussion of the considerations controlling the development of an algebraic formulation of the variation of the Joule-Thomson effect for carbon dioxide over the field which the writer's experiments, combined with some others, cover; to presentation in tabular and graphical form many of the relations disclosable by the formulation; and to comparisons with the results of previous attempts by others to determine these relations.

PRELIMINARY EXPERIMENTAL APPARATUS

15. Experimental work on the porous plug effect in fluids was commenced by the writer in the year 1905 in the physics laboratory of the University of Wisconsin. In all of the preliminary work compressed air was the fluid experimented with. A three-stage belt-driven Ingersoll-Sargeant air compressor

of 30 cu. ft. per minute capacity afforded an ample supply of air compressed to 200 atmospheres, or more if necessary. It was intended to investigate the Joule-Thomson effect for air over this range in pressure and over as wide a range of temperature as could reasonably be covered, but particularly over the lower temperature range covered by the cycle of an air liquefier. The writer obtained through the courtesy of Mr. F. Wittenmeier of Kroeschell Bros. Ice Machine Co., Chicago, the gratuitous use for a period of two years, of a three-ton carbonic anhydride refrigerating machine of their make to assist in these investigations, especially for those on carbon dioxide. This machine was therefore available for pre-cooling the compressed air to initial temperatures of from -30° to -40°C as well as for direct use over a limited pressure range for driving CO₂ gas or liquid through a porous plug.

16. In initial attempts at experimental investigations it is good policy to keep the apparatus as simple as possible. As experience is gained the complication found advisable can be introduced into the simpler arrangements without making them unusable. Hence considerable time was spent in experimenting with simpler apparatus before the advantages of the "radial flow" plug were hit upon. Many forms of "axial flow"⁴ porous plugs made of numerous materials were tried, some singly, and some arranged in series. Various guard ring and other schemes for avoiding detrimental heat interchange, or for compensating for it, were investigated. Several schemes of pressure control were developed and tested. A large amount of work of this nature was done and a mass of data accumulated in learning what "not to do" as well as what to do to get fairly consistent results with plugs of the axial flow type.

17. It may be said as a result of this experience as well as of that of others that in order to obtain data of any value it is essential to secure accurate control of the initial temperature of the fluid approaching the plug, and accurate control of both the initial and final pressures of the fluid approaching to and receding from the plug, respectively. It is necessary to control the initial temperature of the fluid to nearly the order of variation allowable in reading the temperature difference across the plug; and to control both the initial and final pressures to within the variation that will cause the allowable variation in the temperature difference.

18. Adequate control of the initial temperature is most readily assured by passing the approaching high pressure fluid through a coil of metal tubing immersed in a stirred bath of liquid which also bathes the plug container, the temperature of the bath being controlled by a thermostat operating on a heating system preferably electric and immersed in the bath. Automatic apparatus for the adequate control of both the initial and final pressures is required in order also that a single observer may be able to gather the data. It is not always a simple task to maintain these devices in satisfactory operation while taking observations; it is however the only practicable method of getting consistent and reliable data, and many previous attempts have been unsuccessful due largely to lack of adequate and automatic control of these important factors.

19. No attempt will be made here to describe in detail these various efforts by the writer, or by others, that preceded his adoption in 1908 of the radial flow type of plug discussed by Burnett and Roebuck⁵ early in 1910. The

⁴ For a detailed discussion distinguishing between various forms of axial and radial flow porous plugs and their relative advantages experimentally determined as applied to steam, see paper by Trueblood, H. M., *Proc. Am. Acad. Arts and Sci.* 52, 733, 1917.

⁵ Burnett and Roebuck, On a Radial Flow Porous Plug and Calorimeter, *Phys. Res.* 30, 529, 1910.

inherent advantages of this type of apparatus are so obviously far superior to those of any other form previously tried or suggested that further effort along other lines was dropped at once. The essential principles of this arrangement, with improvements in detail, appear to have become adopted as standard for experimental work of this nature,⁶ in spite of Regnault's utter disgust over results from his early attempt⁷ at radial flow porous plug investigations to which the writer's attention was called⁸ subsequent to this present development of that type.

The advantages characteristic of the radial flow type of porous plug which led to its adoption for this work are stated in the announcement and description by Burnett and Roebuck,⁹ as follows:

Phys. Rev., Vol. 30, p. 529, 1910. ON A RADIAL FLOW POROUS PLUG AND CALORIMETER. In any investigation involving the accurate determination of the thermal properties of fluids, the freedom of the experimental apparatus and methods from inherent contributory errors of more than negligible value is of the utmost importance. Where, as is usually the case, the fluid under investigation must pass through several parts of the same or successive pieces of apparatus whose temperatures may differ, the external variants can seldom be under proper control nor can their disturbing influence be sufficiently determined. Undesired and frequently indeterminate interchange of heat by conduction and radiation between the fluid and its surroundings and between the temperature-measuring devices and their surroundings, affect the temperature so measured to an uncertain extent. In the ordinary form of porous plug and fluid calorimeter where the flow is continuous and linear, the most serious errors arise from the complicated radiation and conduction losses unavoidably involved with any temperature gradient between the points at which the temperatures are measured. It is the purpose of this note to describe an experimental arrangement in which these errors are avoided by the use of a radial fluid flow and accompanying radial temperature gradient whose inherent freedom from all detrimental heat interchange is demonstrated in detail below.

For several years past Mr. Burnett has been engaged in an investigation of the Joule-Thomson effect in various gases, and much of the time has been spent in overcoming experimental difficulties of the disturbing nature above indicated. Dr. Roebuck has more recently undertaken some work elsewhere noticed (*Phys. Rev.*, 1, 30, 129, 1910) involving apparatus of a similar nature. During one of the many discussions of our respective researches we evolved a scheme of apparatus and procedure which are almost ideal and of wide application.

The fluid under investigation is brought to any desired initial temperature, and maintained there by passing through a coil immersed in a bath controlled by a suitable thermostat. After passing through

⁶ Roebuck, J. R., A Porous Plug Method for the Determination of the Mechanical Equivalent of Heat, *Phys. Rev.* 2, 79, 1913; The Reversible Expansion of Water, 2, 299, 1913; The Joule-Thomson Effect for Air, *Proc. Am. Acad. Arts and Sci.* 60, 537, 1925.

Trueblood, H. M., (Ref. No. 4).

Hoxton, L. G., The Joule-Thomson Effect for Air at Moderate Pressures, *Phys. Rev.* 2, 13, 428, 1919.

⁷ Regnault, H. V. *Relation des Experiences*, 3, 700.

⁸ In 1910, by the late E. B. Ross of the Bureau of Standards.

the coil its temperature is measured by a thermometer immersed in the fluid; under these conditions the thermometer, fluid and walls being continuously at the same temperature, radiation and conduction errors are avoided.

The next part of the apparatus which may be described as the "radial flow" portion constitutes the essential improvement. It is situated in the bath with adequate thermal insulation therefrom. Figs. 5 and 6 represent diagrammatically the principal features of two cases. In the porous plug form this consists of a round-ended porous tube of suitable non-conducting material, *e.g.*, porcelain. In calorimetry it consists of two concentric opaque tubes offering, however, practically no resistance to the flow of the fluid which is electrically heated as it

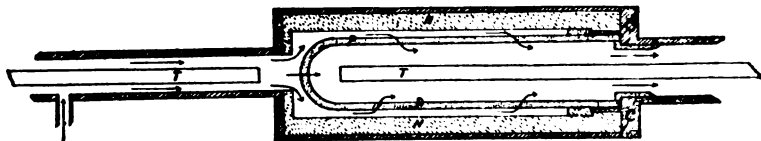


Fig. 5.

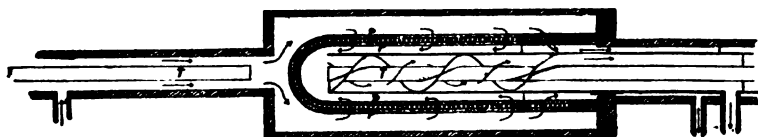


Fig. 6.

crosses the space between them. The fluid is led toward the outer surface of these non-conducting tubes, flows radially through them, gathers within, and is led away through a passage preferably insulated from the bath. The second temperature is measured within the porous tube by a thermometer whose stem is thus protected by the outflowing fluid.

Two cases are to be distinguished, first when the fluid passes through the porous partition with a rise, and second with a fall of temperature. In the first case the conduction of heat by the porous walls is opposed to the flow of fluid and only very small flows would be necessary to completely offset this minute conduction; as a consequence both outer and inner surfaces of the plug would be exactly at the temperature of the fluid and thus both radiation and conduction losses would be entirely avoided. In the second case the conduction takes place in the same direction as the flow, but conduction from the outside can only carry heat from the fluid entering to the fluid leaving the walls and therefore cannot influence the thermometer readings. Its total effect will be to shift slightly the temperature of the plug surfaces from that of the passing fluid, and so might give rise to appreciable radiation errors. The minute conductivity of porous material and the intimate contact of the fluid with the wall render this error negligible in most cases. Insulation of the porous tube from its container in the bath by suitable non-conducting material, confining the fluid flow to the region adjacent to the porous wall, will minimize any possible radiation loss from the plug, or other interchange of heat after its initial temperature has been measured.

In both cases the support of the porous tube should be arranged to prevent radiation to or from the second thermometer. When used as a porous plug the effect of axial conduction along the porous wall from its support is without influence on the second thermometer reading because the fluid which passes close to the support does not come in contact with the bulb of the thermometer. When used as a calorimeter the fluid must be mixed before passing the thermometer. The end effects near the tube support can be eliminated by a guard ring arrange-



Fig. 7. View of air compressor as left by explosion.

ment. This can be accomplished by respective division of the heat supply and the fluid flow as it passes through the porous walls and leading the divided flows away independently.

Either apparatus may be so designed as to be compact, self-contained, yet flexible, and adapted to a wide range of temperature and pressure without departing from the essential features of merit.

20. Considerable time and effort were devoted to the design, construction and testing of radial flow porous plug apparatus before these arrangements were perfected to the usable stage presented in the following pages, and again many series of observations were made before consistent results could be obtained. A host of minor time-absorbing annoyances incident to the perfection of the more complicated radial flow plug apparatus had to be ferretted out and eliminated: the methods of its mechanical support in the apparatus; the control of its porosity, etc., were problems that had to be solved before continuity of final porous plug investigations was possible.

21. Then in the summer of 1909, just as the apparatus and the method of handling it had apparently reached that desired stage of development, the air compressor, situated in the other end of the same room as the experimental

set-up, exploded some of its connections, put itself and a part of the apparatus out of commission and sent the writer to the hospital for repairs.* Later in the year the work was resumed; the damaged apparatus was repaired; but the offending compressor had been banished and the time of its replacement was indefinite. It was therefore decided to utilize the uninjured carbonic anhydride ice machine compressor, operate with CO₂ instead of air and investigate the porous plug effect for CO₂ over the limited pressure range which the single small cylinder of the compressor, with its double-acting leather-packed piston, would permit. This outfit limited observations practically to



Fig. 8. Fragments collected resulting from explosion.

within the pressure range of about 50 atmospheres, extending from 20 or 25 atmospheres to 70 or 75 atmospheres, a limitation which resulted in increasing the difficulties of interpretation of the data acquired.

RADIAL FLOW POROUS PLUG APPARATUS

22. The apparatus described in the following paragraphs is that actually used by the writer in 1910 to secure the data herewith presented. It is illustrated in detailed general assembly on Plate I drawn substantially to the scale thereon shown except for the electrical circuits which are largely diagrammatic. Fig. 9 is a diagrammatic sketch of the principal part of the apparatus; it shows the heat interchanger, thermostatic bath arrangements, the porous plug, and the fluid circulation. The simplest plan of description is to outline the path of the fluid, following it from the compressor through its circuit of the several parts of the apparatus back to the compressor again, and to describe with sufficient detail each element as it is reached, or in later paragraphs when advisable.

*See page 96 for note regarding effects and probable cause of explosion.

23. From the compressor the fluid was conducted through the valve V_0 to a drying tube A that contained calcined calcium chloride through which it passed in an upward direction: thence through a high pressure regulator B_n , the operation of which is discussed in detail under the heading, **Pressure regulation**, in paragraphs 45 to 50 inclusive: thence to a heat interchanger (so labelled). The heat interchanger consisted of parallel coils of small copper

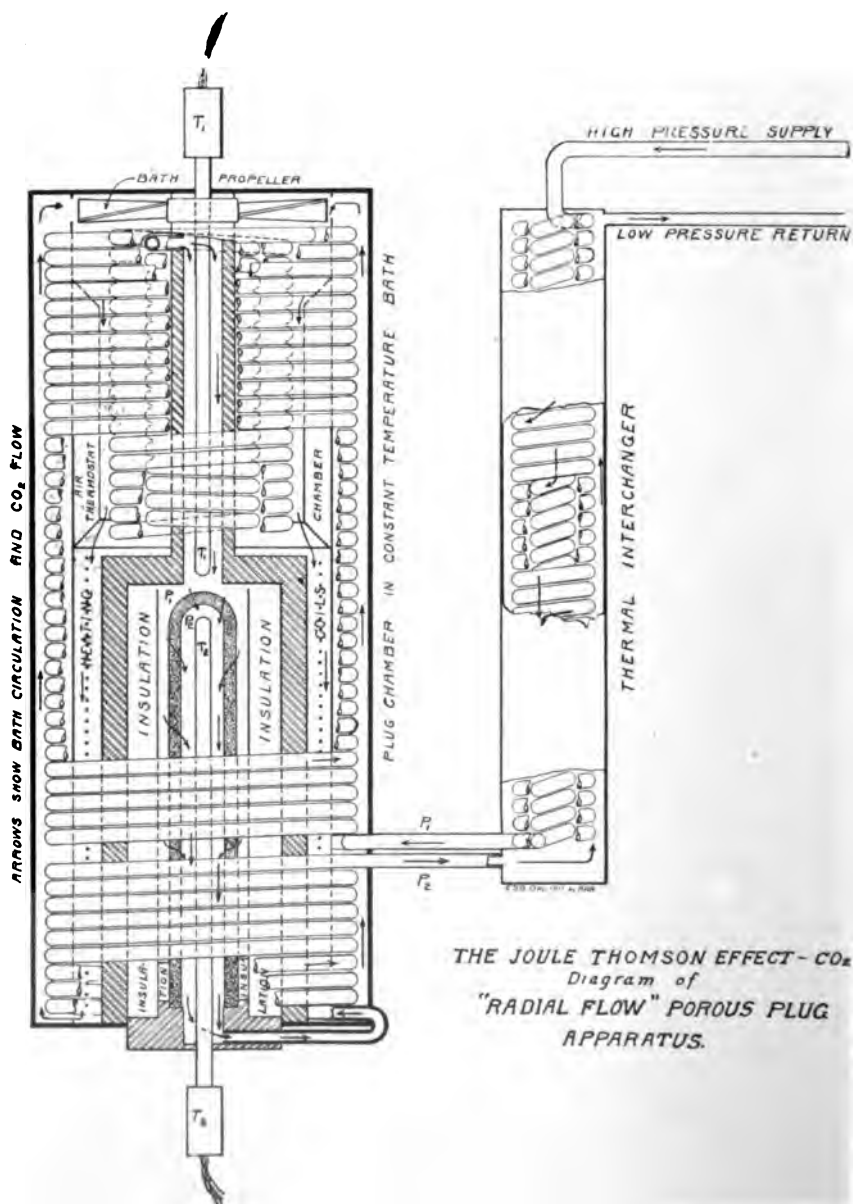


FIG. 9.

tubing **W** (only one coil is shown) located inside of a steel tube **C** (of extra heavy inch and a half steel pipe) which carried the return flow counterwise up past the incoming high pressure fluid going down within the tubes **W** on the so-called "regenerative" or heat-interchanging system of an air-liquefier. The object of this interchanger was to bring the incoming high pressure fluid from the temperature that it had attained after leaving the compressor and arriving at the interchanger to the temperature of the outgoing low pressure fluid that was leaving the bath (described in the next paragraph) after its expansion through the plug.

24. Having passed through the interchanger the fluid arrived at the principal part of the apparatus the parts of which can easily be traced from the illustration on Plate I. The central portion **EDF**, shown as one piece, consisted of two cold-drawn seamless steel tubes **E, F** threaded and soldered to a connecting steel block **D** at their junction. The larger of these tubes, **E**, was about four inches (10 cm) outside diameter by three inches (7½ cm) inside diameter, and about twelve inches (30 cm) long. It was threaded and soldered at its lower end to a steel base **G**. A groove turned in the upper side of this base **G** near its periphery received a casing **H** made of a two foot (60 cm) length of six-inch (15 cm) steam pipe which supported in a similar manner at its upper end a steel head **I**. Lead packing was placed in the grooves and the whole drawn tightly together by the central nut **J** threaded onto the upper end of the smaller, **F**, of the two steel tubes mentioned. A lead packing ring, shown black, under the nut **J** served to seal the joint between the steel tube **F** and the head **I**. The space between the casing **H**, the tubes **E, F**, the base **G**, and the head **I** served as a container for the liquid of the constant temperature bath for the apparatus.

25. Within this bath chamber were two long coils **U, L**, of one-eighth inch pipe-size copper tubing measuring about 1.0 cm outside diameter by 0.6 cm inside diameter. The upper coil **U** led from the high pressure connection **K** from the interchanger coils **W** and wound spirally upward along the casing **H** to the top of the bath chamber, then crossed over toward the center, wound downward along the central steel tube **F** to its junction **D** with the larger one **E**, then outward and upward again toward the top where it connected with the interior of the steel tube **F** near its upper end, at **M**. The total length of this coil **U** was approximately 90 ft. (27.5 metres). As the fluid passed downward within tube **F**, after flowing through the long copper coil **U** which was immersed in the bath liquid, it surrounded the thermometer **T₁** which, located at the axis of tube **F**, measured its initial temperature; the fluid then emerged at **N** into the plug chamber, the interior of the tube **E**.

26. The porous plug, **P**, an unglazed tube of porous porcelain of about 4½ cm outside diameter by 2½ cm inside diameter by 25 cm length, with upper end closed and dome-shaped, was copper plated at its open base **O** and there soldered into a steel socket **S** that was pressed upward against shoulders cut in the large steel tube **E** which formed the plug chamber. A second block of steel **R** which was pressed against this socket **S** by contact with an intermediate ring of lead, shown black, supported the socket **S** from below and was itself in a similar manner supported by the outer threaded steel block **Q** that could be screwed up by application of a spanner wrench thus insuring gas tight joints at all surfaces in contact with the lead ring gaskets. The intermediate block **R** served a triple purpose; besides supporting the steel plug socket **S** in place it also served to support at **T₁'** the thermometer **T₁** (not shown) which entered axially into the plug tube **P** for the measurement

of the final temperature of the fluid at T_2 ; and it, R , was so shaped and drilled that in connection with the socket S , above, it formed a radial passage (indicated by the arrows above T_1') for conducting the low pressure fluid discharging from the plug to an annular space V turned in the steel base G from which annulus V the fluid was gathered and conducted out through the passage X shown at the right of G .

27. From the passage X the fluid entered a copper coil L about 27 ft. (8 meters) long and of the same size piping as coil U , located in the lower part of the bath chamber; in passing through coil L the fluid was brought back approximately to the temperature of the bath. Thence it was delivered through the connections Y, Z and the valve V_2 to the interchanger tube C in which it passed upward outside of the high pressure coils W within the tube C and in a direction counter to the flow of the high pressure fluid within those coils. This allowed an interchange of heat between the counter currents of fluid; it brought the temperature of the high pressure flow toward that of the bath and the temperature of the low pressure return flow toward that of the room (or toward that of the high pressure fluid arriving at the interchanger from the high pressure regulator B_H). This feature was especially useful when operating with bath temperatures below that of the room, for since the plug effect for the experimental range covered is a cooling effect the apparatus served as its own refrigerator by accumulating the refrigeration until that was offset by the heating effect of the electric current applied by the temperature controlling apparatus.

28. From the interchanger the fluid leaving through valve V_3 passed through a low pressure regulator B_L similar to the one B_H for the high pressure flow both of which are described in paragraphs 45 to 50 below. The low pressure regulator B_L served to maintain a constant final pressure on the inside of the plug tube P . From B_L the fluid was conducted back to the compressor for recompression thus completing the circuit of the apparatus.

29. **Insulation of the porous plug.**—The plug P was held axially within the plug chamber, the interior of steel tube E . A brass tube a , shown by heavy lines in the figure, telescoped into tube E and carried within it a roll of asbestos paper wrapped about a glass tube b concentric with and a little larger than the porous porcelain plug tube P . Both the brass tube a and the glass tube b were secured to a common brass base c , and together with the asbestos roll constituted a unit which could be handled independently. The asbestos projected slightly beyond the ends of the tubes a, b which were a trifle shorter than the plug chamber so that the pressure of the plug socket S , below, crowded the heat insulating asbestos against the end D of the plug chamber thus effectually precluding any free space along its walls where the fluid might have wandered on its way to the plug after its initial temperature had been measured. The flow was thus confined to a space (shown by the divergent arrows at N) immediately adjacent to the walls of the porous plug and, presumably, the fluid was thereby protected from appreciable cooling influence of the lower part of the walls E which influence will be discussed later. Of course the fluid permeated the whole plug chamber at the pressure of supply, but since that was constant there could have been no appreciable convection currents therein.

30. **Temperature regulation.**—The bath chamber—the space between the casing H , the tubes E, F , the base G and the head I —was divided by a long brass tube, d , shown black in section, that extended the full height of the chamber but had openings at each end to allow circulation of the bath liquid.

This circulation, shown by arrows, was downward within the tube *d* and upward without it, or vice versa, as determined by the rotation of the propeller *p* which, located at the top just below the upper head *I*, was driven by means of the spur gear *g* and pinion *e* from the pulley *f* above.

31. Along the upper half of this brass tube *d*, and within it, was built an annular space *od* with brass walls. This space, labelled "air chamber of thermostat," was hermetically sealed except for its connection *o* at the top with a fine brass capillary tube *h* which was connected through the atmospheric valve block *V_a* to the glass capillary *i* of the barometric thermostat (so labelled) shown on the left on Plate I. The annular space *od* constituted an air chamber of considerable volume and of surface large in proportion; it functioned as the bulb of an air thermometer the pressure in which was balanced against a barometric column of mercury so that a slight change of temperature of the air in the chamber caused a corresponding change in the level of the mercury meniscus at *m* which effect was utilized to make or break at *c*, a relay circuit that controlled the electric current applied to the heating coils. By means of the mercury reservoir and stop-cock at *n* the level of the mercury meniscus at *m* could be adjusted when necessary or desirable.

32. **Adjustment of the thermostat.**—In use the atmospheric valve *V_a* was left open until the bath had attained the desired temperature thus permitting atmospheric pressure to prevail within the air chamber of the thermostat and over the mercury meniscus at *m*; the latter was brought to the level of the upper of two sealed-in electrical contact points, depending from above, by adding or removing mercury through the stop-cock *n*; valve *V_a* was then closed and the thermostat was in condition to function provided that the electrical heating circuits and currents were properly adjusted.

33. **The scheme of automatic control of the heating currents.**—The scheme of automatic control of the heating currents was so arranged that the operation of the relay threw into or out of circuit a part *c₁c₂* of the external resistance *c₁c₃* of the bath heating circuit; both this part *c₁c₂* and the remainder *c₂c₃* of the external resistance were independently variable. The internal resistance of the heating circuit, shown as dots between the steel tube *E* and the brass tube *d*, consisted of coils of nichrome wire disposed without the plug chamber and directly immersed in the oil baths used. It was divided into two circuits of unequal resistance which could easily be thrown into either single, series or parallel connection. (Only one circuit is shown on the diagram, but three terminals are indicated.) The thermostat and heating circuits were further so arranged that by operating switch *sw* either a part *c₁c₂* or all *c₁c₃* of the external resistance *c₁c₃* could be automatically thrown into or out of circuit. All of these arrangements served to render the adjustment and automatic control of the system very flexible, convenient and reliable.

34. Kerosene oil was used as the bath liquid for the lower range of temperatures and a heavier grade of mineral oil for the higher range of temperatures. An expansion tank (marked "oil reservoir" on the Plate) was arranged above the bath chamber to take care of temperature changes, leakage and evaporation of the oil, and to receive in the manner shown the oil that crept up the shaft and shaft housing *k* of the pulley *f*.

35. **Pressure measurement.**—Pressure leads were taken off from the lower part of the steel base *G*, as shown at its left, and brought to a twin-valve-block (labelled "pressure switch-block") so made that either pressure could be switched to the pressure gauge and free-piston manometer. The change from

high pressure to low pressure connection, or the reverse, was accomplished by throwing a bale handle (labelled "valve reversing handle") which closed one valve V_H and opened the other V_L at the same time, or vice versa. The operation should be evident from the figure.

36. The high pressure lead was brought from the lower part of the plug chamber, as shown; it was fed by fluid that permeated the asbestos insulation and traversed the slight clearance between the brass container a of the insulation and the large steel tube E about it. As there was no flow through the lead at the time that a pressure measurement was being made the pressure measured at the manometer was necessarily the same as that prevailing at the outer plug surface and therefore the same as that of the fluid having the initial temperature.

37. The low pressure lead was taken from the annular space V in the base G that collected the flow as it was delivered radially outward from the lower end O of the plug P in a manner that is evident from the sketch. Its point of connection to the annulus V was directly opposite the exit X , which is shown on the right, where the flow was a maximum. From the drawing it may seem that the apparently tortuous passages between the end O of the plug and the exit pipe X might have introduced an appreciable pressure gradient. This was suspected and tests to determine its magnitude were made by connecting a differential mercury manometer between those two places. The maximum difference of pressure at a time of greatest flow* in these experiments with CO_2 was found to be of the order of 0.01 of an atmosphere and hence of negligible account at all times.

38. For much of the preliminary work with this apparatus a Bourdon spring gauge of the usual bent tube type was used, one made expressly for this work by Schaffer and Budenberg. As the work progressed this gauge was found to be unsatisfactory, partly because of the difficulty of calibrating it accurately, and partly, it was thought, because of inaccuracies of its mechanism which later was rebuilt. In place of the toothed sector and pinion and rather crude linkage system of its indicator there was substituted a system consisting of pivot-bearing smooth-faced sectors and pinion connected by thin phosphor-bronze ribbons. Mechanically these improvements were all that could be desired, but they served to bring out very distinctly the most serious defect inherent in instruments of this type which becomes important when attempts are made to use them for the purpose of precise pressure measurement, namely, the elastic lag or hysteresis of the material of the tube. It was found that the uncertainties of pressure measurement due to this defect, especially with reference to the determination of relatively small differences between large pressures, could not be satisfactorily overcome.

39. The free-piston manometer[†] is inherently free from elastic lag effects, at least for the pressures involved in these experiments, and although cumbersome and somewhat complicated its indications are very reliable; it was therefore adopted. The device used is shown to the left on the Plate and is drawn to scale. As it was originally made for a somewhat different method of application than the one indicated and here described its design may seem a little peculiar; it was adapted for the sake of economy and as remade gave excellent results.

40. It consisted of a series of three pistons arranged to rotate about a common vertical axis in cylinders carefully ground to size. The upper piston

* After E. H. Amagat, *Annales de Chimie et de Physique*, Series 6, Vol. 29, 1893.

† See par. 75 and 76 pp. 27-29.

u, the smallest of the three, received the (high) pressure to be measured. Its load was transmitted to the large "H-sectioned" piston w in the middle of the apparatus; this piston w had bearing surfaces toward each end of its length but was reduced slightly in diameter there-between. Connected to w from below was a third and smaller piston v that extended downward through a close fitting cylinder; by means of the slender stem s and the pulley attached thereto, piston v could be rotated. All three pistons were connected together by universal joints as suggested by the sketch, and therefore all rotated together when the pulley was turned. It will be observed from the sketch that the upper part of the device was originally symmetrical with the lower part, both of the smaller pistons having had the same diameter as the lower one shown; in making the change mentioned above, the lower one, except for the addition of the stem and pulley, was left intact since its form and size were definite: a sleeve y was inserted in the upper part to carry the reduced small piston u there shown.

41. The area of the lower piston v was intended to have been one-half square inch; that of the upper small piston u one-fifth square inch; and that of the large H-sectioned central piston w five and one-half square inches. The difference between the area of the large piston w and that of the smaller one v below it was, therefore, approximately fifty times the area of the smallest and uppermost piston u.

42. Castor oil was placed above the upper piston but separated from the fluid the pressure of which was to be measured by a mercury seal (so indicated on the sketch) which prevented absorption of the fluid by the oil and its subsequent dissemination throughout the connecting tubes when the pressure was lowered. Castor oil was also placed in the annular space x between the large cylinder surrounding the piston w and the small piston v below. This space x communicated through the connection l with a glass-walled cylindrical cup z of approximately the same net sectional area as that of the annular space x. The lower half of the cup z was filled with mercury upon which rested the castor oil. The weight of the three pistons u, v, w, with the stem s and the pulley, plus any pressure superimposed upon the small piston u above, was transmitted through the oil to the mercury in the cup z and was effective in raising the mercury in the connecting manometer tube to such a height as to balance the pressure causing the displacement. When in use the three pistons were kept in slow rotation by a small motor driving the pulley. Motion was communicated to the pulley by means of a linen tape belt running on a vertical counter shaft parallel to the axis of the manometer and at some considerable distance therefrom, precaution always being taken to avoid any disturbing effects from possible misalignment.

43. Careful measurements of areas and other calibration factors which are discussed under the sections dealing with the calibration of apparatus gave the relation that a change of level of the mercury meniscus of the mercury column of 100 cm meant a change of 64.95 atm. of applied pressure.

44. **Pressure regulation.**—In work of this nature dealing with the flow of fluid between differing pressures it is absolutely essential to be able to maintain those pressures constant for considerable periods of time. In the preliminary work much difficulty was experienced on this account and many attempts were made to devise automatic pressure controlling apparatus. Finally, however, the devices shown on the Plate were evolved and were found

to give eminent satisfaction. The principle and construction of each are simple and when properly proportioned the operation is effective.

45. Consider first the high pressure regulator B_H on the Plate. A diaphragm of thin flexible sheet metal (copper sheet about 0.003 inch, 0.075 mm, was used) was pinched gas tight between two steel flanges that were "dished," somewhat less than shown, on their faces adjacent to the diaphragm; the latter supported at its center a valve-stem a trifle smaller than the drilled hole in the upper flange in which the valve stem operated. The proportions of the diaphragm and the "dishing" were such that at the extremes of motion of the former it was entirely supported by the latter; at the same extremes of motion of the diaphragm the base of the valve-stem at its attachment to the diaphragm was supported by recessed seats in the flanges; these provisions eliminated the liability of excess pressure on either side shearing the diaphragm from its attachment to the valve-stem. Near its upper end the valve-stem was provided with guides, not indicated in the figure, for the purpose of maintaining it in proper operating alignment with its housing. The valve-stem terminated in a tapered pin which extended through a small hole above that at times could be closed by a small steel or bronze ball confined to the space above the hole which connected directly with the fluid supply. The device functioned as a mechanically operated ball-valve between the fluid supply above the ball, the pressure of which was variable, and the fluid delivered to the outlet at the side of the valve-stem (toward V_i) the pressure of which was to be kept constant. The fluid delivered had free access along the valve-stem to the space above the diaphragm and its pressure of delivery prevailed there. The space below the diaphragm was connected to a receiver of suitable capacity, such as a 20 pound CO_2 drum, as shown attached to the low pressure regulator B_L , in which a constant gas or vapor pressure could be maintained but subject to variation at will. Connections for charging these drums without disconnecting from the regulators were provided but are not shown on the drawing.

46. The operation of the high pressure regulator B_H was as follows: When the pressure delivered became less than that prevailing below the diaphragm the latter was lifted and the ball valve opened thus admitting more fluid from the supply above. When enough had passed to reverse the balance of pressure on the diaphragm the latter was depressed which permitted the ball to drop over the hole and interrupt the supply. This operation was repeated at a very rapid rate for a relatively small difference in pressure distributed over the large area of the diaphragm was immediately effective. It is possible that for a steady condition of flow the ball valve was held just enough open to have passed the required amount; at times a constant delivery pressure would be maintained accompanied by a humming of high frequency, and again, under apparently the same conditions, the action would be inaudible. When everything went well the delivered pressure was observed to remain constant to the order of but one or two hundredths of an atmosphere variation at seventy atmospheres pressure, for an hour at a time, which was usually longer than required.

47. The low pressure regulator B_L operated on a similar principle but since the direction of flow of fluid was reversed the valve action also had to be reversed. A ball valve could not easily be used and more care had to be taken to secure accurate adjustment of valve to seat at the extreme upward movement of the diaphragm. As constructed, though not so shown on the dia-

gram, the valve seat in B_L could be raised or lowered by use of a screw-driver inserted through the lower part of the pipe union at the top of the upper flange when that union was uncoupled.

48. These two devices gave splendid results and they supplied a great necessity, that of accurate and automatic control of pressures; one moreover that could be easily adjusted by varying the control pressures in the receiver. It is of course obvious that the receivers and all connections below the diaphragms had to be absolutely tight, and that the compressed gas or vapor in the receivers had to be kept at an unvarying temperature. A slowly varying temperature was not seriously disturbing because its effect could be offset by adjustment of some of the valves between the two regulators as described in the following paragraph. When either CO₂ gas, or CO₂ vapor in contact with its liquid was used in the receivers for control pressures as was sometimes done, it was found both desirable and necessary to control these pressures by controlling the temperatures of the receivers and their contents. This was done by immersing the drums in a stirred water-bath subject to easily adjustable thermostatic control. Details of these arrangements are, however, of non-essential mention here. The use of a saturated vapor in contact with its liquid for control pressures in the receivers of these pressure regulators was advantageous for two reasons worth noting; the effect of a slight leak in the system between the diaphragm and the receiver was reduced or entirely eliminated so long as sufficient liquid remained in the receiver, and the pressure in the control system could easily be varied by varying the temperature of the water bath around the receivers. It should perhaps be noted here that some modifications of the design here shown are necessary to render the action of these devices entirely satisfactory as regulators for the flow of liquids.

49. **Auxiliary pressure control.**—In order to obtain experimental data to satisfactorily establish isenthalpic curves for porous plug flow of fluids it is essential that all the flows for a given curve should either take place from the same initial condition of temperature and pressure, or that they should be consecutively contiguous to each other in regard to their respective initial and final conditions of temperature and pressure. In these experiments the attempt was made to make all flows designed to establish such a curve start from a fixed initial temperature and pressure; consequently the amount of flow, measured in mass of fluid passing through the plug in unit time, would vary more or less directly with the pressure drop established across the plug, and also with the porosity of the particular plug in use or with any change in its porosity which usually gradually decreased with use. A pressure gradient between the high pressure regulator and the plug necessarily existed which was subject to variation with the flow; on this account, and whenever, for any reason, the control pressure below the diaphragm of the high pressure regulator changed appreciably during a run, it was necessary to adjust a valve V₁ (located, on the Plate, at the delivery from the high pressure regulator) between that regulator and the plug in order to maintain a constant initial pressure at the plug surface as determined by observations of the mercury column of the free piston manometer. Once set, this valve did not require further adjustment until it was desired to change conditions. Valves V₂ and V₃ respectively located at entrance and exit of the return flow through the interchanger tube C, permitted adjustment of the pressures at the discharge surface of the plug, and of the pressure drops at either end of the interchanger for the purpose and in the manner described in the following paragraph.

50. When the pressure drop across the plug was small compared to the pressure drop from the plug to the low pressure regulator, valves V_1 and V_2 could be so set as to throttle the flow from the plug to the low pressure regulator and thus lessen the demands on that device, which tended to improve its action. Also, the setting of valves V_2 and V_3 determined whether the cooling effect made available by the expansion of the fluid from its state on leaving the tempering coil L in the lower part of the bath chamber to its state after passing these valves should be operative within the interchanger for pre-cooling the incoming high pressure fluid in the coils W as would occur if it were throttled at V_2 and unrestricted at V_3 , or whether this cooling effect should be discarded beyond the interchanger by throttling the flow at V_3 and not restricting it at V_2 . Obviously a part of this "after-cooling" beyond the plug could be applied in the interchanger and the remainder discarded beyond it, in any proportion, by throttling more or less at either or both valves as desired. When operating at low bath temperatures, below room temperature, most or all of the throttling was done at V_2 so as to return to the bath through the interchanger the cooling-effect so made available; in fact no other means was used for reducing bath temperatures below room temperature. When operating at high bath temperatures, above room temperature, most or all of the throttling beyond the plug was done at V_3 since it was undesirable to throw back onto the bath heating arrangements the load of off-setting this excess cooling. The cooling due to the flow through the plug however had in any case to be off-set by the bath, mostly immediately in coil L before passing to the interchanger, or later in coil U after its transfer in the interchanger to the incoming high pressure fluid. No provision was made for by-passing around the interchanger any part of the discharge from the plug as from the point X directly to the low pressure regulator; such an arrangement would have been desirable under certain conditions, particularly for the higher bath temperatures and for the larger pressure drops at those temperatures, since it would have materially decreased the load on the temperature regulating devices.

51. **Sampling and analysis arrangements.**—A valve V_4 was provided at the base of the interchanger through which samples of the fluid soon after it left the low pressure side of the plug could be drawn off for analysis. In the work with carbon dioxide a modified Orsat gas analysis apparatus was used. Tests were made to determine the content of carbon dioxide, carbon monoxide, oxygen, and occasionally for hydrogen. Further reference to these analyses and the method of making them will be made later. (See pars. 72 to 74 p. 27.)

52. **Temperature measurements.**—The initial and final temperatures of the fluid before and after the plug were measured at the points shown on the Plate at T_1 and T_2 respectively, by two platinum resistance thermometers secured from the Cambridge Scientific Instrument Company. The thermometer cups or cases were tubes of brass made as thin as would safely withstand an external pressure of 200 atmospheres. Each thermometer "head" was provided with a device for insuring that only dry air could enter its tube and surround its platinum coil. One of the thermometers, the initial and upper one, T_1 , is shown in place, on the Plate; the other, the final and lower one, T_2 , is not shown, but its location and method of support within the plug is apparent from the drawing. Each thermometer was secured in its respective socket by a swivelled head screwed against a lead gasket for tightness; thus

twisting of the soldered-in lead-wires on inserting or removing thermometers was avoided. The resistances of the thermometers were adjusted by the makers to give fundamental intervals of almost exactly ten ohms each.

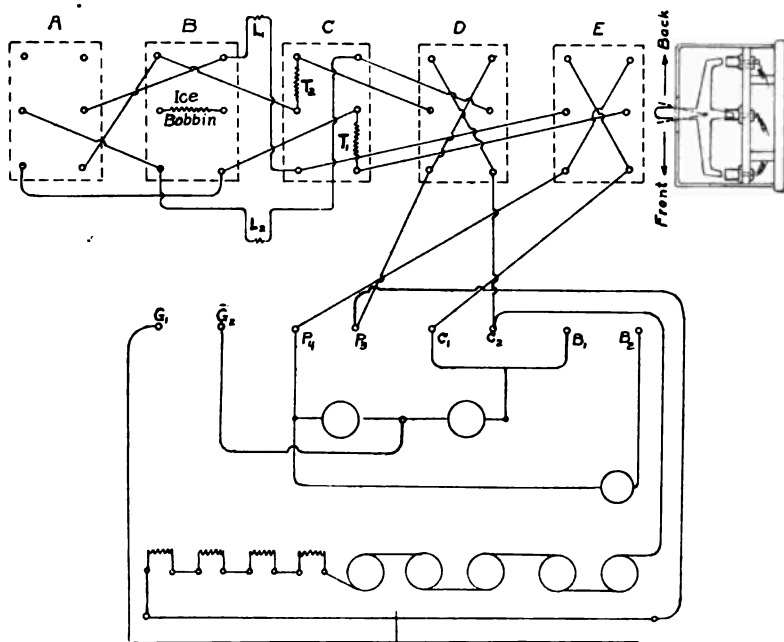


FIG. 10. Diagram of Callendar and Griffiths Wheatstone bridge circuits, mercury-cup-contact double-pole, double-throw switches, and interconnections.

53. A special mercury-cup-contact Callendar and Griffiths Wheatstone bridge (Fig. 10) with binary scale resistance "box" was supplied by the Cambridge company; it was designed to read directly "platinum" temperatures when the fundamental intervals of the thermometers used with it were either one ohm or ten ohms. The bridge box contained an oil bath for immersion of the resistance coils in order to keep them at a uniform temperature; an oil stirring attachment and a thermostat, both built at the laboratory, were used a part of the time for maintaining a constant resistance-coil temperature. An "ice-bobbin" made at the laboratory was wound with resistance sufficient to balance the resistance of each thermometer at the ice point; this was placed in the bridge box and was used alternately with each thermometer; it was not in use when the thermometers were differentially connected.

54. Four-ply cable-leads, of copper, not shown on the drawing, were soldered to fusible terminal connections on the thermometers and brought to a special mercury-cup-contact switch-board, Fig. 10, that was made at the laboratory shop; leads from this were carried to the bridge, battery and galvanometer circuits, all junctions being either soldered or mercury-cup. All leads and switch-board connections were of copper. By means of the switch-board either thermometer could be thrown into proper connection with the bridge, or both could be differentially connected therewith so as to measure

the difference of their resistances and therefore directly the difference between the temperatures of the fluid approaching and leaving the plug.

55. A d'Arsonval type galvanometer with telescope and scale was arranged for use so that one mm of scale deflection corresponded to a temperature change of approximately 0.001°C, though much of the time a less sensibility was used.

CALIBRATION OF APPARATUS

56. **Resistance bridge and platinum resistance thermometers.**—Four calibrations of the binary scale resistances of the Callendar and Griffiths' bridge, by their method of substitution with the bridge wire, were made during the course of these experiments. The corrections, in terms of equivalent bridge-wire resistance, to be applied to the nominal values of the bridge box coils in interpreting the observed resistances of the platinum resistance thermometers T_1 and T_2 were obtained by averaging the results from these four calibrations. The fundamental intervals of the two thermometers were also checked several times during the same period by comparison of their resistances at the ice and steam points.

57. From these data a table was prepared based upon the nominal values of the bridge box coils arranged in ascending series increasing by steps of one "box unit" from -28 (bridge reversed) to $+128$, giving the corresponding centigrade temperatures for each thermometer; all box coil corrections, corrections for differences of fundamental intervals, and the " δ " correction to transform "platinum temperatures" to equivalent centigrade temperatures were incorporated in this table. The value 1.57 was supplied by the makers for the " δ " of the platinum wire of these thermometers; and since the range of temperature covered by these experiments was only from -20°C to $+120^\circ\text{C}$ no third point calibration was considered.

58. There were some variations in values shown by these several calibrations of both bridge and thermometers that are difficult to account for but their effect is of small moment compared to those due to variations in experimental results arising from other causes that occurred during the experimental runs. It seems therefore unnecessary to present either the detailed figures of these standard routine calibrations or the extended table mentioned in the preceding paragraph for transforming observed thermometer resistances into corresponding centigrade temperatures.

59. **Differential free-piston manometer.**—Careful measurements of the effective diameters of the three pistons in their corresponding cylinders gave the following values:

upper piston, u , effective diameter, 0.3595 inches
 lower piston, v , effective diameter, 0.7980 inches
 middle piston, w , effective diameter, 2.6463 inches

These values yield the ratio, $[(\text{area of } w) - (\text{area of } v)]/(\text{area of } u) = 49.26$. Cathetometer measurements showed that a vertical displacement of the rotating pistons following a change of pressure superimposed on the upper piston u caused a displacement of mercury level in the well z equal to 0.0073 of the corresponding displacement of the mercury level in the mercury column, but of course in the opposite direction. The total vertical displacement of the mercury levels was therefore 1.0073 times the displacement observed to occur in the mercury column of the manometer.

60. The specific density of mercury at the latitude and elevation of the laboratory and at an average summer temperature of 27.5°C (81 to 82°F) was taken as 13.528 gms. per cu. cm. Correction for elevation was negligible as it amounted to but one part in 26400; correction for latitude was practically negligible also as it amounted to but one part in 5500. Maximum variations from the above average temperature called for corrections of the order of $\pm 1/2000$; in reducing the observations this temperature correction was also ignored for it usually amounted to less than $\pm 1/2500$, which is considerably less than uncertainties due to some other variations resulting from difficulties experienced in the control of pressures.

61. The piston ratio derived above, 49.26, multiplied by 13.528/13.596 $\times$ 5499/5500, the ratio of the specific density of mercury at 27.5°C, corrected for latitude, to the specific density of mercury under standard conditions, and by (1.0073)/76 yields the value 0.6495 as the calibration factor to change observed displacements of mercury in the manometer column, measured in cm into equivalent changes of pressure in atmospheres acting on the rotating piston *u* of the free-piston manometer.

62. To these pressure differences thus determined it was necessary to add the barometric pressure prevailing at the time of the observations to further reduce them to equivalent absolute pressures. Slight variations in barometric pressure were insignificant in comparison with other variations of pressure that occurred during an experimental run; hence reduction of manometer readings to equivalent readings of corresponding absolute pressures was made by adding uniformly the value 1.50 cm to the observed mercury column displacements. This is equivalent to adding 0.975 atm. ($= 1.50 \times 0.6495$ atm.) or 73.1 cm barometric pressure which for the purpose is sufficiently close to the normal 73.5 cm barometric pressure at the location of the laboratory.

EXPERIMENTAL PROCEDURE

63. **General order of experiment.**—Having started the compressor and the bath stirrer, and established a circulation of fluid through the circuit of the apparatus, the control pressures in the regulator receivers were adjusted to give the desired initial high, and lowest final pressures respectively before and after the plug unless this adjustment had remained satisfactory from a previous run. The next step was to adjust the bath thermostat to maintain any desired initial temperature. Two situations were here confronted; first, when the initial temperature was below the room temperature, and second, when it was above room temperature.

64. **Below room temperature.**—In this case a part or all of the expansion of the fluid was caused to take place either through the plug, or through the plug and through valve *V*₂ at its return to the interchanger. The cooling effect in this case was thus transferred to the bath liquid either by the passage of the fluid through the coil *L* in the lower part of the bath chamber, or by this method and by its precooling of the high pressure fluid approaching the bath through the coils *W* of the interchanger. This action resulted in a progressive lowering of the temperature of the bath and of the parts affected by it in precisely the same manner that the initial cooling of the expansion-valve section of an air liquefier is effected. When the bath temperature was thus reduced to a point at which it was desired to operate, electric current was applied to the bath-heating coils and adjusted with the aid of the thermostat

to just off-set the cooling effect and so to maintain a constant and low bath temperature.

65. **Above room temperature.**—In this case the expansion of the fluid was caused to occur either entirely through the plug, or through the plug and through the valve V_3 at its departure from the interchanger, so as to avoid cooling of the bath as far as possible with the arrangements shown. The heating current was applied immediately, or sometimes previously, and usually in excess of ultimate requirements in order to hasten the heating of the bath to the temperature desired; thermostatic adjustments were made to maintain this desired temperature when or as it was reached or approached.

66. **Thermostatic adjustments.**—In either of the above cases, while the bath temperature was approaching the desired value, the valve V_a , connecting the brass capillary h with the glass capillary i , was left open to the atmosphere so that the pressure of the air over the mercury meniscus m and in the brass air-chamber od in the bath remained atmospheric. When the desired bath temperature was nearly reached, as could be ascertained by following the indications of a mercury thermometer inserted into the bath through the tube supporting the oil reservoir mounted on the head I of the bath chamber, valve V_a was closed. The level of the meniscus m was then adjusted, if necessary, by admitting or removing mercury through the stopcock cock n to or from the barometric tube of the thermostat which was then ready to assume automatic operation.

67. The arrangement in single, series or parallel connection of the two heating circuits immersed in the bath liquid, and the proportions of the external resistances c_1, c_2, c_3 of the heating circuits were then adjusted as rapidly as possible so as to secure the most even control of the bath temperature with the least variation of heating current. Practically this was indicated by the most frequent and uniform time interval between the "make" and "break" of the relay circuit at m and between the "on" and "off" swings of the contact breaker at c_3 . To assist in rapidly securing these best adjustments the timing of the relay swings was made doubly annunciative by means of a pilot light and a bell neither of which are indicated on the Plate.

68. When all went well these several adjustments required usually but a few minutes to make. As soon as constancy of conditions had become established observations were recorded until it was thought that enough had been obtained to insure sufficient data to cover the conditions prevailing. These were then adjusted to new ones and when constancy was again obtained recording of observations was resumed.

69. **Specific order of experiment.**—Specifically an experimental run was carried through as follows: Some initial condition of fixed temperature t_1' and high pressure p_1' for the fluid approaching the plug was selected and it was endeavored to maintain this condition constant throughout the run; the final pressure p_2 on the low pressure side of the plug was set a few atmospheres lower than the initial pressure p_1' and the temperature t_2 of the fluid after expansion through the plug was observed. The final pressure p_2 was then set several atmospheres still lower and the adjustments were made as found necessary to maintain the same initial high-side conditions t_1', p_1' . Observations were recorded as soon as constancy of conditions prevailed. This procedure was continued until the lowest final pressure obtainable had been reached; the next run would then be commenced with some different initial temperature but with preferably the same initial pressure. In some of the

runs the order of change from one low-side pressure to another was reversed and in some it was irregular.

70. **Order of taking observations.**—The procedure involved in taking observations was usually carried through in this manner: Switch high pressure p_1 to manometer; switch initial thermometer T_1 into resistance bridge and balance bridge for its resistance r_1 ; observe height of mercury in manometer column for high pressure reading h_1 ; reverse manometer connections throwing low pressure p_2 to manometer; switch T_1 out of bridge and T_2 into bridge and balance for resistance r_2 of final thermometer T_2 ; observe height of mercury in manometer column for low pressure reading h_2 ; switch thermometers into differential connection with bridge and balance bridge for their differential resistance r_Δ , a check measurement on $(r_1 - r_2)$: record the time at each observation.

71. Repeat the above series several times in rotation until observations indicate sufficient constancy of experimental conditions. Note and record the nominal value of each bridge box coil in use when balance is obtained for each arrangement of thermometer connection to the bridge; note the zero of the mercury column; observe as required the bridge box temperature, the mercury column temperature, and the zero of the mercury column. Change adjustments to secure some new condition and when steadiness is attained continue again the above procedure.

72. **Sampling and analysis of fluid.**—While waiting for the automatic devices to obtain control, with a little manual assistance to get the best regulative action, samples of the fluid were drawn off through the valve V_4 which connected with the delivery or low pressure side of the plug, and analyzed in the Orsat gas analysis apparatus. Data and results of a partial analysis are given in Table I, B, for the run represented by the sample set of observations presented in Table I, A.

73. In this run samples of CO₂ were drawn off every fifteen minutes, about 293 cc at a time. Each such lot as drawn was passed into the CO₂ absorbing reagent, potassium hydroxide. The residue from three lots was then transferred into the oxygen absorbing reagent, potassium pyrogallate, and its oxygen content was determined. The remainder was then transferred into the carbon monoxide absorbing reagent, cuprous chloride, for determination of its CO content. As this was very small the remainders from several runs were allowed to accumulate before CO determinations were made; these indicated about 1/40 of one per cent, by volume, of CO in the original CO₂. Finally the residue accumulated from the above tests during a number of runs was mixed with a measured quantity of air and the mixture passed over hot palladium for determination of its hydrogen content. This test indicated an average hydrogen content of about 0.003 of one per cent, by volume, in the original samples of CO₂.

74. The general conclusion from these various analyses of the CO₂ is that the volatile impurity present was mainly air in which the oxygen content, by volume, was either normal or less than normal, ranging from 20 per cent down to 9 per cent, although one determination indicated 24 per cent.

75. **Rate of flow of the carbon dioxide through the plug.**—No direct measurements were made of this quantity. The cylinder of the compressor which circulated the CO₂ was of two-inch bore by eight-inch stroke, double-acting, and was operated at 160 R.P.M. The cylinder displacement was therefore about 0.08 cu. ft. per second, or, assuming 75 per cent volumetric

efficiency, about 0.06 cu. ft. per second. The suction pressures varied with conditions from 17 to 35 atmospheres; the suction temperatures from 20 or 30 degrees below to that much above room temperature, usually above. The "free CO₂" displacement ranged therefore very roughly between one and two

TABLE I, A

SAMPLE SET OF OBSERVATIONS, (CURVE "J").

Resistances are expressed in bridge box units, bridge wire readings in hundredths of a bridge box unit; pressures in cm of mercury.

Date: Wed. P. M., Aug. 17, 1910.

Zero of manometer column, +1.50 cm

Coils								Temps. °C	
r_1	r_2	r_3	Time r_1 (bw)	Time r_2 (bw)	Time r_3 (bw)	Time λ_1 (cm)	Time λ_2 (cm)	Bridge box	Hg column
32	16		2:51 0.08	2:56 1.19	2:58 0.83	2:56 113.04	2:54 63.54		
16	8		3:01 0.12	3:03 1.24	3:04 0.84	3:00 113.04	3:06 63.65		
4	2	16	3:06 0.11	3:08 1.40	3:09 0.95	3:08 113.06	3:12 63.80		
1	1	8	3:10 0.11	3:12 1.39	3:13 0.91	3:15 113.05	3:18 63.84		
			3:15 0.15	3:17 1.39	3:18 0.75	3:23 113.05	3:21 63.86		
53	27	24	3:21 0.15	3:22 1.43	3:24 0.75	3:24 113.04	3:27 63.90		
			3:25 0.13	3:27 1.45	3:29 0.68	3:29 113.08	3:30 63.90		
Means			53.12	28.36	24.82	113.05	63.78	26.6	29.0
32									
16	32		3:38 0.17	3:42 0.57	3:44 0.57	3:38 113.05	3:43 75.58		
4	2	16	3:45 0.17	3:47 0.58	3:48 0.57	3:48 113.04	3:50 75.58		
1	1	1	3:49 0.17	3:50 0.58	3:51 0.57	3:53 112.95	3:54 75.55		
			3:53 0.16	3:55 0.60	3:56 0.56	3:59 112.94	3:56 75.55		
53	35	17	3:58 0.16	4:00 0.62	4:03 0.56	4:03 112.86	4:00 75.52		
Means			53.17	35.59	17.57	112.97	75.56	26.8	29.0
32									
16	32	8	4:08 0.18	4:12 0.82	4:14 0.36	4:13 113.05	4:11 86.70		
4	8	2	4:16 0.17	4:17 0.82	4:18 0.36	4:19 113.05	4:17 86.70		
1	1	1	4:19 0.17	4:20 0.82	4:22 0.36	4:22 113.05	4:23 86.70		
53	41	11							
Means			53.17	41.82	11.36	113.05	86.70	26.8	29.5
32									
16	32		4:32 0.17	4:36 0.17	4:38 0.84	4:32 113.00	4:35 95.20		
4	8		4:45 0.18	4:48 0.37	4:50 0.90	4:43 112.94	4:46 94.82		
4	4	4	4:53 0.21	4:54 0.23	4:55 0.85	4:51 112.62	4:53 95.02		
1	2	2	4:58 0.14	4:58 0.23	4:59 0.87	4:56 113.00	4:57 95.02		
			5:00 0.14	5:01 0.19	5:03 0.98	5:03 112.97	4:59 95.03		
			5:04 0.15	5:05 0.20	5:06 0.86	5:07 113.00	5:04 94.82		
53	46	6	5:07 0.11	5:08 0.17	5:10 0.92	5:10 112.93	5:09 95.02		
Means			53.17	46.22	6.87	112.89	94.99	27.25	29.05
32									
16	32		5:22 0.10	5:25 0.84	5:27 0.29	5:23 113.10	5:26 102.40		
4	16	2	5:28 0.15	5:29 0.84	5:30 0.29	5:28 113.06	5:30 102.35		
1	1	1	5:33 0.16	5:34 0.84	5:35 0.29	5:35 113.05	5:36 102.28		
			5:36 0.16	5:37 0.84	5:38 0.29	5:38 113.05	5:40 102.22		
53	49	3	5:40 0.16	5:41 0.84	5:42 0.30				
Means			53.14	49.84	3.29	113.06	102.33		

cu. ft. per second, corresponding, also roughly, to from one-eighth to one-quarter of a pound weight per second, or from 50 to 100 grams per second.

76. Computations from these figures showed that the maximum velocity effects at the points where the temperatures were measured, on either side of

the plug,—taking into account the pressure and temperature conditions at those points and the probable values of the constant-pressure specific heats of CO₂ under those conditions—were generally of a negligible order; they

TABLE I, B

SAMPLE LOG AND RESULTS OF CO₂ ANALYSES, (CURVE "J")

Date: Aug. 17, 1910.

Time of sampling	Amount of sample	Total to analyse	Residues after passing through successive absorbing reagents			Per cent impurity	Per cent O ₂ in impurity	Per cent CO in sample
			After KOH cc	After Pyro cc	After CuCl cc *			
3:00 P. M.	293.5							
3:15	292.5							
3:30	293.5	879.5	10.0	8.2		1.137	18.0	
3:45	294							
4:00	294							
4:15	297	885	9.9	7.9		1.119	20.2	
4:30	293.5							
4:45	293							
5:00	293.5	880	10.3	8.2		1.170	20.4	
5:30	295							
5:45	293	588	6.5	?		1.165	?	
Totals	3232.5		36.7	24.3				
Means	293.9		3.34	2.7		1.136	19.3	0.025*

* No tests for CO were made during this run; analysis of the residues accumulated from several runs indicated about 1/40 of one per cent of CO in the original CO₂.

amounted usually to less than a few thousandths of a degree centigrade, seldom to more than a hundredth of a degree. Hence in the treatment of the data no attempt has been made to make temperature corrections due to velocity of flow.

REDUCTION OF THE OBSERVATIONS

77. **Sample set of observations.**—Table I, A shows a sample set of observations as recorded in the log book. In this table the figures in the columns under $r_1(\text{bw})$, $r_2(\text{bw})$ and $r_{\Delta}(\text{bw})$ are bridge wire readings expressed as hundredths of a bridge box unit, the bridge wire being so graduated. The values in the columns marked "coils" are the nominal values in bridge box units of the bridge box coils in use for the corresponding r_1 , r_2 and r_{Δ} resistance measurements. The sum of these values under any r column, plus a corresponding bridge wire reading, is the resistance measurement for the particular observation represented by that bridge wire reading. The means of these measurements are also given; under $r_1(\text{bw})$ is the average observed resistance of thermometer T_1 , under $r_2(\text{bw})$ is the average observed resistance of thermometer T_2 , and under $r_{\Delta}(\text{bw})$ is the average observed difference of resistance of thermometers T_1 and T_2 when differentially connected, all expressed in uncorrected bridge box resistance units.

78. Since the fundamental interval of each thermometer was nearly equal to 100 bridge box units, variations of observed resistances expressed in these units are practically the same as variations in temperatures or temperature differences expressed in degrees of the centigrade scale. A perusal of the bridge

wire readings under column $r_1(\text{bw})$ shows the extent and nature of the initial temperature variations of this particular run; on the whole they were not disturbing. Under columns h_1 and h_2 a variation of 0.10 cm corresponds to a pressure variation of approximately one lb. per sq. in. (0.065 atm. actually). On the whole the pressure variations of this run were also not disturbing. Vertical comparisons of the means under $r_1(\text{bw})$ and h_1 show but slight deviations from a fixed initial condition defined by t_1' and p_1' remaining constant, (Par. 69).

79. Not always, however, was the control either of temperature or of pressure as good as was desired, in spite of the apparent simplicity of the automatic controlling apparatuses. Often the initial temperature would vary but a few hundredths of a degree, sometimes less than one hundredth, for intervals sufficiently long; at other times some disturbing influence, difficult to locate, would cause it to vary by several tenths of a degree, or to drift continuously in a very disconcerting manner. Similar variations of either or both pressures frequently occurred. As a consequence of these irregularities it was necessary to plot many series of observations against the time at which they were taken in order to determine properly the best single or average values to be used in later treatment of the data. Averages representative of the observations were, therefore, obtained either by inspection or by computation or by a comparative study of their time plots.

80. Table II assembles these averages for the complete run the log of which is given in Table I, A. It also gives the complete corrected values of temperatures, temperature differences, and pressures in degrees centigrade and atmospheres, respectively, according to the calibrations; and it gives the average initial temperature t_1' and the average initial pressure p_1' for the

TABLE II
MEAN VALUES FROM TABLE I, (CURVE "J")
Temperatures in degrees C; pressures in atmospheres

r_1	t_1	r_2	t_2	r_Δ	t_Δ	$t_1 - t_2$	h_1	p_1	h_2	p_2	$p_1 - p_2$
53.12	53.26	28.36	27.47	24.82	25.75	25.79	113.05	73.48	63.78	41.46	32.02
53.17	53.31	35.59	34.80	17.57	18.16	18.51	112.97	73.43	75.56	49.12	24.31
53.17	53.31	41.82	40.93	11.36	12.42	12.38	113.05	73.48	86.70	56.35	17.13
53.17	53.31	46.22	45.35	6.87	8.00	7.96	112.89	73.38	94.99	61.74	11.64
53.14	53.28	49.84	49.00	3.29	4.32	4.28	113.06	73.49	102.33	66.51	6.98
Means	53.29							73.45			

whole run. In this table column t_1 gives the average observed initial temperature for each portion of a run, each portion being represented by the data of one horizontal line of Table II and by one group or series of Table I, A; column p_1 gives the corresponding average observed initial pressure. Column headed $(t_1 - t_2)$ is added for comparison with the values under t_Δ which is $(t_1 - t_2)$ differentially observed and should agree with the difference between t_1 and t_2 when separately observed.

81. **Adjustment to standard initial point.**—Variations of initial temperature averages and of initial pressure averages, when they exist, indicate the possibility, almost the certainty, that each pair of points (p_1, t_1) , (p_2, t_2) , represented by a line of Table II may not belong to the same isenthalp as that passing through the point (p_1', t_1') , the average initial point for the whole run. It is therefore necessary to adjust the values of temperature and pressure of each of the pairs of experimental points represented by the lines of Table II so that they shall all conform to the same initial point, either the average initial point of the whole run or one very near to it.

82. The first approximation to this corrective adjustment is to shift each pair of points, first in a direction parallel to the pressure axis by an amount $(p_1' - p_1)$ equal to the difference between the initial pressure p_1 of the pair under consideration and the initial pressure p_1' selected as average or standard for the curve in question; and second, in a direction parallel to the temperature axis by the corresponding difference of temperature, $(t_1' - t_1)$.

83. This adjustment, however, is not necessarily sufficient for the lower point (p_2, t_2) . A study of a plot of the experimental points as shown on Chart I discloses that in general the shift of the low-pressure point of a pair must be different from the shift of the high-pressure point to insure that the resulting values shall correspond, subject to experimental error, to the isenthalp through the initial point, (p_1, t_1') , selected as standard for the experimental curve in question.

84. The total amount of shift of either pressure or temperature which it is or may be necessary and legitimate to apply to any of the several experimental "low-side" points of a given curve for the purpose of effecting such conformable correspondence is obviously a function of the relative location of the point along the curve with respect to the initial point as well as of the absolute values of the temperature and pressure of the standard point itself. This function is, a priori, unknown, for it is involved in the very relations which are sought to be determined by these experimental investigations.

85. If the experimental data be sufficiently extensive and "smooth" it may be possible to determine by graphical means alone a close approximation to this total adjustment. Unfortunately the data of these experiments, assembled as in Table II, are too limited, and in certain places too uncertain to permit a satisfactory graphical treatment. It will be made evident later that the relations involved are of so intricate and variable a nature that only exceedingly smooth and extensive experimental data will admit of satisfactory graphical treatment.

86. Table III shows the results of adjusting the data by the first approximation method described in paragraph 82 in which both points of a pair of points are shifted alike, the shifts of each pair being such as to make identical the initial points of all pairs of a given curve. In this table no use has been made of t_{Δ} , the differentially observed difference of temperature across the plug, in fixing the low side temperature which has been taken as that directly measured as t_2 of Table II.

TABLE III
REDUCTION OF VALUES IN TABLE II TO $t_1' = 53.30$; $p_1' = 73.50$
Date: 8-17-'10 Impurity: 1.14 per cent by volume, N.T.P. (Curve "J")

t_1	$t_1' - t_1$	t_2	t	p	p_2	$p_1' - p_1$	p_1
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			53.30	73.50			
53.28	+0.02	49.00	49.02	66.52	66.51	+0.01	73.49
53.31	-0.01	45.35	45.34	61.86	61.74	+0.12	73.38
53.31	-0.01	40.93	40.92	56.37	56.35	+0.02	73.48
53.31	-0.01	34.80	34.79	49.19	49.12	+0.07	73.43
53.26	+0.04	27.47	27.51	41.48	41.46	+0.02	73.48

87. In columns 1 and 3 of Table III have been assembled the observed initial and final temperatures, and in columns 8 and 6 the observed initial and final pressures, respectively, all transferred from Table II but arranged in descend-

ing order of magnitude. The values in the top lines of columns 4 and 5 give the initial temperature t_1' and the initial pressure p_1' respectively, selected as standard for this run. Column 2 gives $(t_1' - t_1)$, the difference between this standard initial temperature and the initial temperature of each of the several pairs of points represented by the data of the horizontal lines of this table; column 7 gives the corresponding difference of pressure, $(p_1' - p_1)$. The remaining values of columns 4 and 5 give the adjusted values of temperature and pressure of the consecutively lower points of the curve through the standard initial point, (p_1', t_1') . Values of temperature and pressure, as given in columns 4 and 5 of Table III are the coordinates of the points plotted on Chart I which are connected by heavy-lined curves. Those given in Table III are for the particular curve marked *J* on Chart I.

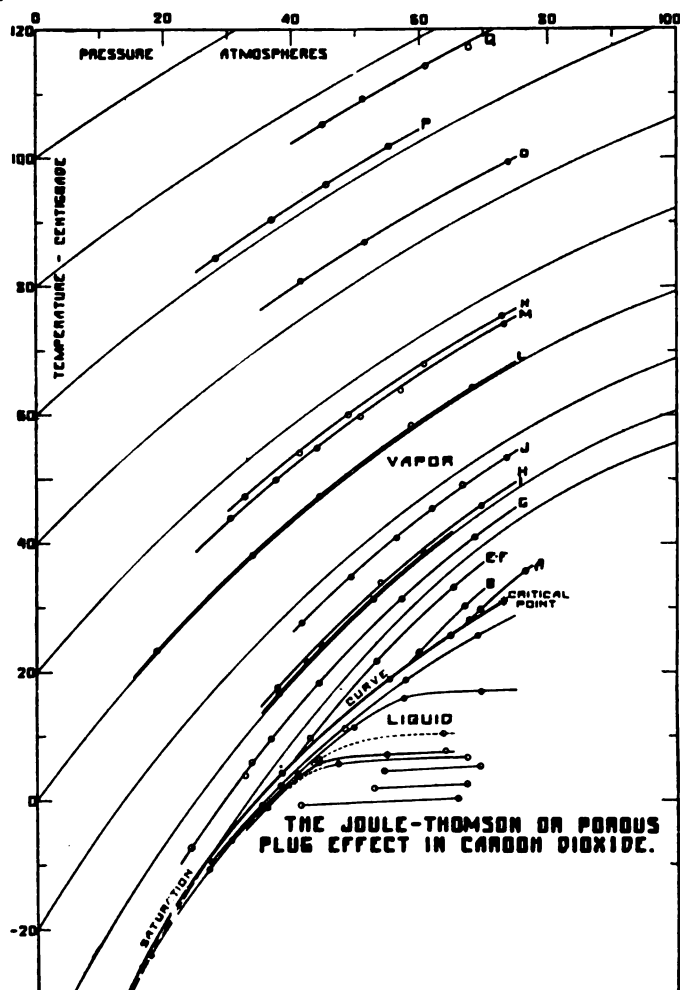


Chart I. Experimental isenthalpic or throttling curves; also curves computed from empirical formulation of 1913.

88. Intersections of the isenthalps with the saturation curve.—Some of these curves intersect the saturation curve for CO₂ near its upper end. Experimentally the observed values of temperature and pressure on the low side of the plug that plot below these points of intersection are distributed along the saturation curve. In Table IV these values are assembled into groups on the basis of average impurity content; to avoid confusion they are omitted from Chart I; on Chart VI, p. 44 they are compared with saturation values determined from other sources.

TABLE IV
SATURATION VALUES; EXPERIMENTAL

Average Impurity (by volume, N.T.P.)	Pressure atm.	Temperature degrees C
1.11%	65.54	25.55 ± .01
	58.34	20.68 .02
	57.65	18.79 .04
	48.90	11.12 .08
	47.56	12.05 .14
	46.68	12.78 .05
	40.31	6.00 .38
	38.00	+ 2.41 .11
	35.80	+ 0.80 .25
	35.09	− 0.72 .15
	34.67	− 0.25 .17
	34.60	0.0 .16
	32.76	− 1.16 .34
	31.22	− 4.28 .17
	28.48	− 7.67 .19
	27.25	− 9.65 .30
0.87%	44.08	6.46 ± .20
	40.69	+ 4.16 .14
	35.94	− 1.10 .02
	30.38	− 6.22
	26.97	−10.78 .26
	23.21	−15.30 .86
	17.63	−24.21 .17
0.60%	56.68	20.00 ± .01
	55.08	18.77 .05
	52.08	16.28 .06
	47.86	12.84 .10
	46.33	11.39 .06
	45.02	10.23 .11
	42.28	7.73 .05
	34.00	− 1.35 .02
0.30%	55.53	18.77 ± .14
	53.68	17.83 .59
	43.38	8.65 .10

89. The courses of those isenthalps that intersect the saturation curve show that when porous plug expansion of CO_2 vapor takes place from a "superheated" condition represented by points near enough to the upper reaches of the saturation curve, and is carried far enough, a part of the vapor condenses so that the observed pressures and temperatures must be those of saturated CO_2 vapor. It is probable that as the expansion is continued sufficiently far toward the region of low pressures the condensation of vapor gradually decreases, re-evaporation ensues until the portion condensed is completely evaporated, the isenthalp emerges from the saturation curve at a certain low pressure and diverges therefrom toward lower pressures in the superheat region. This latter course of an isenthalp was not experimentally observed on account of the limitations set by the single-stage compressor available for the work; the conclusion that such a course is undoubtedly the most likely one is, however, supported by experimentally determined throttling curves for initially "wet" steam in the corresponding region of the steam field.¹⁰ The chart shows that there must be one isenthalp which will just graze the saturation curve, its superheat having become zero with condensation incipient at the "grazing point," where, it may be shown, the enthalpy of dry saturated CO_2 vapor is a maximum.

90. All of the results, treated as in Table III, were plotted to a large scale, 1/4 inch per atm. and per degree C, for the purpose of study and graphical differentiation. The results for liquid CO_2 appeared to be much less satisfactory than were those for its vapor. It was much more difficult to control the experimental conditions when working with liquid, and consequently much more initial-point variation occurred, requiring larger and more uncertain adjustments to group corresponding pairs of points into the several curves shown in the liquid field of Chart I. In the light of more recent data to be presented later in this paper the more probable courses of the curves for pure CO_2 liquid in respect to the manner of their approach to intersection with the saturation curve is shown to be quite different from that represented on this chart. It is probable, however, as will be shown later on in discussing the effect of volatile impurities on the courses of these liquid isenthalps, that the curves here shown do represent very closely the true courses of such curves for those mixtures of CO_2 and volatile impurities that constituted the fluid actually used in these experiments.

EMPIRICAL FORMULATION OF RESULTS FOR SUPERHEATED CO_2 VAPOR (1913)

91. Because of the previously mentioned difficulty of graphical treatment of these data, attempts were made to express the results of the experiments by means of empirical formulae, fitted to the experimental curves of Chart I. This was not an easy task, for although it is not difficult to determine the coefficients of equations of various forms, any one of which can be made to satisfy with sufficient closeness the coordinates of a smooth curve assumed to best represent the experimental points among which it threads its way, it is difficult to correlate the corresponding coefficients of equations of the particular form adopted, in such a way as to be able to compute those coefficients and reproduce the curves in satisfactory agreement with experimental values.

92. If it be desirable or necessary to determine certain interrelations of the

¹⁰ See particularly the discussion and application of the throttling experiments on steam of Peake, Grindley, Griesman, and Dodge, by H. N. Davis, *Proc. Am. Acad. Arts and Sci.* 45 241-311, 1910.

properties of the fluid over the field covered by the experiments, it is practically imperative to accomplish such a formulation. Furthermore, by taking into proper account certain known or rationally indicated limits beyond the range of the experimental field, it may be possible, by empirical means, to greatly extend the knowledge disclosable by the data over the limited field of their immediate determination. Conclusions based upon such extension, however, should be discussed with due regard to the inherent limitations of the bases of the extension.

93. The degree of success with which an empirical formulation of the data covering the superheated vapor field of these experiments was accomplished, can be fairly well adjudged by comparing the lightly drawn curves extending across Chart I, which are spaced 20° apart on the temperature axis, with the heavy-lined curves through the experimental points. The agreement can be considered satisfactory over that part of the experimental range extending from its upper temperature limit, 120°C, down to the curves which intersect the saturation curve; below this lower limit equations of the form found agreeable above it could not be made to fit.

94. The equations for the individual curves shown in light lines on Chart I have the form:

$$t = t_0 + \mu_0 p + m p^n; \quad (8)$$

$$\left(\frac{\partial t}{\partial p}\right)_g = \mu = \mu_0 + n m p^{n-1} \quad (9)$$

in which t is the temperature in degrees centigrade at any pressure p atmospheres; t_0 is the temperature at the intercept of the curve with the temperature axis, where $p=0$; μ_0 is the slope of the curve at that intercept in degrees per atmosphere; n is a positive constant for each curve, its value being greater than unity and having here the value 5/3 for all the curves of this formulation, m is a parameter, negative for the field covered.

95. The several equations of this form are connected by the following relations between their coefficients, in which $T_0 = t_0 + 273$;

$$\mu_0 = \frac{121000}{T_0^2} - \frac{53}{T_0}; \quad (10)$$

$$m = - \frac{3350}{T_0^2} + 0.0155. \quad (11)$$

96. These comparatively simple equations, together with the curves in the liquid and vapor fields of Chart I that lie below the lower limit of agreement with this formulation, represent the extent to which by 1913 the writer had succeeded or had failed to succeed in formulating a general relation covering the full range of his experiments as hereinbefore presented.

97. Considerable study was made with these empirical relations extrapolated to several times the critical pressure of CO₂ (72.9 atm.), for it was observed that the maximum points of these extended curves, where their slopes, that is the Joule-Thomson effects, change sign by passing through the zero value, lie close to the locus of "inversion points" as computed by Porter¹¹ from Dieterici's equation of state in its "reduced form." (See Chart II.)

¹¹ Porter, A. W., *Phil. Mag.* 6, 11, 554, 1906; 6, 19, 888, 1910.

98. It was also noted that equations of this form (8) represent satisfactorily the approximate isenthalpic curves for air, Chart III, that the writer obtained

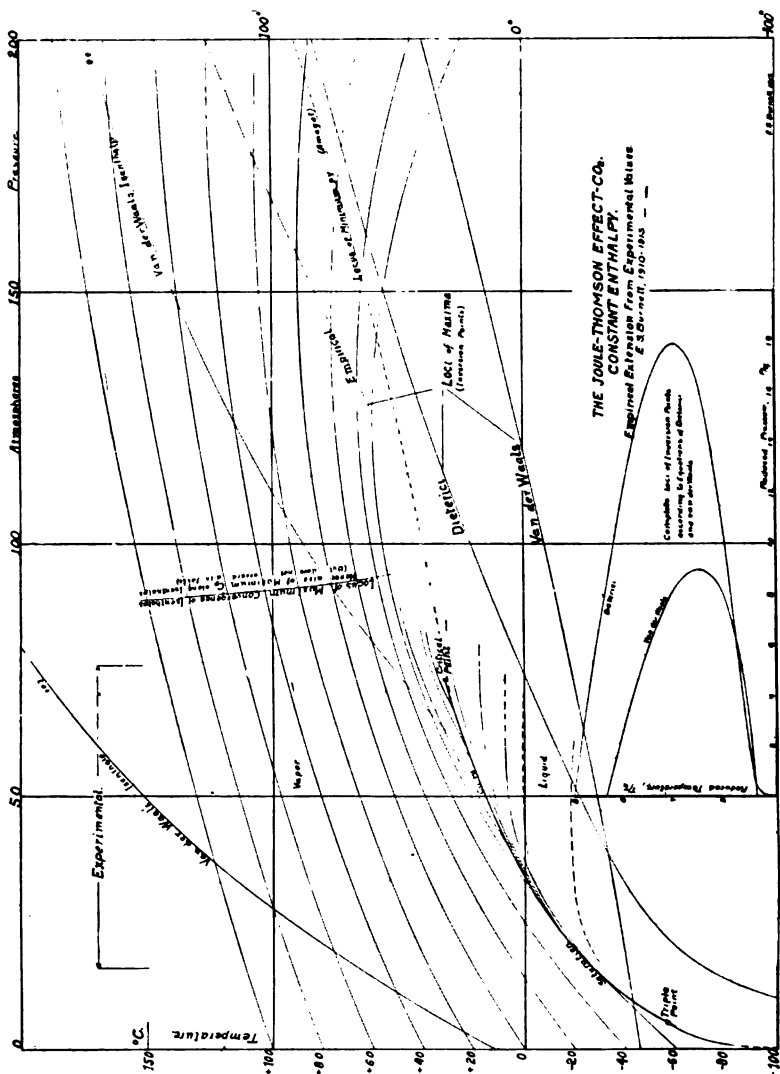


CHART II

Throttling curves for carbon dioxide; empirical extension from experimental range. Also, in reduced units, the complete locus of inversion points according to the equations of Dieterici and of van der Waal.

by cross-plotting data secured by Bradley and Hale¹² on the integrated cooling effect observed when air at different initial pressures and temperatures was

¹² Bradley and Hale, *Phys. Rev.* 29, 258, 1910.

expanded to atmospheric pressure in a Hampson type air liquefier; and, further, that such equations for these air curves also yield maximum or inversion

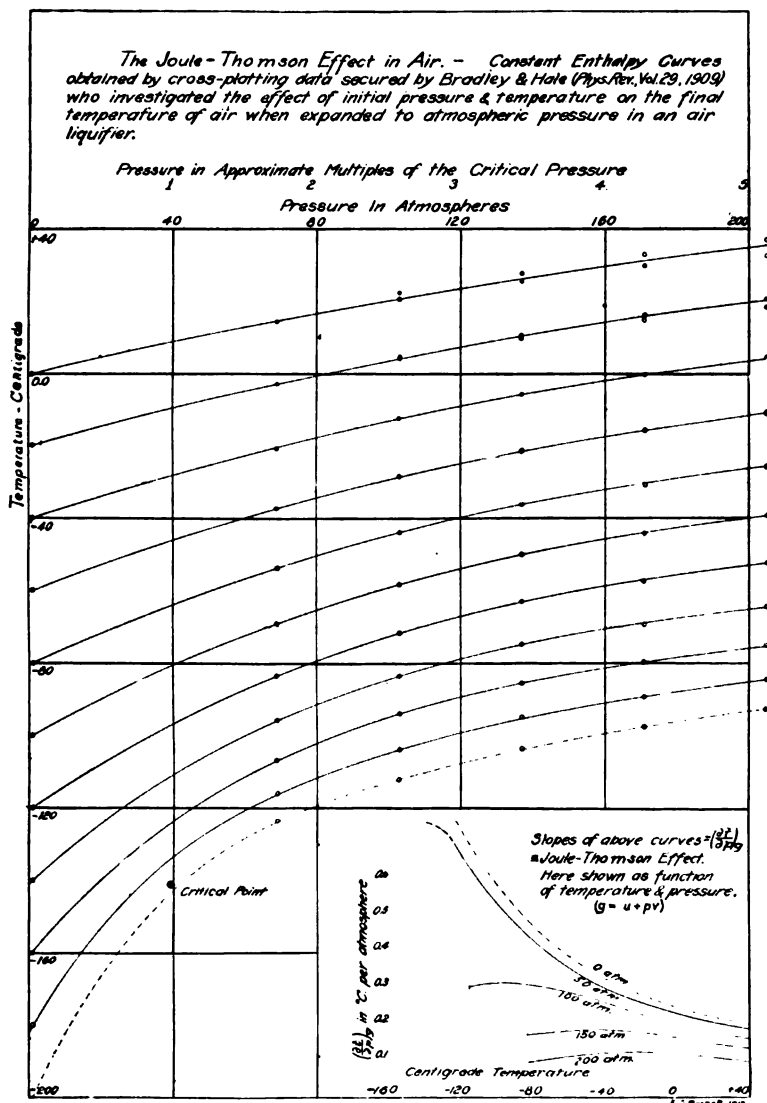


Chart III

Throttling curves for air, from experimental data of Bradley and Hale; also, the slopes of throttling curves.

points practically coincident with the Dieterici locus. But with air as with CO₂, equations of this form fail to represent the data for curves that approach the liquefaction region.

99. Chart II depicts this empirical extension of the curves from the experimental vapor field of CO_2 to the 200 atmosphere line; also the locus of inversion points—according to the Dieterici equation, according to the van der Waal equation, both as computed by Porter,¹¹ and according to equations (8) to (11) derived from these experiments. The two curves marked van der Waal isenthalps, crossing the family of isenthalps represented by equations (8) to (11) indicate the courses of isenthalps which van der Waal's equation predicts as computed by Dickson.¹² The disagreement between the experimentally determined isenthalps and those derived from van der Waal's equation is seen to be extreme. The writer makes no claim as to the accuracy of the course of his experimental curves when empirically extended toward higher pressures, for he knows of no direct experimental confirmation therefor. This presentation of the curves of Chart II is given principally because of the bearing which study of its indications had on later treatment of the data.

100. Values of the Joule-Thomson effect, μ , the slopes of the isenthalpic curves, computed from the empirical equations (8) to (11) over the experimental field which they cover (see Chart VII) were supplemented by values obtained by graphical differentiation of the remaining vapor and liquid curves of Chart I and arranged in graphical form for study. Other charts were prepared showing both isobaric and isothermal variations of μ with temperature and pressure respectively. These charts are not reproduced here (except in part, as Charts VII and VIII) for their showings have since been replaced by the results of a complete formulation covering and extending the whole experimental field shown on Chart I.

101. A comparative study was made of the variations of μ as shown by these charts (see Chart VII), and of μ for air as shown by corresponding charts prepared by the writer on the basis of the relatively more extended data of Bradley and Hale¹³ (see Chart III); also of Witkowski's¹ brilliant though involved attempt to determine graphically from other properties of air its isenthalpic curves up to 130 atmospheres in pressure and from 0°C down to the liquefaction region (see Chart 0); and of Dalton's¹⁴ researches on the Joule-Thomson effect for air and hydrogen, partly experimental and partly analytical and empirical, which yield similar curves. This study yielded important clues as to the characteristics of the most probable form of the variation of μ with temperature and pressure in the region contiguous to the critical point where experimental data is meagre, uncertain or lacking.

102. The writer was not satisfied with the results of his efforts thus far to determine and to express the relations involved. He felt that further effort was warranted to formulate an expression that would enable one to compute over their entire range the relations disclosed by the data, for the advantage of having such means of determining these and associated relations would be great. He therefore decided to make this effort before publishing the results of the experiments and of the efforts at their interpretation as thus far developed, (1913).

LATER TREATMENT OF ORIGINAL AND ADDITIONAL DATA (1922)

103. It was not until early in 1922 that the writer had further opportunity to devote to this work the uninterrupted time necessary to again attempt a complete formulation of the available data on the Joule-Thomson effect for

¹¹ Dickson, J. D. Hamilton, *Phil. Mag.* 6, 15, 126, 1908.

¹⁴ Dalton, J. P., *Konink. Akad. Wetensch. Amsterdam*, Proc. 11, 863, 1908-9.

CO₂. The task proved to be far more difficult and laborious than had been anticipated, but a formulation was eventually evolved covering the whole field of the writer's 1910 data already herein presented, extended by additional data in both the liquid and vapor fields that had in the meantime appeared.

104. **Effect of volatile impurities.**—In reviewing his previous treatment of the writer's experimental data at the start of this fresh attack, an effort was made to take into some sort of account the effect of impurities in the fluid, a factor that had been neglected before. The effect of volatile impurities is really a part of the problem, almost unlimited in scope, of determining the Joule-Thomson effect for mixtures of fluids. The effect of non-volatile impurities in the small amounts that may have been present in the CO₂ used by the writer is undoubtedly very small in comparison with that of the volatile impurities and is here assumed to have been without discernible effect throughout these experiments. The effect of impurities more volatile than that of the pure fluid under investigation, which is the case here, is of greater moment in the section of the field contiguous to the saturation curve than it is in sections more remote in the region of superheated vapor or in that of under-cooled liquid; near the saturation curve it is considerably greater for the liquid than for the vapor. In the writer's experiments with CO₂ the impurity was shown by analysis to have been essentially air which is known to have a smaller Joule-Thomson effect than has CO₂ vapor. Consequently one would expect the resultant effect for a mixture of CO₂ vapor and air to be less than that for pure CO₂ and therefore the isenthalps observed for a mixture of these fluids (in the vapor field of CO₂) to be less steep than those for pure CO₂. Definite evidence that this relation is so will be presented later.

105. **Experimental evidence of the effect of impurities.**—The only direct experimental evidence bearing upon the effect of impurities in CO₂ vapor on the Joule-Thomson effect, which have come to the writer's notice, are those of Kester¹⁶ on CO₂ in recent times, and the original experiments of Joule and Thomson³ on the porous plug effect for various mixtures of gases, done between 1854 and 1862.

106. Kester was forced to make effort to determine the magnitude of this effect because the impurities in the CO₂ that he used were so variable, ranging from about 0.3 per cent to 2.2 per cent by volume. He concludes that from 0° to 60°C the effect of the impurities at pressures up to 40 atmospheres decreases with increasing temperature. His observations at higher temperatures, and some of those within the range from 0° to 60°, are contradictory to that relation as a general trend. His results appear to indicate that, for each one per cent of impurity, the Joule-Thomson effect for CO₂ at the temperatures given is increased by nearly 5 per cent at 0° down to less than 0.7 per cent at 60° and back up to about 2½ per cent at 100°C. These figures can hardly be taken as reliable indications either of the magnitude or of the variation of the effect of impurities on the Joule-Thomson effect for CO₂, particularly when the nature of those impurities was specified only in a negative sense, for Kester reports that "complete analyses of the CO₂ gas were not made"—"tests for H₂S and SO₂ gave negative results—also a test for oxygen gave negative results, showing that the impurity was not air."

107. Joule and Thomson, the original investigators of the porous plug effect for gases, especially air and carbon dioxide, investigated that effect for various

¹⁶ Kester, F. E. *Phys. Rev.* 21, 261, 1905.

binary mixtures of gases, namely air and carbon dioxide, air and hydrogen, nitrogen and oxygen; they also reported observations of this effect in three

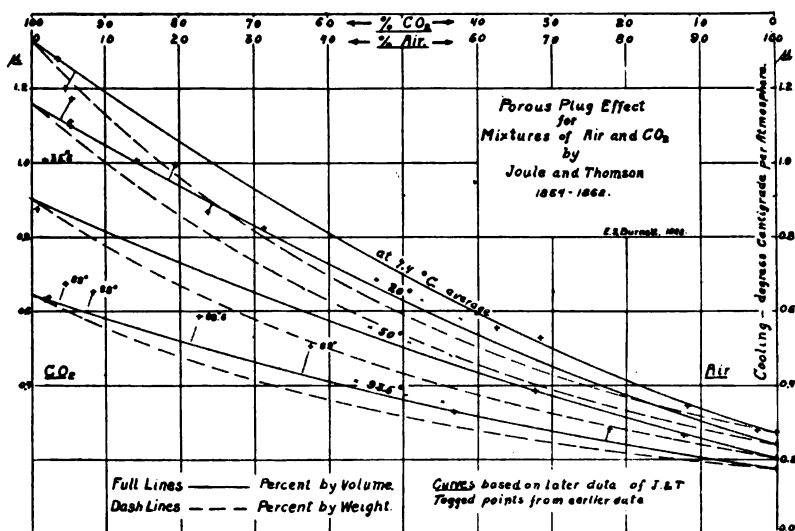


Chart IV

widely different ternary mixtures of air, carbon dioxide and hydrogen, in which the proportions of air were very small.

108. Their results, for the air and CO_2 mixtures, grouped by the writer into four series on the basis of the average temperatures of their experiments,

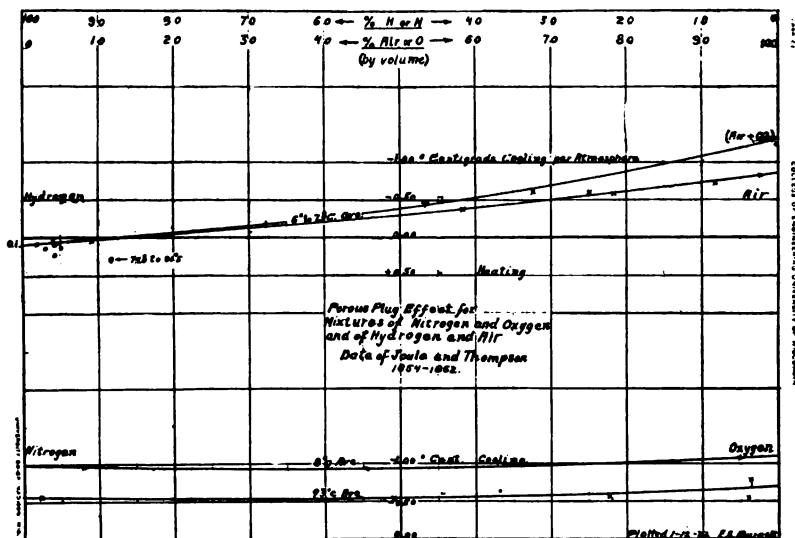


Chart V

Note; ordinates of Chart V should be labelled, "degrees centigrade per 100 inches of mercury."

are shown by the four pairs of curves of Chart IV, in which the ordinates are the observed cooling-effects in degrees centigrade per atmosphere pressure drop, and the abscissae are the relative percentages of the two components of the mixtures; the full-lined curves are drawn on the basis of the volume percentages of two components, and the broken-lined curves on the basis of their mass percentages.

109. Chart V shows similar curves, drawn on the volume basis only, for the other mixtures above mentioned, air and CO₂, being grouped together against hydrogen in the ternary mixture. The writer has not seen these data arranged in this manner elsewhere, and presents them in this form as of general interest, aside from their immediate bearing on the correction for impurities. In both Charts IV and V the curves as drawn are based upon the data of Joule and Thomson announced in 1862 which are more extensive and probably more reliable than those announced in 1854; the scattering points tagged to the curves of Chart IV are from their earlier data. Incidentally, attention is called to the rather slender basis on which the porous plug effect for hydrogen was shown to be a heating instead of a cooling effect as with the other gases at the same temperature (Chart V). The maximum pressure involved in any of their experiments rarely exceeded six atmospheres, and then but slightly.

110. For none of these mixtures do the curves of Charts IV and V approximate straight lines; hence the resultant effect for these mixtures is certainly not to be determined by assuming that the partial effect of either constituent is independent of the presence of the other, a decision ultimately reached by Joule and Thomson themselves. Referring to the air-CO₂ mixtures they said, "We might expect the two to contribute each its own proportion of cooling effect according to its own amount and its specific heat, volume for volume. But do the mixtures exhibit such a result? No!" They found by trial an expression applicable throughout the range of pressure and temperature of their experiments that enabled them to compute the cooling effect for pure CO₂ from their observations of the cooling effects of its mixtures with air and from their previous determination of that effect for air alone. This expression "gives results as consistent with one another as the probable errors of the experiment justify us in expecting," they remark. The probable errors of their experiments are, however, very considerable. Their expression follows:

$$\frac{\left[\left(\frac{\text{vol. of air} +}{0.7 (\text{vol. of CO}_2)} \right) \times \left(\frac{\text{observed cooling}}{\text{effect for mixture}} \right) \right] - (\text{vol. of air}) \times \left(\frac{\text{cooling ef-}}{\text{fect for air}} \right)}{0.7 (\text{vol. of CO}_2)} \quad (12)$$

The significance of this relation is probably more obvious when it is rearranged, as follows:

$$\left(\frac{\text{observed cool-}}{\text{ing effect for}} \right)_{\text{mixture}} + \frac{1.43 \times (\% \text{ of air})}{100} \left[\left(\frac{\text{observed cool-}}{\text{ing effect for}} \right)_{\text{mixture}} - \left(\frac{\text{cooling effect}}{\text{for air alone}} \right) \right] \quad (13)$$

111. For small percentages of air mixed with CO₂, a simpler expression can be obtained from a study of the indications of Chart IV, namely:

$$\left(\frac{\text{observed cooling}}{\text{effect for mixture}} \right) \times \left(\frac{\text{The cooling effect for pure CO}_2 = \text{ordinate at the CO}_2 \text{ end of proper temperature curve}}{\text{ordinate on same curve corresponding to per cent of air}} \right) \quad (14)$$

which is equivalent to

The cooling effect for pure $\text{CO}_2 =$

$$\left(\frac{\text{observed cooling}}{\text{effect for mixture}} \right) - \left(\frac{\Delta\mu}{\% \text{ of air}} \right) \times \% \text{ of air} = \left(\frac{\text{observed cooling}}{\text{effect for mixture}} \right) - fh \quad (15)$$

in which h is the per cent of air in the mixture and f is the initial slope of the appropriate temperature curve, that slope being measured as the change in μ for each one per cent of contained air. Thus measured the initial slopes of the four full-lined isotherms of Chart IV, reading from the uppermost to the lowermost—that is from lower to higher temperatures—have as drawn the values, -0.014 , -0.0115 , -0.0085 , and -0.006 , which are approximately proportional to the reciprocals of the cubes of the corresponding absolute temperatures thereon indicated.

112. Dividing these values by the corresponding values of μ for pure CO_2 as read from the chart yields the ratios -0.0106 , -0.0101 , -0.0095 , and -0.0093 , respectively, which indicate that the values of μ for CO_2 containing one per cent of air are less than the corresponding values of μ for pure CO_2 by a little more than one per cent of the latter values at the lower temperatures to a little less than one per cent of them at the higher temperatures of this chart. This variation is in the same direction as that observed by Kester between 0° and 60° but of considerably less magnitude. Disregarding its variation with temperature these figures indicate that *an approximate correction for the influence of small amounts of air in CO_2 on its porous plug effect can be made by increasing the observed porous plug temperature drops by one per cent of their values for each one per cent of contained air.* This also disregards any variation of this influence with pressure for neither Kester's results nor those of Joule and Thomson disclosed any relation between variation of pressure and the influence of volatile impurities on the porous plug effect for a pure fluid.

113. The writer felt that this original work of Joule and Thomson, despite the great variation existing between individual values recorded for nearly identical conditions, does when arranged as in Charts IV and V afford more definite information as to the influence of known volatile impurities such as air on the porous plug effect for CO_2 than any other investigation yet available. He has, therefore, for the corrective purposes in this paper, assumed that the effect of such impurities as were present in the CO_2 vapor of his experiments, which analyses showed to be principally air, usually less than one per cent by volume and seldom more, is directly proportional to the percentages which those analyses showed, is independent of the pressure, and is related to the temperature in approximate accordance with the figures given above.

114. These relations can be expressed as follows: the factor f can be represented with sufficient closeness for the present purpose by the equation

$$f = \frac{a}{T^3} = \frac{-296000}{T^3} \quad (16)$$

and the total correction F to be added to an observed porous plug temperature change from T_1 to T_2 in CO_2 containing h per cent of air, by

$$F = \int_{T_1}^{T_2} fh \cdot dt = ah \int_{T_1}^{T_2} \frac{dt}{T^3} = \frac{-ah}{2} \left(\frac{1}{T_2^2} - \frac{1}{T_1^2} \right) = 148000h \left(\frac{1}{T_2^2} - \frac{1}{T_1^2} \right) \quad (17)$$

On the basis of the available information Eq. (17) probably gives the more certain impurity corrections particularly for large porous plug temperature

changes, and for those lying wholly above or wholly below about 20°C. In most instances the approximate one per cent correction suggested two paragraphs above is sufficiently close. (See par. 134, p. 52.)

115. It may be well to note in passing that if these correction items be estimated on the basis of the impurities measured in per cent by weight or mass instead of by volume they will be the same; the constants in the preceding several equations will however be somewhat different from those given, for, in the case of CO₂ and air, the impurity percentage by weight is smaller than by volume; the resultant temperature correction must necessarily be the same.

116. All of the observed temperature drops through the plug as hereinbefore recorded for CO₂ vapor should then be increased in accordance with the relations expressed in Eq. (17). As to what may be the magnitude of the influence of small percentages of air or other volatile impurity in liquid CO₂, undergoing porous plug expansion there appeared to be no direct clue. Consequently no attempt has been made to apply an impurity correction to the data of the liquid phase. (A possible approximate evaluation of this correction may appear from comparisons to be made later in this paper.)

117. **Effect of impurities along the saturation curve.**—As the writer, working largely alone, eventually obtained his data under stress of the approaching termination of a limited tenure, his effort was to accumulate data covering as wide a range as equipment, conditions and time would permit. There was not opportunity to work over the observations of each run, plot them, determine wherein they were deficient, and then repeat or fill in by further experiment. Consequently, particularly in the region adjacent to the saturation curve, the usable data acquired are too meagre for certain determination of the true courses of the isenthalpic curves in that region.

118. Experimental points on either vapor or liquid curves, taken below the intersections of these curves with the saturation curve, yield values essentially coincident with that curve except as influenced by the impurities in the CO₂. These points served to indicate to some extent the magnitude and character of such influence but they were of little value in determining the courses of the isenthalps approaching intersection with the saturation curve, and the time spent in securing them was, for this purpose, lost. On the liquid side, the writer's data definitely show that the impurities present had the effect of yielding "individual" saturation curves appreciably lower in temperature or higher in pressure, particularly toward their upper limits than that assumed for pure CO₂, an effect to be anticipated for a mixture of CO₂ with a more volatile impurity; note Chart I. On the vapor side a similar effect is to be expected and it is supported by the data.

119. The courses of these "individual" saturation curves are functions of the relative amounts of the volatile impurities in the CO₂ of the corresponding runs and of their distribution between the varying phase proportions of the CO₂ when more than one phase appears. These saturation curves may intersect each other if their impurity proportions are different. In fact, for any given proportion of a definite impurity present there exists a whole family of isenthalpic curves and saturation curve which are different presumably from those corresponding to any other proportion of the same impurity, or to any proportion of any other impurity admixture. Consequently, in a series such as the data of this paper represent, varying amounts of impurity will determine curve courses corresponding to different families of curves all having the same general characteristics but differing more or less in their orientation; the

interferences observed may therefore correspond to true variations of differing multi-component mixtures.

120. The mechanism of the inter-relations by which the volatile impurities present in either liquid or vapor CO_2 shift the courses of the pure CO_2 isenthalps approaching their true saturation curve is so involved with the relative solubilities of the impurities in both liquid and vapor CO_2 , and with the variation of these solubilities with both pressure and temperature and with the relative proportions of each phase present; as well also as with the heats of solution of the impurities in both phases under these varying conditions, and with the heat of vaporization of CO_2 , the specific heats of the two phases and of the impurities; all varying simultaneously with the Joule-Thomson effect for the different components, that a complete analysis thereof would be extremely difficult were all these relations known; it is well nigh impossible with only our present limited knowledge of these various factors.

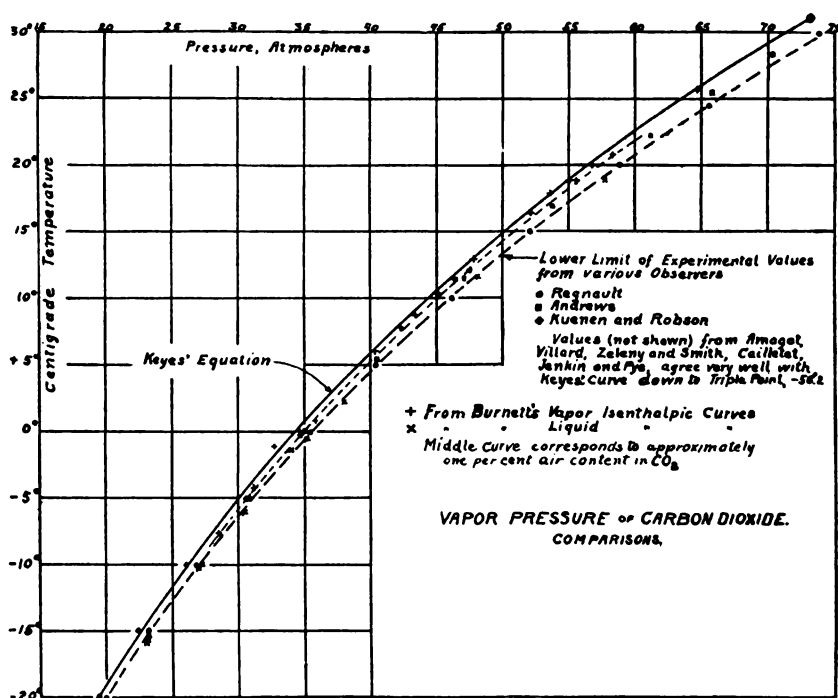


Chart VI

121. Saturation vapor pressures and temperatures.—On Chart VI are plotted the observed saturation vapor pressures and temperatures, assembled in Table IV from the writer's data, taken from points below the intersections of the vapor isenthalps with the observed saturation curves, and also from similar observed values correspondingly taken from points of the liquid isenthalps approaching closely to the saturation limits.

122. On Chart VI are also plotted various experimental determinations of the saturation pressures and temperatures for CO_2 , and a smooth curve from

an empirical equation by Keyes¹⁶ designed to represent the best of the data then available for supposedly pure CO₂. The lower limit (in temperature) of these various determinations is shown by the broken curve. Most of the writer's saturation data yields points lying between these two curves. An intermediate curve is shown determined by points corresponding to approximately one per cent air impurity in the CO₂.

123. **Discrepancies between the two methods of measuring temperature differences.**—In the earlier study of the data, discrepancies, some minor, others considerable, were noted between the values of the temperature drop across the plug as measured differentially, (t_{Δ}) , and as determined by subtraction of the observed final from the observed initial temperature, $(t_1 - t_2)$. Some of these discrepancies were undoubtedly due to variations in the constancy of initial temperature or pressure or of the difference of pressure across the plug which introduced varying time-lag factors into the observed values of the thermometric resistances. Some were difficult to reconcile. A study of the magnitudes and distribution of these discrepancies showed that about seventy-five per cent of the values, $[(t_1 - t_2) - t_{\Delta}]/2$, are positive with an average value slightly less than one per cent of the mean temperature-drops, $[(t_1 - t_2) + t_{\Delta}]/2$, and twenty-five per cent negative with an average value slightly less than one and a half per cent of the mean temperature-drops. The maximum variation of these discrepancies in the differences from the means of the differences is from three to five per cent except in a few instances for liquid CO₂ where the temperature drop across the plug was very small. The corresponding absolute discrepancies are mostly less than 0.1° to 0.2° and seldom more than 0.3° either way. In this present and final treatment of the data, except as mentioned below, the observed temperature-drops are taken as $[(t_1 - t_2) + t_{\Delta}]/2 \pm [(t_1 - t_2) - t_{\Delta}]/2$.

124. The combined resistance of the two smallest bridge box coils was equivalent to the graduated bridge-wire resistance of 1.5 box units, and balances of the bridge could be made at times either with or without the use of one or the other or occasionally of both of these coils. An error in recording the exact arrangement in use would almost exactly account for the few outstanding discrepancies of 0.5° and 1.0° between $(t_1 - t_2)$ and t_{Δ} . In plotting the curves of Chart I (and Chart IX) a few scattering points were observed to be displaced horizontally (parallel to the pressure axis) from smooth curves through the associated points by amounts corresponding almost exactly to an error of one cm in reading or recording the height of the mercury manometer column. In view of the numerous details that required constant attention during the course of an experimental run of the nature of these, and the many observations that were taken and recorded by the writer working alone, sometimes nearly all night after a day's preparation, it is not unlikely that a few such errors of observation or of record may have occurred.

125. **Correction for initial point variation.**—In paragraph 83, p. 31, it was pointed out that the parallel shifts of the coordinates of the several pairs of points of an experimental curve which are required to make their initial points identical may not, and generally will not be sufficient to make their lower points correspond to the isenthalp through this identical initial point. The proper relation between the temperature shifts at the upper and lower points of a pair may be closely approximated for all points lying within the field covered by the empirical relations given by the Eqs. (8) to (11) previously

¹⁶ Keyes, F. G. and A. W. Kennedy, *Thermodynamic Properties of Carbon Dioxide*, *Am. Soc. Refr. Engr. Jour.* 3, 17-43, 1917.

derived to represent the data for those vapor isenthalps that do not intersect the saturation curve. The function required is $(\partial t/\partial t_0)_p$ along the isenthalps. The ratio of the values of this function at different points along an isenthalp is the ratio of the corresponding temperature shifts required to move in a direction parallel to the temperature axis, corresponding points of a curve from one isenthalp to another closely adjacent. Differentiation of Eq. (8) combined with Eqs. (10) and (11) yields the relation,

$$\begin{aligned} (\partial t/\partial t_0)_p &= 1 + p(\partial \mu_0/\partial t_0)_p + p^n(\partial m/\partial t_0)_p \\ &= 1 - p \left[\frac{242000}{T_0^3} - \frac{53}{T_0^3} \right] + \frac{6700}{T_0^3} p^{2/3} \end{aligned} \quad (18)$$

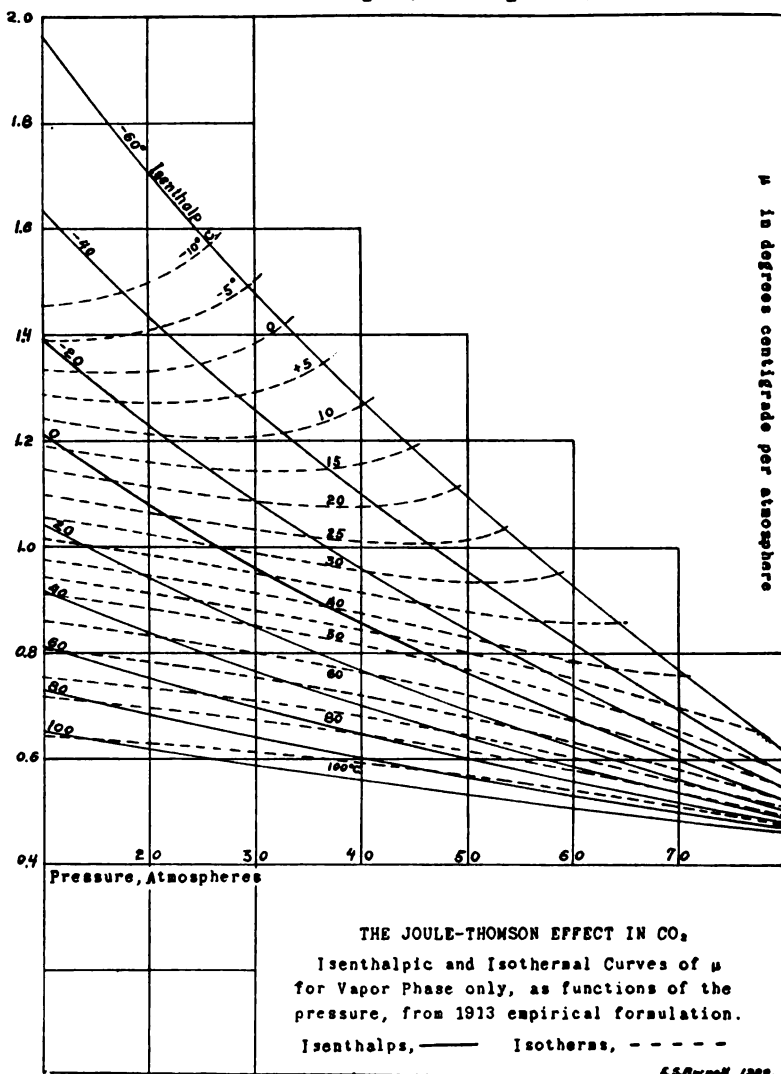
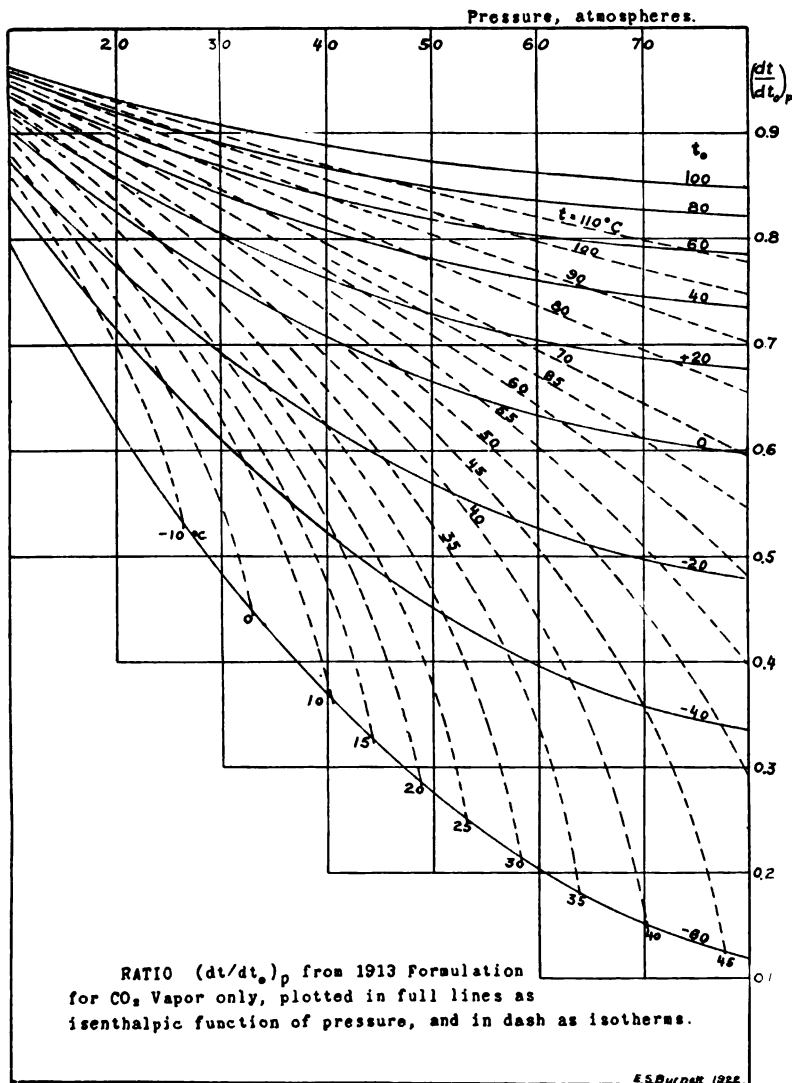


Chart VII

126. Values of μ , from Eqs. (9) to (11) have been plotted on Chart VII as isenthalpic functions of the pressures of each light-lined isenthalp of Chart I, and cross-connected by isothermal lines. The numbers near the margin of the chart label the μ isenthalps in terms of the p_0 -temperature intercepts of the calculated isenthalps of Chart I. On Chart VIII have been similarly plotted and labelled the corresponding values of $(\partial t/\partial t_0)_p$ as computed from the approximate Eq. (18).



127. With the aid of these charts and of Eq. (17) it is now possible to make all complete and final first order adjustments permissible to effect the required conformation of experimental points covered by the agreeable range of these several equations. Outside of that range the adjustments must be either omitted or supplied by estimate from the trend of the smooth curves of the region under consideration.

128. **Final corrections and adjustments of experimental data.**—In this later and final treatment of the writer's data all of the original observations have been subjected to the closest re-scrutiny; many more time-plots have been made and studied; where possible better representative values have been selected than those previously chosen. Subject to these slight modifications the original data as represented in Table II is now ready for the several corrections or adjustments discussed in the preceding paragraphs. The steps necessary to arrive at the proper corrections and the method of applying them are presented in Table V, illustrated by the diagrams, Figs. 11 and 12, and explained in the following paragraphs.

129. In the temperature-pressure plane of these diagrams let A_0B_0 be the observed initial and final points of a pair with coordinates $(p_1t_1)_{obs}$ and $(p_2t_2)_{obs}$, respectively. Let A' be the standard initial point with coordinates $(p_1't_1')$. It is required to locate the point B' to which to shift B_0 after shifting A_0 to A' , so that B' shall lie on the isenthalp through A' for the impure CO_2 actually used; then to apply a correction F , moving the point B' to B such that B shall lie on the isenthalp through A' for pure CO_2 . Two methods are available.

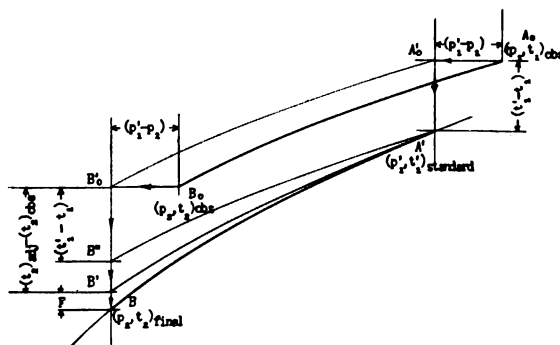


Fig. 11. Adjustment for initial point variation, first method.

130. First method, Fig. 11: Shift the observed points, A_0, B_0 , parallel to the pressure axis by an amount $(p_1' - p_{1obs})$ to the position A'_0, B'_0 ; then shift the latter points parallel to the temperature axis by an amount $(t_1' - t_{1obs})$ to the position $A'B''$. [These were the only adjustments made in the first formulation for variation of observed initial points from standard initial point, discussed in paragraphs 79 to 85 inclusive and recorded in Table III.] Because of the divergence of these curves the temperature shift $B'_0 \rightarrow B'' = A'_0 \rightarrow A'$ is not enough by the amount $B'' \rightarrow B'$; the total shift required at the lower end is determined by the relation

$$\frac{B'_0 \rightarrow B'}{A'_0 \rightarrow A'} = \frac{[(\partial t / \partial t_0)_p]_{at B'}}{[(\partial t / \partial t_0)_p]_{at A'}} = \frac{(t_2)_{corrected} - (t_2)_{observed}}{(t_1)_{standard} - (t_1)_{observed}} \quad (19)$$

These ratios are read from Chart VIII using the coordinates of A' and B' or of A_0 and B_0 for entry, preferably the former. The now adjusted temperature drop from A' to B' requires a correction F due to impurity as given by Eq. (17) [Note that the adjustments by this first method do not affect the pressure drop $(p_1 - p_2)_{obs}$ except to shift its position slightly.]

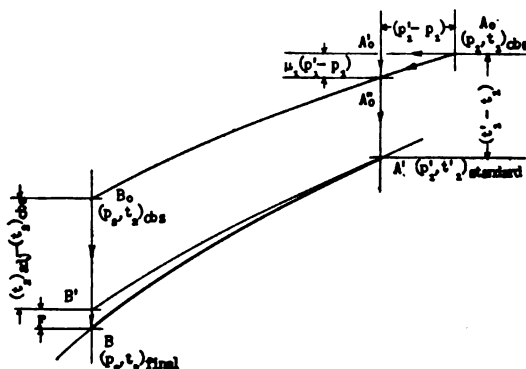


Fig. 12. Adjustment for initial point variation, second method.

131. Second method, Fig. 12: Shift the observed initial point A_0 parallel to the pressure axis by an amount $(p_1' - p_{1obs})$ to A_0' as in the first method; then shift it parallel to the temperature axis from A_0' to A_0'' by an amount $\mu_1(p_1' - p_1)$. This double shift amounts to slipping A_0 along the isenthalp A_0B_0 from A_0 to A_0'' where the pressure is the same as that at the standard initial point A' . Next shift A_0'' to A' by an amount equal to $[(t_1' - t_1) - \mu_1(p_1' - p_1)]$, taking proper account of the signs of these factors. B_0 is then shifted to B' by an amount determined by the relation

$$\frac{B_0 \rightarrow B'}{A_0'' \rightarrow A'} = \frac{[(\partial t / \partial t_0)_p]_{at B'}}{[(\partial t / \partial t_0)_p]_{at A'}} = \frac{(t_2)_{corrected} - (t_2)_{observed}}{(t_1' - t_1) - \mu_1(p_1' - p_1)} \quad (20)$$

These ratios are read from Chart VIII using for entry the coordinates of A' and B' preferably, A_0 , A_0' or A_0'' and B_0 being the alternatives; μ_1 is read from Chart VII entering it with the coordinates of A_0 preferably, or A_0' or A' . Finally apply the correction F , for impurity, over the temperature range from A' to B' as in the first method. [Note that the adjustments by the second method leave p_2 as observed but alter the observed temperature and pressure drops.]

132. These two methods are essentially equivalent, for, within the limits of experimental uncertainty and of the approximations of Charts VII and VIII, both are effective in placing the successive lower points onto the isenthalp passing through the standard initial point. If both methods are used the successive lower points will appear as doublets, occasionally coincident, ranged along the isenthalp.

133. In Table V are recorded the figures showing the applications of these two methods to the data for curve "G," one of the longest in range of pressure and temperature and one with considerable initial point variation. The column headings of this table, combined with the above explanation of these

TABLE V
RECORDING THE STEPS IN THE ADJUSTMENT OF THE OBSERVED DATA FOR VARIATION OF INITIAL POINT FROM STANDARD, AND IN APPLYING CORRECTION FOR IMPURITY
IN THE CO₂, (CURVE "G").
Pressures in atmospheres, temperatures in degrees centigrade.

(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Curve "G" Sept. 15, 1910. Impurity = 0.50% $\mu_1 = 0.75$ (From Chart VII) Standard initial point; $p' = 68.40$ atm. $t' = 41.00^\circ\text{C}$	Observed Values				t_Δ	$t_1 - t_2$ (mean)	$[t_1 - t_2] / 2 = \Delta t$	$t_1 - \Delta t = t_2$ (mean)	Deviations in pres. and tem. of observed initial points from standard		Isenthalpic temperature shift of initial point due to its pressure deviation $\mu_1(p_1' - p_1)$	Residual temperature shift of initial point $(t_1' - t_1) - \mu_1(p_1' - p_1)$
	p_1	p_2	t_1	t_2					$p_1' - p_1$	$t_1' - t_1$		
First Method	68.40		41.00									
	68.22	56.82	40.66	31.10	9.57	9.56	9.57	31.09	+0.18	+0.34		
	68.34	43.98	40.72	18.08	22.71	22.64	22.68	18.04	+0.06	+0.28		
	70.17	38.31	41.78	10.29	31.61	31.49	31.55	10.23	-1.77	-0.78		
	68.25	33.41	41.37	6.07	35.61	35.30	35.46	5.91	+0.15	-0.37		
	68.34	32.39	40.75	3.63	37.38	37.12	37.25	3.50	+0.06	+0.25		
	67.38	23.11	40.51	-7.92	48.85	48.43	48.64	-8.13	+1.02	+0.49		
Second Method	68.40		41.00									
	68.22	56.82	40.66	31.10	9.57	9.56	9.57	31.09	+0.18	+0.34	+0.14	+0.20
	68.34	43.98	40.72	18.08	22.71	22.64	22.68	18.04	+0.06	+0.28	+0.05	+0.23
	70.17	38.31	41.78	10.29	31.61	31.49	31.55	10.23	-1.77	-0.78	-1.33	+0.55
	68.25	33.41	41.37	6.07	35.61	35.30	35.46	5.91	+0.15	-0.37	+0.11	-0.48
	68.34	32.39	40.75	3.63	37.38	37.12	37.25	3.50	+0.06	+0.25	+0.05	+0.20
	67.38	23.11	40.51	-7.92	48.85	48.43	48.64	-8.13	+1.02	+0.49	+0.77	-0.28

	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)
	From Chart VIII $(\partial t/\partial p)_p$	$\frac{(\partial t/\partial p)_p}{(\partial t/\partial p)_p}$	Lower point temperature shifts = $(10) \times (14)$ for first method; $(12) \times (14)$ for second method	$(8) + (15)$ $= t_2$ (adjusted)	Impurity correction	Assuming it amounts to 1% of Δt for each 1% of impurity of t_2 $0.5(t_1' - t_2)$ 100	Known exper- imental uncertainty $\frac{(t_1 - t_2) - t_2}{2}$ ($\pm$)	$(16) - (17)$ $\pm (19) =$ t_2 (final)	(2) + (9) for first method; (2), as ob- served, for second method p_1 (final)	Pressure (One cm of manometer scale = 0.65 atm.)	Temperature (By one of the two smallest bridge box units)
First Method, continued.	0.275										
	0.33	1.20	+0.41	31.50	0.04	0.05	0.01	31.46	57.00		
	0.42	1.53	+0.43	18.47	0.12	0.11	0.04	18.35	44.04		
	0.47	1.71	-1.33	8.90	0.17	0.16	0.06	8.73	36.54		0.50
	0.51	1.85	-0.68	5.23	0.20	0.18	0.15	5.03	33.56		0.50
Second Method	0.55	2.00	+0.50	4.00	0.21	0.19	0.13	3.79	32.41		
	0.63	2.29	+1.12	-7.01	0.29	0.24	0.21	-7.30	24.13	0.65	
	0.275										
	0.33	1.20	+0.24	31.33	0.04	0.05	0.01	31.29	56.82		
	0.42	1.53	+0.35	18.39	0.12	0.11	0.04	18.27	43.98		0.50
	0.46	1.67	+0.92	11.15	0.16	0.16	0.06	10.99	38.31		0.50
	0.51	1.85	-0.89	5.02	0.20	0.20	0.15	4.82	33.41		
	0.52	1.89	+0.38	3.88	0.21	0.19	0.13	3.67	32.39		
	0.64	2.33	-0.65	-8.78	0.31	0.25	0.21	-9.09	23.11	0.65	

applications, do not need further elucidation. The finally corrected and adjusted values of temperature and pressure for this run, together with their known or apparent uncertainties, are given in the last five columns of Table V; they are the coordinates of the points of curve "G" that are plotted on Chart IX.

134. The two methods of arriving at a correction for the effect of volatile impurity in the vapor—Eq. (17) of paragraph 114, and the direct percentage proportion of paragraph 112—are compared in columns (17) and (18) of this table. In this particular case the difference of the corrections determined by the two methods varies from nothing to about $1/800$ of the corresponding mean temperature drop, column (7). A similar comparison carried out for the other curves of the vapor field shows that these differences vary from about $1/200$ of the corresponding temperature drops of the upper curves of this field to about $1/500$ thereof for the lower curves; Eq. (17) gives smaller corrections for the upper, and larger for the lower than does the direct proportion method. The latter more approximate method is therefore satisfactory where the errors of determination of Δt are not more than one half of one per cent of that value.

135. All of the original observations and data have been in this manner re-worked by the writer during the year 1922 in the effort to establish and formulate the characteristic variations of the porous plug effect for carbon dioxide liquid and vapor. For some of the data it was either not possible or not necessary to apply all of the operations outlined in the preceding paragraphs, but no effort has been spared to treat the data by all legitimate means that would assist in clarifying their indications.

136. The finally acceptable values for all the runs determined as above, arranged in the order of the proximity of their graphs to the critical point, are presented in columns (2), (3) and (4) of Table VI, A for CO_2 vapor, and in Table VI, B for CO_2 liquid, in descending order of pressure and temperature magnitudes. These values are plotted on Chart IX, the crosses showing the individual points; where known uncertainties in t_2 are large enough to show on the original large scale plot (see par. 90) the extreme temperature values of a point are indicated by a double cross (‡) on a vertical pressure line; where there is probable error in pressure observation or record that is large enough to show on that plot, the extreme pressure values are indicated by a double cross (++) on a horizontal temperature line. [There are also shown on this chart, by circles (o), experimental points obtained by Jenkin and Pye, discussed below, and smooth curves computed from the empirical formulation developed in the following pages on the basis of their data and the writer's combined.]

137. Throttling experiments of Jenkin and Pye.—In the interim between the writer's earlier attempts at formulating an expression to cover his own experimental data and the present attempt there appeared reports of observations by Jenkin and Pye¹⁷ giving throttling curves for CO_2 liquid and vapor similar to those of the writer but over different sections of these fields except where they overlap, as shown on Chart IX. A tabulation of their values as corrected by them is presented in Tables VII, A and VII, B, expressed here in degrees centigrade and atmospheres. Their values when plotted as isenthalpic curves also show some irregularities, but the close agreement in general trend of the curves through their points with curves through those of the writer, especially in the sections of the fields common to both, indicates that the two entirely independent sets of data are mutually concordant.

¹⁷ Jenkin and Pye, *Thermal Properties of Carbon Dioxide at Low Temperatures*, *Phil. Trans. A* 213, 67-118, 1914; 214, 353-382, 1915.

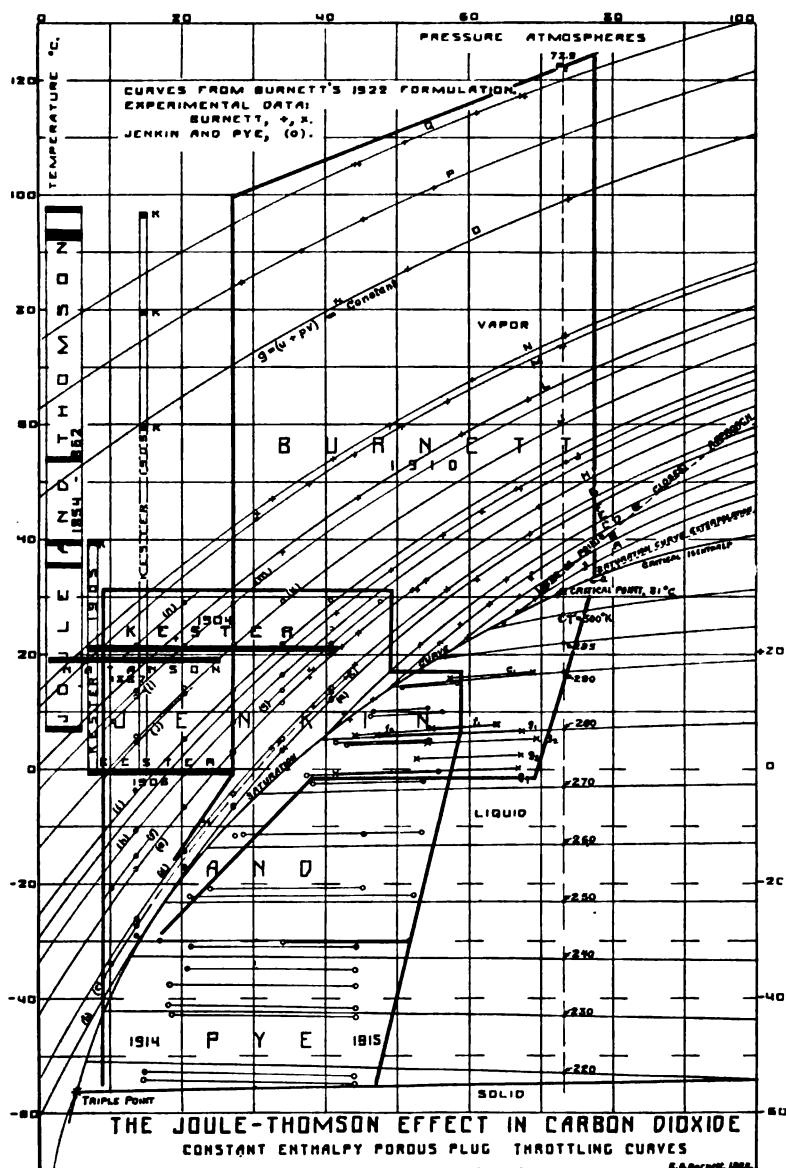


Chart IX

The Joule-Thomson Effect in Carbon Dioxide; throttling curves; location and extent of fields covered by its several investigators.

TABLE VI,A

VAPOR ISENTHALPS. EXPERIMENTAL VALUES AND COMPUTED VALUES

Comparison of Burnett's observed "porous plug" data for CO₂ vapor with corresponding values computed from his 1922 empirical formulation.

Pressures in atmospheres; temperatures in degrees centigrade (except T', in degrees Kelvin); μ in deg. C per atm.

* Values in the lines marked with an asterisk (*) are alternates of those in the lines immediately above; they are derived from the latter by assuming the observed pressure differs from that recorded by ± 0.65 atm. (equal to one cm on the manometer), or that the observed temperature differs from that recorded by one degree C (equal to one "bridge box unit"). The values of column (6) show that such assumptions usually place the corresponding point almost exactly onto the computed curve, indicating the probability of an error of observation or of record of pressure or temperature of the amounts here mentioned.

Column (7) shows whether the computed point lies within the experimental temperature range, Col. (4), of the observed point, Col. (3), or not, and if not by how much it differs therefrom [See pars. 187, 188, p. 77.].

1	2	3	4	5	6	7
Curve Date Per cent Impurity	Observed Values			Computed Values		
	p atm.	t_o (°C)	$\pm$	t_c	$t_o - t_c$	Minimum deviation from ex- perimental range (6) $\pm$ (4)
A Sept. 26 1.05%	72.9			32.70		
	76.10	35.70		35.03	+0.67	
	69.14	29.72	0.03	29.70	+0.02	0.0
	66.67	27.49	0.06	27.55	-0.06	0.0
	64.54	25.51	0.01	25.58	-0.07	-0.06
	Saturation					
	64.54	25.54	0.01			
	58.34	20.66	0.02			
	47.56	11.91	0.14			
	34.67	-0.42	0.17			
B Sept. 17 0.60%	72.9			34.80		
	66.90	30.20	0.0	29.90	+0.30	+0.30
	59.78	23.02	0.09	23.11	-0.09	0.0
	59.72	22.98	0.11	23.05	-0.07	0.0
	56.68	19.93	0.01	19.83	+0.10	+0.09
	Saturation					
	56.68	20.00	0.01			
	55.08	18.77	0.05			
	52.08	16.28	0.06			
	47.86	12.84	0.10			
	46.33	11.39	0.06			
	45.02	10.23	0.10			
	42.28	7.73	0.05			

TABLE VI,A (Continued)

1	2	3	4	5	6	7
Curve Date Per cent Impurity	Observed Values			Computed Values		
	p atm.	t_o (°C)	$\pm$	t_c	$t_o - t_c$	Minimum deviation from ex- perimental range (6) $\pm$ (4)
C+D	72.9			37.5		
Aug. 18, 19	68.60	33.20		34.13	-0.93	-0.93
1.12%	*68.60	34.20		34.13	+0.07	+0.07
	62.76	28.83	0.05	29.06	-0.23	-0.18
$T' = 310.6$	58.77	25.20	0.08	25.24	-0.04	0.0
$\mu' = 0.7500$	55.74	22.15	0.06	22.11	+0.04	0.0
$-b = 0.02024$	46.68	12.34	0.12	11.56	+0.78	+0.66
$t_a = 74.55$	*47.33	12.34	0.12	12.38	-0.04	0.0
	Saturation					
	46.68	12.34	0.12			
	40.31	6.00	0.38			
	35.80	0.80	0.25			
	32.76	-1.16	0.34			
	31.22	-4.28	0.17			
E						
Sept. 14						
1.25%						
$T' = 312.55$	72.9			39.45		
$\mu' = 0.7383$	65.10	33.10		33.25	-0.15	-0.15
$-b = 0.01880$	53.12	21.70 $\pm$ 0.10		21.76	-0.06	0.0
$t_a = 78.74$	42.60	9.58	0.13	9.31	+0.27	+0.14
	Saturation					
	34.60	0.00	0.16			
	28.48	-7.40	0.19			
F						
Sept. 14						
1.25%						
$T' = 315.75$	72.9			42.65		
$\mu' = 0.7173$	61.00	32.95	0.16	33.26	-0.31	-0.15
$-b = 0.01710$	34.25	3.63		3.46	+0.17	+0.17
$t_a = 84.60$						

TABLE VI,A (Continued)

1	2	3	4	5	6	7
Curve Date Per cent Impurity	Observed Values			Computed Values		
	p atm.	t_o (°C)	$\pm$	t_c	$t_o - t_c$	Minimum deviation from ex- perimental range (6) $\pm$ (4)
G	72.9			44.3		
Sept. 15	68.40	41.00		41.01	-0.01	-0.01
0.50%	56.82	31.29	0.01	31.32	-0.03	-0.02
	43.98	18.27	0.04	18.18	+0.09	+0.05
$T' = 317.4$	38.31	10.99	0.06	11.43	-0.44	-0.38
$\mu' = 0.7065$	33.41	4.82	0.15	5.18	-0.36	-0.21
$-b = 0.01639$	32.39	3.67	0.13	3.68	-0.01	0.0
$t_a = 87.40$	23.11	-9.09	0.21	-10.10	+1.01	+0.80
	*23.76	-9.09	0.21	-9.05	+0.04	0.0
H	72.9			48.10		
Aug. 21	60.5	37.00		38.80	-1.80	
1.06%	52.66	31.17	0.03	31.96	-0.79	-0.76
	*52.01	31.17	0.03	31.35	-0.18	-0.15
(Combines with I)	42.25	21.45	0.05	21.53	-0.08	-0.03
	37.71	16.09	0.13	16.45	-0.36	-0.23
I	72.9			48.10		
Aug. 12	69.6	45.90		45.8	+0.10	+0.10
1.50%	53.90	32.95	0.02	33.09	-0.14	-0.12
	44.44	23.86	0.07	23.86	0.0	0.0
$T' = 321.2$	37.66	17.28	0.07	16.40	+0.88	+0.81
$\mu' = 0.6825$	*38.31	17.28	0.07	17.31	-0.03	0.0
$-b = 0.01505$						
$t_a = 93.45$						
J	72.9			52.90		
Aug. 17	73.50	53.30		53.29	+0.01	+0.01
1.14%	66.52	48.94	0.02	48.54	+0.40	+0.38
	*67.17	48.94	0.06	49.00	-0.06	0.0
$T' = 326.0$	61.74	44.89	0.06	45.01	-0.12	-0.06
$\mu' = 0.6546$	56.35	40.73	0.02	40.72	+0.01	0.0
$-b = 0.01387$	49.11	34.68	0.17	34.46	+0.22	+0.05
$t_a = 100.1$	41.46	27.18	0.03	27.12	+0.06	+0.03

TABLE VI,A (Continued)

1	2	3	4	5	6	7
Curve Date Per cent Impurity	Observed Values			Computed Values		
	p atm.	t_0 (°C)	$\pm$	t_c	$t_0 - t_c$	Minimum deviation from ex- perimental range (6) $\pm$ (4)
K Aug. 21 1.25%						
$T' = 334.6$	72.9			61.5		
$\mu' = 0.6112$	60.5	52.4		53.3	-0.9	
$-b = 0.01238$	36.35	35.0	0.02	34.2	+0.8	
$t_a = 110.87$						
L Sept. 16 0.83%	72.9			67.00		
	68.20	64.30		64.18	+0.12	+0.12
	58.87	58.34	0.04	58.07	+0.27	+0.23
	44.32	47.20	0.04	47.12	+0.08	+0.04
$T' = 340.1$	33.87	37.86	0.05	38.04	-0.18	-0.13
$\mu' = 0.5865$	19.09	22.90	0.05	23.12	-0.22	-0.17
$-b = 0.01166$						
$t_a = 117.30$						
M Aug. 16 1.25%	72.9			73.80		
	73.10	73.60		73.91	-0.31	-0.31
	*72.55	73.60		73.55	+0.05	+0.05
	56.89	64.02	0.04	63.98	+0.04	0.0
$T' = 346.9$	50.59	59.58	0.02	59.63	-0.05	-0.03
$\mu' = 0.5613$	43.96	54.73	0.04	54.77	-0.04	0.0
$-b = 0.01097$	37.62	49.59	0.04	49.63	-0.04	0.0
$t_a = 125.0$	30.47	44.62	0.05	43.50	+1.12	+1.07
	*30.47	43.62	0.05	43.50	+0.12	+0.07
N Aug. 27 0.97%	72.9			75.20		
	73.40	75.40		75.48	-0.08	-0.08
	60.42	67.91	0.04	67.76	+0.15	+0.11
	48.91	59.65	0.15	59.96	-0.31	-0.16
$T' = 348.3$	40.92	54.08	0.07	53.94	+0.14	+0.07
$\mu' = 0.5564$	32.51	47.06	0.02	47.02	+0.04	+0.02
$-b = 0.01084$						
$t_a = 126.52$						

TABLE VI,A (Continued)

1	2	3	4	5	6	7
Curve Date Per cent Impurity	Observed Values			Computed Values		
	p amt.	t_0 (°C)	$\pm$	t_0	$t_0 - t_c$	Minimum deviation from ex- perimental range (6) $\pm$ (4)
O Aug. 27 1.50%	72.9			98.80		
	73.80	99.30		99.24	+0.06	
	$T' = 371.9$	51.43	87.05 0.12	87.13	-0.08	0.0
	$\mu' = 0.4912$	41.36	81.54 0.12	80.79	+0.75	+0.63
	$-b = 0.00928$	*42.01	81.54 0.12	81.22	+0.32	+0.20
	$t_a = 151.73$					
P Sept. 29 0.90%	72.9			110.30		
	55.10	101.20		101.27	-0.07	-0.07
	$T' = 383.4$	45.25	95.79 0.00	95.67	+0.12	+0.12
	$\mu' = 0.4683$	36.69	90.35 0.05	90.37	-0.02	0.0
	$-b = 0.008715$	28.36	84.69 0.17	84.83	-0.14	0.0
	$t_a = 164.04$					
Q Sept. 23 0.90%	72.9			120.00		
	67.70	117.30		117.60	-0.30	-0.30
	*67.05	117.30		117.30	0.0	0.0
	61.00	114.32 0.06		114.42	-0.10	-0.04
	$T' = 393.1$	51.06	109.22 0.03	109.15	+0.07	-0.04
	$\mu' = 0.4530$	44.78	105.28 0.14	105.66	-0.38	-0.24
	$-b = 0.00831$	*44.13	105.28 0.14	105.28	0.0	0.0
	$t_a = 174.52$					

138. The experimental throttle-valve arrangements by which the observations of Jenkin and Pye were obtained are, in the opinion of the writer, inherently much more likely to introduce uncertainties in the accuracy of the determinations than are those of the radial flow porous plug used by the writer, crude as some of these were in comparison with improvements in the detail of its design and operation that have since been made in it.† In view of the concordance noted between the two sets of data there appears, however, to be no reason for assuming that either set is less reliable than the other aside from the fact that the volatile impurity in the CO₂ which Jenkin and Pye

See references Nos. 4, 5, 6.

used never amounted to more than 0.11 percent by volume according to their analyses which showed it to be air, as against the 0.25 to 1.50 percent impurity, substantially air, that the writer had to contend with. Consequently, from this point on, both are used as a guide to the complete formulation hereinafter presented.

TABLE VI, B
LIQUID ISENTHALPS; EXPERIMENTAL VALUES
Pressures in atmospheres; temperatures in degrees centigrade

Curve	Per cent impurity	Adjusted values		Observed values, unadjusted			
		<i>p</i>	<i>t</i>	<i>p</i> ₁	<i>t</i> ₁	<i>p</i> ₂	<i>t</i> ₂
<i>a</i>	0.80 Sept. 18			71.92	31.25	55.53	18.77±0.14
				72.22	31.22	53.68	17.83 0.59
				72.22	30.38	43.38	8.65 0.19
<i>b</i>	1.08 Aug. 19 combined with Sept. 18	68.90	25.70				
		57.65	18.79±0.04	68.92	25.65	57.65	18.73±0.04
		48.90	11.12 0.08	68.84	26.07	48.09	11.52 0.08
		38.00	+ 2.41 0.11	67.68	24.80	38.00	+ 2.23 0.11
		35.09	- 0.72 0.15	69.26	26.16	35.09	- 0.54 0.15
		27.25	- 9.65 0.30	69.72	25.70	27.25	-10.06 0.30
<i>c</i>	0.90 Sept. 25	69.30	17.00				
		57.32	15.09±0.00	68.93	17.09	57.32	15.97±0.00
		49.55	11.35 0.08	68.96	17.09	49.55	11.37 0.08
		40.69	+ 4.16 0.14	69.62	16.99	40.69	+ 4.16 0.14
		30.38	- 6.27	68.83	16.41	30.38	- 6.22
		26.97	-10.66 0.26	69.22	14.64	26.97	-10.78 0.26
		17.63	-24.17 0.17	70.02	15.94	17.63	-24.21 0.17
<i>d</i>	0.50	Sept. 19		56.30	11.70	34.00	- 1.35±0.02
<i>e</i>	0.80	Sept. 23		63.50	11.40	44.10	+ 6.45 0.20
<i>f</i>	0.85 Sept. 23	63.80	7.75				
		54.77	7.10±0.05	63.84	7.87	54.77	+ 7.26±0.05
		44.03	+ 5.93 0.12	63.73	7.69	44.03	+ 5.84 0.12
		35.94	- 1.10 0.02	63.81	7.75	35.94	- 1.10 0.02
		23.21	-15.30 0.86	63.82	7.74	23.21	-15.30 0.86
<i>g</i>	0.4 to 0.2 Sept. 23			67.30	6.70	47.10	5.60±0.03
				69.35	5.40	54.25	4.60 0.04
				67.20	2.55	52.60	+ 1.90 0.07
				66.80	+0.25	41.25	- 0.70 0.03
				62.35	-0.20	23.05	-15.95 0.10
				65.25	-0.40	31.55	- 5.70 0.03
				61.45	-0.90	23.25	-15.80 0.21

139. Indications of the Jenkin and Pye data for liquid, near the saturation curve.—The data of Jenkin and Pye on the throttling of liquid CO₂ consist of pairs of points that fix chords of isenthalps as shown on Chart IX by the circles connected by short straight lines. The first two points only of some of the writer's liquid curves—taken as observed, without any attempt at adjustment into groups or correction for impurity content—are shown by those crosses of Chart IX that also are connected by straight lines. [The remaining points of these curves (at lower pressures) as well as the second and lower points of the rest of the writer's liquid curves, when more than two points were

observed, fall either on the saturation curve or near enough to it to be appreciably depressed by the effect of the air impurity, as it evident from Chart I, and are therefore not serviceable to the comparisons of this and the next paragraph.] These first two points of the writer's data are generally farther from the saturation region than are the points of the Jenkin and Pye data; they therefore correspond to a condition where the volatile impurities are present in, presumably, an undersaturated solution in liquid CO₂, so that the courses of the observed isenthalps in that region differ but little if any from those corresponding to pure CO₂ in the same region.

140. The more nearly pure the CO₂ the less should be the divergence between the observed isenthalps and the "pure" ones as they approach the saturation region. The Jenkin and Pye data having been obtained from CO₂ that had not over 0.11 percent of volatile impurity must be considered as better indicating the probable courses of isenthalps near the saturation curve, for pure CO₂, than do those of the writer when more than the first two points of his liquid curves—as limited in the preceding paragraph—are considered, since the liquid CO₂ which he used contained from 0.2 to 1.1 percent of volatile impurity. The close agreement, therefore, between these two sets of data that is shown by comparison of the slopes of the writer's "first-two-point" chords with the slopes of chords from the data of Jenkin and Pye, as plotted on Chart IX, must be taken as evidence that the trend of the isenthalps for pure CO₂ on the liquid side of the saturation curve is of the order of that shown on Chart IX rather than of that shown on Chart I. Further developments herein are based upon that conclusion.

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141. Having now available this double set of data, mutually agreeable, covering fairly well an adequate temperature range from +120°C in the field of superheated vapor down to -55°C, close to the liquid-solid phase-boundary in the liquid field, but unfortunately too limited in pressure range for satisfactory handling, the writer set himself the task of again attempting a formulation which should represent the data within the limits of their experimental uncertainty; a formulation which could serve for computation of the Joule-Thomson effect and its valuable associated relations over the whole field involved.

142. This task proved to be far more difficult than was at first anticipated; it involved an immense amount of laborious and time consuming computation, plotting, curve-drawing—and patience; so much that many times it was doubted that the result which might be obtained would justify the effort. Eventually, however, a formulation was evolved which, while not entirely satisfactory from several standpoints, does, with a reasonable degree of faithfulness, represent the experimental data over their whole range. Its use enables him to show very definitely in the remaining charts of this paper the characteristic variations of several of these relations with a completeness that otherwise would be impossible.

143. Every effort was made to keep this formulation simple and still faithfully represent the facts as disclosed by the data. But as the study progressed and the intricate nature of the relations became more and more evident, especially in the region near the critical point, it became manifestly certain that no simple formulation would be adequate. It is possible that if the writer

could have afforded to spend a few more months of such effort he might have arrived at a relation both less cumbersome and more nearly exact than that expressed by the intricate equation herewith presented. Until further and more exact data be available covering a greater range in temperature and particularly a much wider range in pressure further effort in this direction is inadvisable. He hopes that the relations which this formulation enables him to present for the consideration of others will have justified the effort by the result. A detailed account of these efforts at formulation will not be given, only the salient points of probable general interest will be discussed; they may be of assistance to others engaged in similar effort.

144. **Choice of equation for isenthalps.**—The most direct method of attack is to select some one form of equation between pressure and temperature to represent the course of each individual smooth curve drawn through the several points corresponding to each of the various runs represented by the plots of Chart IX; to determine the coefficients of these several equations, and then to find the cross-relations between these coefficients and the temperature intercepts of their graphs on some constant pressure line such as $p=0$, $p=1$, $p=p_c$ (the critical pressure), or $p=\text{some pressure greater than the critical pressure}$ and therefore not involved with abrupt phase changes.

145. One great deficiency which increased the difficulty of determining the coefficients of equations of several of the forms that were tried was evident from the start; the experimental curves are too limited to permit, with any approach to certainty of location or of slope, their graphical extension to intersection with either the zero or the one-atmosphere pressure line. Most of the writer's vapor curves could be extended to intersect the critical pressure line with a fair degree of certainty as to their probable courses at these intersections, except those near the critical point where the data were meagre and curvature apparently changing rapidly both along the curves and from curve to curve. These deficiencies were especially difficult to handle in dealing with the liquid curves, each of which was practically represented only by a chord through two points. The advantage in knowing the point of intersection of a curve with the zero pressure line and its course there is that the temperature of this intersection, and the corresponding curve slope furnish two of the coefficients of equations of many forms that are applicable to curves of this type.

146. Preferably the type equation for each of the curves should have properties such that it may conform to the probable courses of the curves when extended far beyond the present experimental limits, so that when these limits be extended by future investigations the same type of equation may be applicable, either by slight modification of the coefficients, or by addition of another term, or both. This preference immediately raises the questions as to what the "outer" courses of the curves, and their limits, probably are. To neither question can definite answer now be given, though some idea thereof may be gained by a process of rational conjecture based upon other properties of this or other fluids, associated through thermodynamic relations more or less rigorously expressed, with the properties disclosed by available sections of the experimental curves under consideration.

147. **Isenthalpic equations deducible from equations of state.**—From the various forms of equations of state for fluids that have been from time to time suggested there may, theoretically at least, be derived equations between temperature and pressure that are presumed to represent the courses of

isenthalpic curves on a temperature-pressure plane; or other equations expressing μ as functions of temperature and pressure independently. In some cases involving transcendental relations these equations cannot be cleared of other variables and so are not directly available.

148. Hoxton,⁸ for instance, has derived from van der Waal's equation of state and simplified by approximations an expression for μ as a function of pressure and absolute temperature and finds it applicable over the very limited range of his experiments on the porous plug effect for air. Dickson¹² has treated van der Waal's equation in reduced form and obtained a general equation for isenthalps (isopotentials, he calls them) which he has evaluated and variously displayed in reduced coordinates on several charts. But his equations do not fit the facts disclosed by the data for CO₂ nor do they represent the facts for air as disclosed by the data of Bradley and Hale¹³ and others. The divergence is very great in both cases (Cf. Charts II and III and the discussions in par. 97-99, pp. 35-38).

149. It was noted during the earlier efforts at formulation that the empirical equations for the curves of Charts I and II for CO₂, and similar equations that the writer fitted to throttling curves (Chart III) derived from the data of Bradley and Hale for air, yielded, or could be made to yield maximum or inversion points practically coincident with the locus of such points as predicted from Dieterici's exponential equation of state by Porter¹¹ (cf. Chart II). It was thought therefore that equations for isenthalps derived from Dieterici's reduced equation of state might be especially appropriate to apply to the experimental isenthalps. Unfortunately, because of the transcendental nature of the mathematical relations involved, his equation could not be made to yield usable relations.

150. In general the various forms of equations of state did not yield satisfactory expressions for application to the isenthalpic curves or their derivatives; they were therefore discarded entirely from further such consideration, and effort was concentrated on purely empirical forms, but with some regard to the probable extensions before mentioned.

151. **High pressure isenthalpic curve limits.**—As to the probable courses of the curves extended in the direction of increasing pressures: there is reason to believe that they rise at a decreasing rate to a maximum or inversion temperature where their slopes, μ , change sign by passing through zero; that beyond this point they fall, slowly at first, but possibly at an increasing rate until they meet some branch of the liquid-solid phase-boundary curve, or pass beyond its extremity. Several of the characteristic equations of state investigated by Porter¹¹ indicate such a probable course.

152. In the low temperature of the liquid field for CO₂ the present data disclose such inversion temperatures, but are insufficient to trace their variation with pressure. The original experiments of Joule and Thomson indicate a negative slope of the hydrogen isenthalps at low pressures, but at temperatures high with respect to its critical temperature. (See Chart V, p. 40.) Olszewski¹⁴ secured experimental evidence in 1902 of a maximum or inversion temperature for hydrogen, and later, in 1907, also for air and for nitrogen. Within the present year (1922) J. R. Roebuck¹⁵ at the University of Wisconsin, continuing, with radial flow porous plug apparatus, investigations commenced

¹² Olszewski, K., *Ann. d. Phys.*, 4, 7, 818, 1902; *Phil. Mag.*, 6, 13, 722, 1907.

¹³ Roebuck, J. R. *Proc. Am. Acad. Arts and Sci.* 60, 537-596, 1925.

there by the writer in 1905 on the Joule-Thomson Effect for Fluids, has definitely established a portion of this locus for air (gaseous state) at pressures up to 215 atmospheres.

153. **Low pressure isenthalpic curve limits.**—The question as to the manner in which these isenthalps approach their zero pressure limits is also a factor of importance in selecting a proper form of equation to represent them. The writer is aware of no porous plug or equivalent experiments with fluids at absolute pressures appreciably lower than one atmosphere. It is true that engineers have made many tests of steam with "throttling calorimeters," for the purpose of determining the initial quality, that is, the dryness or wetness of the steam sampled, in which tests the steam discharged from the calorimeter into condenser connections where the absolute pressures were considerably less than one atmosphere; but the temperature and pressure measurements involved were, however, taken not for the purpose of accurate extension of the throttling curves into the sub-atmospheric pressure range, but for use in entering "steam tables" or charts of the thermal properties of steam in which this low pressure range of values had been obtained by some more or less uncertain extension of better known values at the higher pressure ranges. It is doubtful if throttling calorimeter measurements for porous plug investigational work can be made with sufficient accuracy, as witness the efforts of Peake,²⁰ Grindley,²¹ Griessman,²² Dodge,²³ and others with steam at high pressures.

154. Since the porous plug effect is due to redistribution of the enthalpy of the fluid between the two factors, u , the internal energy, and pv , the external energy; and since the isothermal variation of pv with pressure has been shown to approach zero as the pressure approaches zero;* and since the coefficient of constant-pressure volume expansion with temperature, $(1/v)(\partial v/\partial T)_p$, has been shown to become sensibly constant, approaching $1/T$ as a limit at decreasingly low pressures;* it might therefore be assumed that the porous plug effect, $\mu = (\partial t/\partial p)_q = u + pv$, would approach a zero limiting value as the pressure approaches zero, at least for temperatures relatively high; in which case the isenthalpic curves on a p, t plane would meet the temperature axis with a horizontal tangent.

155. It does not necessarily follow, however, that there may not be intramolecular redistribution of intrinsic energy; u , that would result in a porous plug change of temperature as these low pressure limits are approached. Let the thermodynamically rigorous relation for the porous plug effect,* usually written

$$\mu = [T(\partial v/\partial T)_p - v] \div c_p \quad (21)$$

be re-arranged and written as

$$\frac{\mu c_p}{v} = \frac{T}{v} \left(\frac{\partial v}{\partial T} \right)_p - 1 \quad (22)$$

The second member of (22) approaches zero limiting value as the pressure approaches zero and v approaches infinity, if and when the values of the iso-

²⁰ Peake, *Proc. Roy. Soc.*, A 76, 185, 1905.

²¹ Grindley, *Phil. Trans. A*, 194, 1, 1900-1.

²² Griessmann, *Zeitsch. Ver. d. Ing.* 47, 1852, and 1880, 1903; *Forschungsarb. Ver. d. Ing.* 13, 1, 1904.

²³ Dodge, *Jour. Am. Soc. Mech. Engr.* 28, 1265, 1907, and 30, 1227, 1908.

* See Thermodynamic Texts.

thermal pv variation, and of $(1/v)(\partial v/\partial T)_p$ have the limits above expressed; the corresponding limiting value of the product μc_p must therefore be either finite or zero or of a lower order of infinite magnitude than is v at zero pressure: from which either of the factors c_p or μ may have any of these three values provided the other factor has the proper corresponding value to keep the term $\mu c_p/v=0$.

156. Worthing²⁴ has computed values of μ for air and CO₂ at several temperatures and pressures using this relation (21) coupled with various expressions for p v t relations extrapolated to zero pressure and infinite volume and by assuming, apparently, that c_p has finite values for these limiting conditions differing but little from known values at atmospheric pressure has arrived at finite values for μ at zero pressure.

157. Buckingham,²⁵ using this relation in deriving corrections to the constant pressure gas thermometer scale of temperature, apparently assumes that μc_p remains finite when $p=0$ and $v=\infty$; he definitely assumes "in accordance with the indications of experiment" then available, that μ is sensibly independent of pressure so long as the pressure is small; and furthermore assumes "it is known that c_p varies only slightly with the pressure" at small pressures. [Recent c_p measurements for air at atmospheric pressures by Schele and Heuse²⁶ indicate an increase in c_p at low temperatures.]

158. As a matter of fact our experimental knowledge of the pressure (or temperature) variation of either c_p or μ at pressures below one atmosphere is extremely meagre, practically nil, and any estimate as to the character of the variation of either at decreasingly low pressures is largely speculative for the lack of the necessary data. It is highly probable that c_p will remain positive in approaching its limiting value at zero pressure; certain plausible assumptions indicate very definite finite limiting values; but to the writer it seems possible that it may assume infinite limiting values at either extreme of temperature at zero pressure.

159. At atmospheric pressure μ for the vapor phase is positive at low temperatures, and negative at temperatures sufficiently high. From the known trend of the isenthalpic curves approaching the one atmosphere line there is little or no indication of any pending rapid change in their courses at sub-atmospheric pressures which might make them meet the limiting zero pressure line either normally or tangentially, giving limiting values of $\mu_{p=0}$ of either 0 or $\pm \infty$. Some recent work at this laboratory by J. R. Roebuck¹⁹ on the porous plug effect for air does appear to the writer to give slight indications of an inflection point in the isenthalpic curves at low pressures above one atmosphere, but this appearance is yet to be confirmed.

160. In view of these considerations the writer finally selected a form of equation for the isenthalps which continues undisturbed into the low pressure field the apparent trend of the curves approaching from the higher pressure region.

161. It is of interest to note, in passing, the latitude that exists in the selection of the form of equation for a curve if one is interested in representing the data over their experimental range only, and the extraordinary divergence

²⁴ Worthing, A. G., *Phys. Rev.* 33, 217, 1911. Some Thermodynamic Properties of Air and Carbon Dioxide.

²⁵ Buckingham, Edgar, On the Establishment of the Thermodynamic Scale of Temperature, *Bulletin Bureau of Standards*, 3, 237, 1907.

²⁶ Schele and Heuse, *Ann. d. Phys.* 4, 37, 79, 1912; 40, 473, 1913.

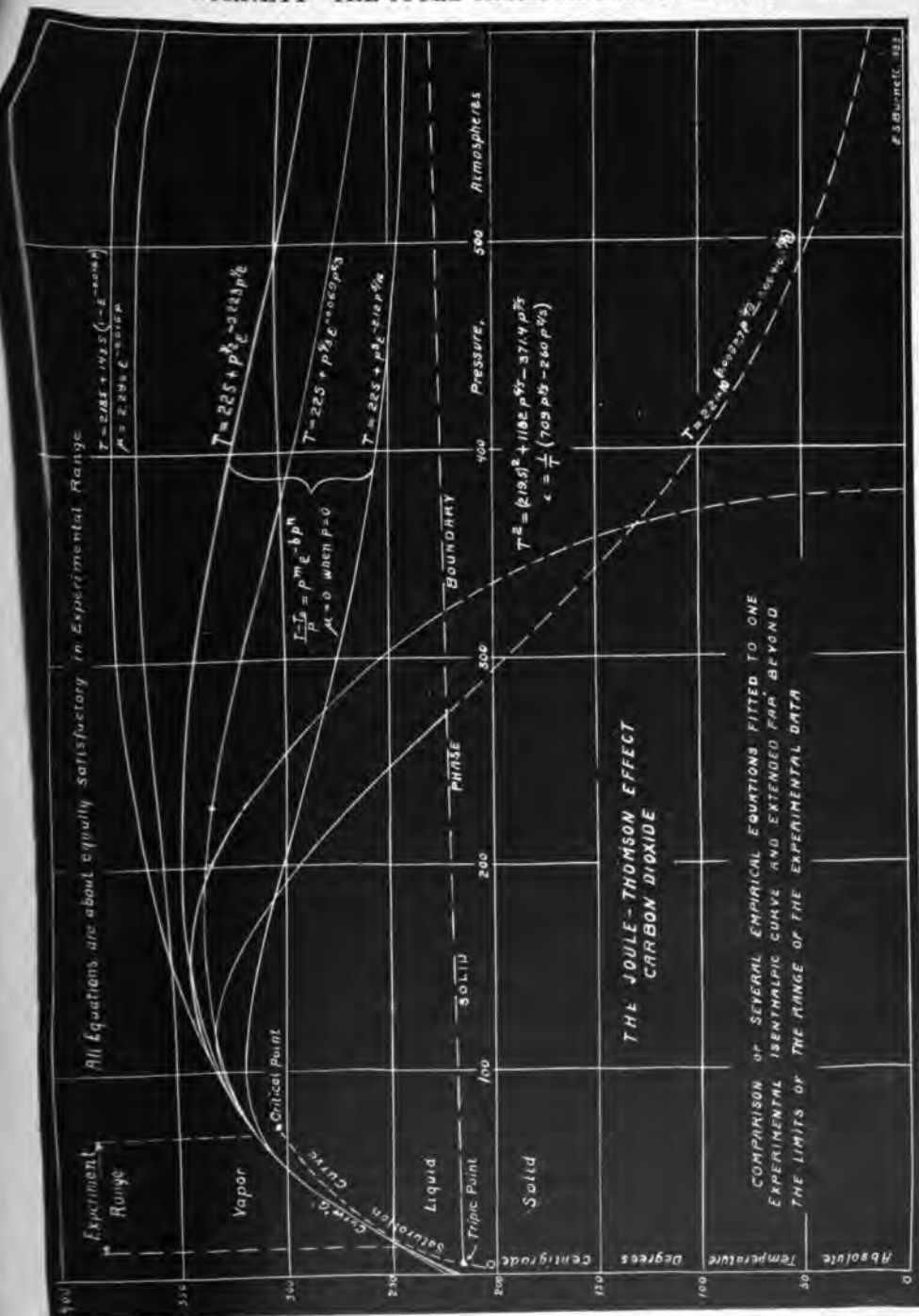


Chart X

Graphical comparison of extrapolated values computed from the several empirical equations listed in Table VIII on the preceding page.

of curves plotted from equations of various forms extended into the field without that range. Of the very many forms of equation that were tried for curve "G" for instance, one of the longest in experimental range, Chart X, p. 67, shows a plot of several such extended far beyond that range within which Table VIII compares observed with calculated values. The agreement of most of them with the data is about equally satisfactory, or could be made so by slight modifications of their coefficients. Three of them have the same general form but different coefficients, showing good agreement with the data, but considerable divergence without their range.

162. This comparison serves also to illustrate very emphatically the futility of expecting to obtain reliable quantitative information as to the variation of an effect outside of the experimental range, from extrapolation of an empirical relation even if it be known to be of the proper form, unless definite controls be available for limiting the "outside" course of the curve through the selection of the proper coefficients. It also illustrates even more emphatically, perhaps, the possibility, the probability, of drawing exceedingly erroneous conclusions from considerations based on even relatively small extrapolation of chance empirical relations that may serve to connect, for lack of more proper relations, limited experimental data. This last procedure has all too often been carried lamentably far beyond the limits of judicious consideration.

163. **Form of isenthalpic equation selected.**—A form of equation having two constants for each isenthalp was ultimately selected which experience had shown would fit both the vapor curves of considerable curvature, and the liquid curves, most of which have less curvature, and some of which are nearly flat. This equation has not the limiting characteristics which one might desire, but it can be modified by the addition of another term to take care of a considerable extension when the controls therefore be available.

164. It was observed that when the logarithm of the graphically determined slopes of the longer experimental curves were plotted against the corresponding pressures, the plots were straight lines, the equations for which have the form or

$$\log \mu = \log \mu' + b(p - p') \quad (23)$$

$$\log (\mu/\mu') = b(p - p') \quad (24)$$

which state that the logarithm of the ratio of the slopes of a curve at any two points of it is proportional to the difference of the corresponding pressures; b , the constant factor of proportionality for any one curve is determined by the slope of the straight line, expressed in units appropriate to the system of logarithms employed. If these be natural logarithms the equations transform to

$$\mu = \mu' e^{b(p - p')} \quad (25)$$

in which μ' is the slope of the isenthalp at its intersection with the pressure line $p = p'$, and e is the base of the natural system of logarithms.

165. Since the critical pressure line is the first pressure line in the direction of increasing pressures that is intercepted at any temperature (except the critical temperature) by one isenthalp only, it seemed logical to select the critical pressure line from which to measure differences of pressure, and along which to correlate the coefficients μ' and b of Eqs. (25) as functions of the absolute temperatures, ($T' = T_{p'} = p_c$), at those intercepts. Eq. (25) then becomes

$$(\partial t / \partial p)_\theta = \mu = \mu_{p_c} e^{b(p - p_c)} \quad (26)$$

in which p_c for CO_2 is taken as 72.9 atm.

166. Retaining the primes of Eq. (25) with the significance of Eq. (26), integration yields

$$T - T' = \frac{1}{b}(\mu - \mu') = \frac{\mu'}{b}[e^{b(p-p')} - 1] \quad (27)$$

from which it is evident that along an isenthalp μ is represented by this relation as a linear function of the temperature, for on re-arranging, Eq. (27) becomes

$$\mu = b(T - T') + \mu' = bT - (bT' - \mu') \quad (28)$$

Further, also, by solving Eq. (27) for p , we obtain,

$$p - p_e = \frac{1}{b} \log_e \left[\frac{b}{\mu'} (T - T') + 1 \right] \quad (29)$$

167. Eq. (27) or its equivalents imposes the following characteristics upon its graph (see Fig. 13):

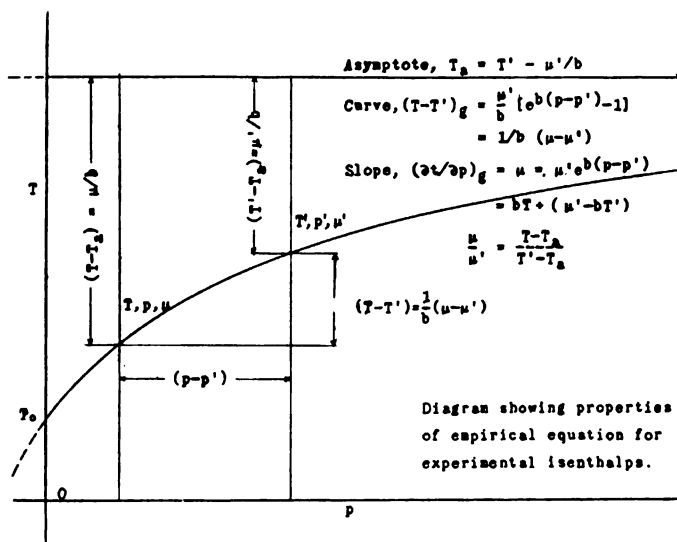


Fig. 13.

When μ' and b are of opposite sign the curve is convex upwards and approaches asymptotically a finite *upper* temperature limit, in the direction of increasing pressures when the slope μ' is positive, in the direction of decreasing pressures when the slope is negative.

When μ' and b are of the same sign, the curve is convex downward and approaches asymptotically a finite *lower* temperature limit, in the direction of decreasing pressures when the slope is positive, in the direction of increasing pressures when the slope is negative.

When both μ' and b are each zero the graph is a constant temperature line.

The temperature difference between a point on the curve and its asymptote is proportional to the slope of the curve at that point, hence the difference of slopes at two points of a curve, and the corresponding differences of temperature are proportional.

The slope μ' at the reference pressure line, p' , may have any value from $+\infty$ to $-\infty$.

When the exponential coefficient, b , equals zero, the slope, μ , is constant and the graph is a straight line with constant slope which may have any value from $+\infty$ to $-\infty$.

When b is infinite, the slope will have but two values, zero and either $+\infty$ or $-\infty$, regardless of any values given to or assumed by μ' ; the graph then becomes the boundaries of a quadrant defined by the intersection of the temperature axis with the temperature asymptote.

168. This type of equation was selected for several reasons additional to those already mentioned: It has no inversion temperature, the slope being uniformly of constant sign for all pressures. It appeared likely that the asymptote of the convex-upward curve presents a limiting temperature,

$$T_a = T' - \mu'/b \quad (30)$$

above which the true isenthalp, which the curve approximates over the experimental range, will not pass, so that by some future modification its course can be turned downward in accordance with the indications of additional experimental data when such be available. It has but one coefficient to be determined if we assume that the temperature intercept of the isenthalp with, and its slope at, the reference pressure line can be read from a pressure-temperature graph of the experimental isenthalp.

169. The actual work of formulation required constant computation, grouping and re-grouping the original data or relations derived therefrom, the plotting and cross-plotting in all sorts of ways various assemblies of numerical values determined by the data and the curve forms, for the purpose of determining their coefficients and their cross relations; all of which had frequently to be checked back to ensure that the formulation being developed adhered to the facts of the data. Many times when the problem seemed to be in a fair way of rapid solution some discrepancy or deficiency would develop that would necessitate starting over again on a new tack.

170. This was usually due to the gradual disclosure of more intricate relations involved in the facts governing the data. The narrowness of the pressure range of the data magnified uncertainties of form and location of the experimental curves and increased the difficulty of arriving at smooth cross relations. In some respects this was an advantage, for the choice of selection was less rigidly fixed than would have been the case if more nearly exact data of wider pressure range had been available: on the other hand, had such been available, the cross-relations would have been smoother and more definite from the start, and on that account easier to handle.

171. In the liquid field the controls for determining the coefficients μ' and b of the equations were very limited, practically to only a two point chord for each curve. The slopes of these chords, presented in Table IX, were plotted against their mean temperatures (Chart XI) and were studied in a variety of ways. At a temperature of about -24°C the chords appeared to have zero temperature gradient; at lower temperatures their slopes are unquestionably negative. This temperature, -24°C , was taken as the inversion temperature of the Joule-Thomson effect for CO_2 liquid at all pressures involved in this formulation. The data are too limited to indicate a variation of the inversion temperature with pressure.

172. The actual determination of the coefficients of the equations of the curves in the liquid field was purely a "cut and try" process tied up with a simultaneous effort to correlate their coefficients by an expression that would also correlate those of the vapor isenthalps. Corresponding values of T' , μ' and b were thus found such that, with these coefficients, a curve through the

TABLE IX

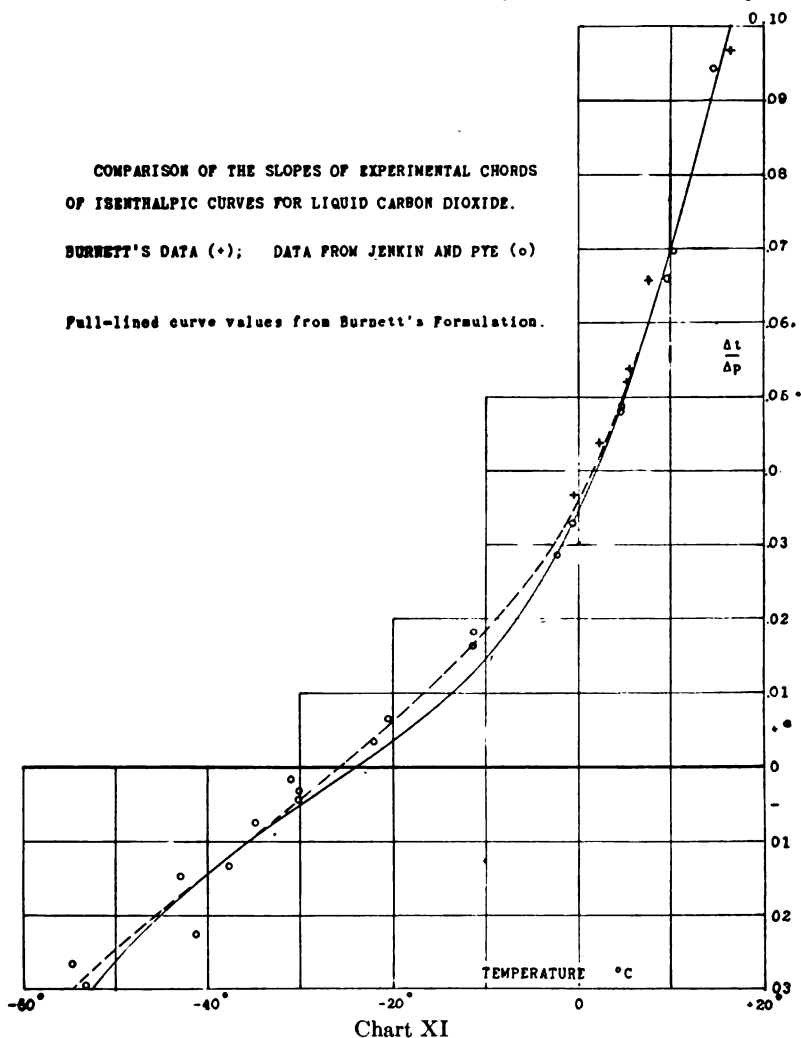
SLOPES OF EXPERIMENTAL ISENTHALPIC CHORDS IN THE FIELD OF LIQUID CO₂
Temperatures in degrees centigrade, pressures in atmospheres.

B, Burnett's data; J, data of Jenkin and Pye.

	Mean tempera- ture	Mean pres- sure	Δt ($t_1 - t_2$)	Δp ($p_1 - p_2$)	$\Delta t / \Delta p$
B	16.53	63.12	1.125 ± 0.005	11.61	0.0968 ± 0.0004
J	14.74	54.48	0.72	7.63	0.0943
J	10.33	49.58	0.53	7.56	0.0700
J	9.67	51.26	0.66	10.00	0.0660
B	7.57	59.30	0.595 ± 0.055	9.07	0.0655 ± 0.0060
B	5.61	57.21	1.088 ± 0.025	20.20	0.0538 ± 0.0012
B	5.00	61.70	0.785 ± 0.035	15.11	0.0520 ± 0.0023
J	4.68	47.96	0.63	12.92	0.0487
J	4.62	48.71	0.56	11.70	0.0479
B	+ 2.22	59.86	0.64 ± 0.07	14.61	0.0438 ± 0.0048
B	- 0.69	53.62	0.94 ± 0.03	25.55	0.0368 ± 0.0012
J	- 0.80	46.60	0.60	18.36	0.0327
J	- 2.32	45.72	0.44	15.38	0.0286
J	-11.22	30.98	0.45	24.83	0.0181
J	-11.43	37.45	0.26	15.99	0.0163
J	-20.63	34.83	0.135	20.69	0.0065 ₂
J	-22.06	36.67	+0.11	31.15	0.0035 ₃
J	-29.94	34.36	-0.11	34.69	-0.0031 ₁
J	-30.26	42.91	-0.08	17.68	-0.0045 ₂
J	-30.98	32.76	-0.04	23.06	-0.0017 ₃
J	-34.91	32.18	-0.18	24.22	-0.0074 ₃
J	-37.73	31.17	-0.345	25.85	-0.0133
J	-41.40	31.09	-0.595	26.12	-0.0228
J	-43.01	31.41	-0.38	25.65	-0.0148
J	-53.20	30.31	-0.81	27.28	-0.0297
J	-54.64	30.28	-0.73	27.49	-0.0266

point (T' , p') would pass at lower pressures along side or over a corresponding experimental chord with essentially the same slope between the pressure limits of the chord as the chord itself had. In this way was built up a set of equations for the liquid isenthalps corresponding to those more directly developed for the vapor isenthalps. (In Chart XI the values from which the full-lined curve is plotted were selected from those shown on Chart XVII, p. 85, computed from the formulation, corresponding most nearly to the mean pressures and temperatures of the experimental chords.)

173. In expressing the relations found for μ' as a function of T' , the critical temperature and the slope of the saturation curve at the critical temperature are both involved. The values here used for the critical temperature, the critical pressure, the vapor pressures and temperatures along the saturation curve, and the slope of the latter at the critical point are taken from Keyes'



formulation¹⁴ which is based upon his consideration of data then available on those values. Keyes' critical values are: $p_c = 72.9$ atm.; $T_c = 31.0^\circ\text{C} = 304.1^\circ\text{K}$; $(\partial t/\partial p)_{sat.}$ at critical point $= 0.6050^\circ\text{C}$ per atm.; Keyes' formulation is used because it is the only one known to the writer which expresses in continuous though empirical assembly from the triple point to the critical point all of the saturation properties of CO_2 .

174. One important controlling condition that had to be satisfied in the writer's formulation concerns the variation of the isenthalps at and adjacent to the critical point. Looking in the direction of decreasing pressures this is the point where isenthalps first meet; their slopes, however, have at this common point a single value and that is necessarily identical with the slope of the saturation curve at the critical point.

175. Isenthalps in the vapor field near the critical point region intersect the saturation curve with slopes steeper than those of the latter at the points of intersection. Isenthalps in the liquid field intersect the saturation curve with slopes less steep than those of the latter at their points of intersection therewith. (See Charts IX and XIII.) Comparison of these isenthalps at their saturation curve intersections shows that the closer the isenthalps pass to the critical point the closer to that point are their intersections with the saturation curve, and the more nearly do their slopes at intersections approach in value that of the saturation curve itself at the intersection, becoming identical therewith at the critical point which is the limiting point of intersections.

176. Following the trend of the isenthalps from the higher temperature field of the vapor phase down past the critical point and saturation curve into the lower temperature field of the liquid phase, they are seen to crowd together (on the critical isobar) as the critical point is closely approached, "pivoting," as it were, about this point or points along the saturation curve; then separating rapidly therebelow, as viewed at their crossings of the critical isobar.

177. The slopes, μ' , of the isenthalps at their crossings of the critical isobar, p' , have, at the higher temperatures, small positive values; at lowering temperatures they have values slowly increasing to a maximum of 0.7635° per atm. at a temperature, $T' = 307.1^\circ\text{K}$, slightly higher than the critical temperature; below this maximum-slope temperature the slopes decrease rapidly with continued lowering temperature, changing at an infinite rate at the critical point where the value of the slope is that of the slope of the saturation curve at the critical point; at about -24°C the slope becomes zero, and becomes increasingly negative at still lower temperature; the rate of change of slope with decreasing temperature below the critical point reaches a minimum value not zero at -24°C .

178. These variations of μ' with temperature along the critical isobar are conditions that had to be satisfied in formulating the relation connecting μ' with T' that is expressed in the following equation:

$$\mu' = \mu_c e^{2.3026 \left(\frac{T_c}{T'} \right)^2 \left[0.1700 - 0.0765 \left(T' - T_c \right)^{3/13} \right] \left(T' - T_c \right)^{1/3} - \left(\frac{T' - T_c}{T'} \right)^{52/15}} \quad (31)$$

in which T_c is taken as 304.1°K ,

μ_c is taken as 0.6050° per atm. (= slope of saturation curve at critical point)

μ' is the value of $(\partial t/\partial p)_h$ at the absolute temperature T' on the critical pressure line $p = 72.9$ atm.

$e = 2.718 \dots$ = the base of the natural system of logarithms.

Values of μ' as this function of T_c and T' , and of $(1/\mu)(d\mu/dT')$ are given in Table X and are plotted on Chart XII.

179. In addition to satisfying the observed variations of μ' outlined in the preceding paragraphs, Eq. (31) gives μ' a value of $-\infty$ at absolute zero of temperature, which, however, is of little significance, since the solid phase is reached at about $T' = 218^\circ$ absolute, (-55°C); it makes $\mu' = 0$ at $T'_i = 249.1^\circ\text{K}$ (-24°C) as mentioned above, and also at $T' = 1656^\circ\text{K}$, which is in the vicinity of the point where we have some reason for expecting to find an inversion temperature at this pressure; at higher temperatures the equation brings μ' asymptotically to the finite limiting negative value of $\mu_c - 1 = 0.605 - 1.000 = -0.395$, which may or may not be of significance. For values of $|T' - T_c| < 10$ the second term of the right hand member of Eq. (31) is of negligible moment.

180. Formulation of "b" as a function of T' .—Using Eq. (27), putting $p = 0$, we have

$$T_0 - T' = (1/b)(\mu_0 - \mu') \quad (32)$$

in which μ_0 is the slope of the isenthalp at its intersection T_0 with the zero pressure line p_0 . The data for the vapor field indicate only a continual increase of μ_0 with decreasing temperature, but set no definite limit to its value at absolute zero of temperature where $T_0 = 0$. For this point, ($T_0 = 0$, $p = 0$), μ_0 has therefore arbitrarily been made to correspond with μ' put equal to μ_c , the critical point value of μ , by selecting a value of b such that the isenthalp that passes through the critical point with the slope there $\mu_c = 0.605$ shall also pass through the point (T_0 , p_0) the absolute temperature and pressure of which are both zero. This imposition requires that $(T' - T_0) = (T_c - T_0) = T_c = 304.1^\circ\text{K}$ from which by trial b must have the value -0.04267 and μ_0 must have the value $+13.580$ to satisfy the above conditions.

181. This relation means that all of the isenthalps of the vapor field that intersect the saturation curve shall do so twice, entering at a higher temperature and pressure and leaving at a lower temperature and pressure. The "critical isenthalp" enters at the critical point and leaves at the null point. Thus the vapor field is entirely covered by all of the curves that lie above the critical isenthalp.

182. In the liquid field the coefficients μ' and b are so related to T' (see Chart XII) that as the T' points of the isenthalps are followed downward from the critical point, the corresponding T_0 points of the isenthalps (extended to intersection with the $p = 0$ line) (see Chart XIII) at first rise rapidly (at an infinite rate) from absolute zero of temperature to a maximum (of about $T_0 = 280$, when $T' = 289$), and then slowly retrograde with decreasing T' , but at a less rate than T' , until at $T = 249.1^\circ\text{K}$ (-24°C) they have the same value. This is the inversion temperature of the liquid field where $\mu = 0$. Below this temperature T_0 lags behind T' as both are further reduced, and the liquid isenthalps, here with negative slopes, eventually meet their limits at the liquid-solid phase-boundary²⁷ which crosses the field along the approximate line extending from the triple point ($p = 5.15$ atm., $T = 216.5^\circ\text{K}$) through the point $p = 72.9$ atm., $T = 218.0^\circ\text{K}$. Thus the curves below the critical isenthalp sweep over and cover the entire liquid field from the saturation curve and the critical isenthalp to the liquid-solid phase-boundary.

183. The relation between T' and the parameter b which was found to satisfy these arbitrary requirements is given by the following equation:^{*}

²⁷ Tammann, *Kristallisieren und Schmelzen*, p. 296; A. J. Barth, Leipzig, 1903.

^{*} In the writer's abbreviated account of this work which appeared in the *Physical Review* 2, 22, 1923, the exponent $4/9$ of the $(T' - T_0)$ term in the exponent of e in Eq. (33) above is erroneously given as $1/2$ in the corresponding Eq. (13) p. 603 of that paper.

$$b = b_c \left(\frac{T'}{T_c} \right)^5 e^{-0.37(T' - T_c)^{1/3}} + 0.0337 \left(\frac{T' - T_c}{T'} \right)^{36/19} \quad (33)$$

in which $b_c = -0.04267$, the value of b for the critical isenthalp through the point where $T' = T_c = 304.1^\circ\text{K}$. Values of b as a function of T' and of $(1/b)(db/dT')$

TABLE X
VALUES DERIVED FROM EQUATIONS (31), (33) and (30)

$T' = p_c$	μ'	$\frac{1}{\mu'} \left(\frac{d\mu'}{dT'} \right)$	b	$\frac{1}{b} \left(\frac{db}{dT'} \right)$	$T_a (= T' - \mu'/b)$
200°K	-0.10394	-0.05070	+0.009137	-0.02958	211.38
210	-0.06174	-0.05385	+0.006580	-0.03436	219.38
220	-0.03528	-0.05626	+0.004527	-0.04126	227.79
230	-0.01867	-0.06956	+0.002803	-0.05700	236.66
240	-0.007736	-0.11568	+0.001291	-0.14190	245.99
245	-0.003376	-0.24875	+0.000578	-0.24540	250.84
249.1	0.0	$\mp \infty$	0.0	$\mp \infty$	249.10
250	+0.000730	+1.1270	-0.000131	+1.0895	255.57
255	+0.004969	+0.17832	-0.000855	+0.17316	260.81
260	+0.009732	+0.10630	-0.001630	+0.09709	265.97
265	+0.01548	+0.08306	-0.002456	+0.07196	271.30
270	+0.02278	+0.07266	-0.003401	+0.05997	276.70
280	+0.04543	+0.06720	-0.005887	+0.05229	287.72
290	+0.09091	+0.07220	-0.010152	+0.05682	298.95
295	+0.13243	+0.08171	-0.013658	+0.06567	304.70
300	+0.21266	+0.11453	-0.019944	+0.09175	310.66
302	+0.27742	+0.15868	-0.02464	+0.12542	313.26
303.5	+0.3804	+0.31550	-0.03156	+0.23482	315.55
303.8	+0.4265		-0.03419		316.27
304.1	0.6050	$+\infty$	-0.04267	$\pm \infty$	318.28
304.2	0.6922	+0.44701	-0.03742	-0.57456	322.70
304.3	0.7081	+0.25817	-0.03572	-0.38557	324.12
304.5	0.7264	+0.07519	-0.03357	-0.25718	326.13
305	0.7468		-0.03045		329.52
305.5	0.7560	+0.16857	-0.02841	-0.12005	332.11
306.5	0.7628	+0.003387	-0.025614	-0.08480	336.28
307.5	0.7630	-0.000646	-0.02385	-0.06694	339.50
308.5	0.7603	-0.004870	-0.02243	-0.05596	342.40
310	0.7533	-0.007072	-0.020792	-0.04559	346.23
315	0.7223	-0.009074	-0.017458	-0.028334	356.38
320	0.6900	-0.009052	-0.015448	-0.020562	364.67
330	0.6331	-0.008696	-0.013108	-0.013190	378.30
340	0.5872	-0.006992	-0.011675	-0.009780	390.30
350	0.5500		-0.010700		401.40
360	0.5200	-0.005252	-0.009982	-0.007630	412.10
370	0.4952		-0.009400		422.70
380	0.4742	-0.004074	-0.008876	-0.005295	433.42
390	0.4560		-0.008470		443.83
400	0.4410	-0.003241	-0.008042	-0.004632	454.84
410	0.4275		-0.007690		465.60
420	0.4154	-0.002785	-0.007308	-0.004289	476.85

are also given in Table X and are plotted on Chart XII for comparison with μ' . It will be noted that the rate of change of b with T' is also infinite at the critical point, but that it changes sign in passing through that value. This

variation corresponds to the infinitely rapid fall and rise of T_0 as T' passes through the critical value T_c , as described in the preceding paragraph and shown on Chart XII. The second term of the right hand member of Eq. (33) is also negligible for values of $|T' - T_c| < 10$. Eq. (33) makes b equal to zero at $T' = 249.1^\circ\text{K}$ (-24.0°C) and at $T' = 1628^\circ\text{K}$.

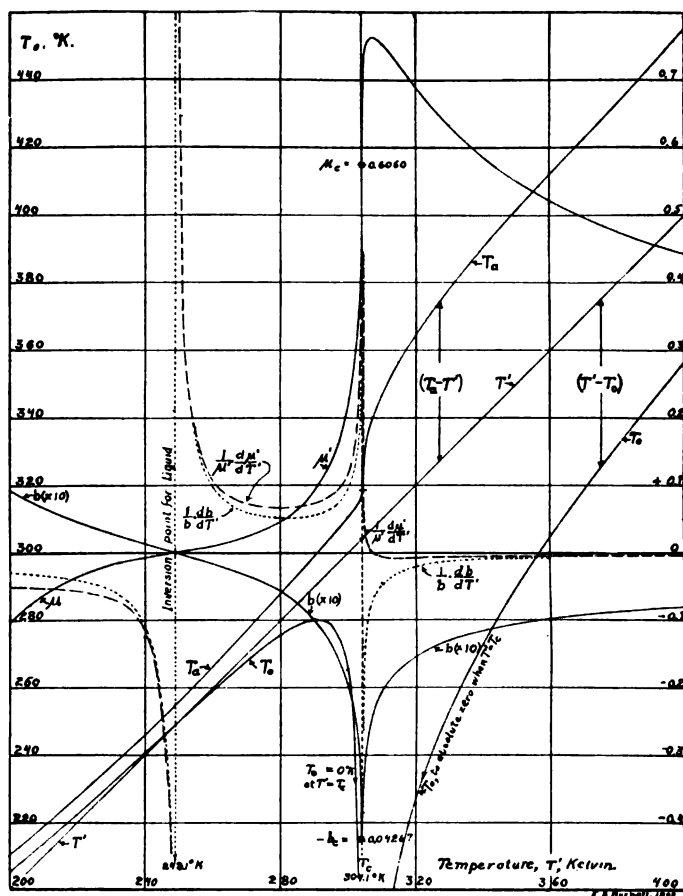


Chart XII

GRAPHS OF VALUES LISTED IN TABLE X.

184. These two equations (31) and (33) for μ' and b respectively, in terms of T' are intended to be so related that both μ' and b change signs at the same two temperatures, namely $T' = 249.1^\circ\text{K}$ in the liquid field, and $T' = \text{about } 1642^\circ\text{K}$ in the field of superheated vapor or gas. This partly accounts for the apparently peculiar values of the several exponents of the $(T' - T_c)$ term, namely: $3/13$, $52/15$, $4/9$, and $96/49$. It was necessary to preserve proper odd or even powers and roots used, in order to obtain the desired positive or negative values of those powers or roots, and thus avoid imaginary values when T' is less

than T_c . Also they made the equation better reproduce the several corresponding values of μ' and b for the various isenthalpic equations which they represent.

185. It has been mentioned that Eq. (31) makes $\mu' = 0$ at $T' = 1656^\circ\text{K}$ (1383°C), and Eq. (33) makes $b = 0$ at $T' = 1628^\circ\text{K}$ (1355°C). The mean of these two values is 1642°K (1369°C), which is 5.43 times the critical temperature of CO₂. It is certain that values for the several exponents of these equations can be found, slightly different from those herein indicated, that will have the proper odd and even relations as mentioned above, but will make both μ' and b zero at the same high temperature. For the present purpose a difference of 14° between the computed inversion temperatures for μ' and b and their average of 1642° is not of importance. It will, however, be interesting to compare these values with those that later may be determined by experiment to be the inversion temperatures for the Joule-Thomson effect in CO₂ over the pressure range involved in the experiments under consideration.

186. **The complete formulation.**—The complete general equations for the whole field involved are, of course, obtained by substituting for μ' and b in equations (26) to (29) their values in terms of equations (31) and (33), which substitution yields admittedly cumbersome and unwieldy expressions that involve transcendental relations. It is a matter of regret that the writer was unable to establish a simpler and more easily usable formulation. His effort however was persistently to make the equations fit as closely as possible the facts disclosed by the data, rather than to warp the data to fit equations—a procedure to which resort has too often been made, in many cases resulting unfortunately in obscuring the true relations otherwise discernible from the data.

187. **Comparison of observed experimental values with values computed from the complete formulation.**—In Table VI, A, for CO₂ vapor, the temperatures listed in column (5) are those computed by means of the present formulation to compare with the observed temperatures of column (3) for the corresponding observed pressures of column (2). Column (6) shows the variation between the observed and the computed temperatures. Column (7) gives the result of combining columns (6) and (4); a zero value in this column indicates that the computed temperature lies within the range of column (3) $\pm$ column (4); other values give the minimum departure therefrom. In column (1), below the designation, date and impurity percent items that apply to each experimental curve, are given the coefficients T' , μ' and b of the respective equation (27) from which the computed values are obtained. There is also given for each computed curve, in $^\circ\text{C}$, the temperature t_a of its asymptote [from Eq. (30)] above which the experimental curve, when experimentally extended, probably will not rise (see Chart XII). Table XI presents a similar comparison of the experimental data of Jenkin and Pye for CO₂ vapor with corresponding values computed from the writer's formulation. Note that their curves d and e appear to be continuations of the writer's curves G and $H-I$, respectively, as presented on Chart IX.

188. The observed values here tabulated are plotted on Chart IX, crosses (+, $\times$) representing the writer's values and circles (o) those of Jenkin and Pye. The smooth curves of this chart are graphs of the computed curves through the several points [p' ($=72.9$ atm.), T'] so selected as to closely represent the experimental curves; they include the comparative computed values given in these tables.

TABLE XI
COMPARISON BETWEEN JENKIN AND PTE'S OBSERVED "THROTTLING" DATA FOR CO₂ AND CORRESPONDING VALUES
COMPUTED FROM BURNETT'S 1922 EMPIRICAL FORMULATION.

Curve	Pressure lbs./in. ² atm.	72.9	700	600	500	400	300	200	150	T' (°K)	μ'	-b	t ₀ (°C)
a	Temp. (°C)												
	t (observed)	42.9		12.2		- 6.6	-17.3			316.0	0.7155	0.01699	85.02
	t (computed)			12.32		- 6.58	-17.82						
b	t ₀ - t _c			- 0.12		- 0.02	+ 0.52						
	t ₀			12.4	3.2	- 6.33	-16.9	-29.05		316.2	0.7143	0.01690	85.37
	t _c	43.1		12.65	3.82	- 6.12	-17.26	-29.76					
c	t ₀ - t _c			- 0.25	- 0.62	- 0.21	+ 0.36	+ 0.71					
	t ₀			13.5				-27.2	-34.0	317.0	0.7091	0.01655	86.74
	t _c	43.9		13.84				-27.6	-34.22				
d	t ₀ - t _c			- 0.34				+ 0.40	+ 0.22				
	t ₀			14.2	5.3	- 4.3	-14.4	-26.0		317.4	0.7065	0.01639	87.40
	t _c	44.3		14.47	5.86	- 3.79	-14.50	-26.52					
e	t ₀ - t _c			- 0.27	- 0.56	- 0.51	+ 0.10	+ 0.52					
	t ₀			20.1	11.6	2.9	- 6.6	-17.3		321.2	0.6825	0.01505	93.45
	t _c	48.1		19.94	12.01	3.26	- 6.48	-17.22					
f	t ₀ - t _c			+ 0.16	- 0.41	- 0.36	+ 0.12	+ 0.08					
	t ₀							-15.05	-20.7	322.25	0.6761	0.01476	94.98
	t _c	49.15						-15.01	-20.67				
g	t ₀ - t _c			- 0.02	- 0.35			+ 0.04	+ 0.03				
	t ₀		29.2	21.9	13.85					322.6	0.6728	0.01466	95.40
	t _c	49.5	28.90	21.92	14.20								
	t ₀ - t _c		+ 0.30	- 0.02	- 0.35								
	t ₀												
	t _c												

TABLE XI (Continued)

Curve	Pressure lbs./in. ² atm.	Temp. (°C)	700 47.6	600 40.8	500 34.0	400 27.2	300 20.4	200 13.6	150 10.5	T' (°K)	u'	b	t _g (°C)
h	Temp. t _g t _g -t _c	72.9	47.6	40.8	34.0	27.2	20.4	13.6	10.5	324.3	0.6642	0.01423	97.87
i	Temp. t _g t _g -t _c	51.2	47.6	40.8	34.0	27.2	20.4	13.6	10.5	327.9	0.6444	0.01349	102.58
j	Temp. t _g t _g -t _c	54.8	47.6	40.8	34.0	27.2	20.4	13.6	10.5	332.6	0.6206	0.01268	108.44
k	Temp. t _g t _g -t _c	60.1	47.6	40.8	34.0	27.2	20.4	13.6	10.5	332.2	0.6179	0.01259	109.18
l	Temp. t _g t _g -t _c	64.6	47.6	40.8	34.0	27.2	20.4	13.6	10.5	337.7	0.5671	0.01197	114.55
m	Temp. t _g t _g -t _c	65.0	47.6	40.8	34.0	27.2	20.4	13.6	10.5	338.1	0.5650	0.01190	115.00
n	Temp. t _g t _g -t _c	70.25	47.6	40.8	34.0	27.2	20.4	13.6	10.5	343.35	0.5743	0.01131	121.02

189. Table VI, B presents the writer's observed values for liquid CO_2 . Those of Jenkin and Pye for liquid CO_2 are given in Table VII. No values have been computed from the formulation for presentation here in direct comparison with either set of liquid data, but the close agreement of the slopes of the writer's "first-two-point" chords with those of Jenkin and Pye is evident from a glance at Chart IX, as is also that of the general trend of these slopes with the variation of the slopes of the empirical curves interdrawn at uniform T' intervals of 5 or 10 degrees on the critical isobar as thereon designated. The general "graphical agreement" between the observed and the computed values in both the vapor and the liquid fields may be considered satisfactory.

190. The "adjusted values" for curves b , c , and f of Table VI, B are shown in the liquid field of Chart I; also the first four pairs of "unadjusted observed values" of group g . The points of curve a , unadjusted, are nearly coincident with the saturation curve from close to the critical point downward; they are in fact the results of an attempt to trace a curve, experimentally, through the critical point. On Chart IX the writer's liquid chords shown are from the "unadjusted observed values" of Table VI, B; the values charted are given in the first lines of groups c and f and in the first four lines of group g .

191. On Chart IX, for comparison, there are also shown in outline and by name and date the location and relative extent of the experimental fields covered by the present and previous investigators of the Joule-Thomson effect in carbon dioxide; it is superfluous to remark that the previous investigations were very limited in scope compared to those developed in this paper. Further comparisons will be made in later paragraphs.

GRAPHICAL PRESENTATION OF ISENTHALPIC RELATIONS FROM 1922 FORMULATION

192. From the equations of the present formulation have been computed corresponding values of p , T , and μ for a series of isenthalps spaced at ten degree intervals along the critical isobar except near the critical point where they are more closely spaced to better show the character of their variation in that region. These values are tabulated in Table XII pp. 98-101 and plotted on Chart XIII. Table X, p. 75, gives the corresponding values of T' , μ' and b .

193. When viewed in the direction of increasing pressures the curves crossing the vapor field are seen to converge, those at the lower temperatures distinctly more than those above. The convergence of the former reaches a maximum beyond which the curves are divergent toward higher pressures. The locus of maximum convergence, that is, of nearest approach of these curves to each other when measured along constant-pressure lines (Chart IX) rises toward increasing pressures from a point on the saturation curve about 6 atm., 4° below the critical point. It will be shown in a subsequent paper that this is one of several loci of maximum constant-pressure specific heats; and conversely, that loci of points of maximum divergence of the isenthalps are loci of minimum constant-pressure specific heats. Along any particular isenthalp the constant-pressure specific heat has a maximum and a minimum value at intersections with these respective loci. In the liquid field the isenthalps are all divergent in the direction of increasing pressures and neither type of locus appears on the charts. (See last section of Table XIII, A, p. 103).

194. Chart XIV shows the slopes, μ , of the isenthalps of Chart XIII, plotted as isenthalpic functions of their corresponding pressures. The upper envelope

of the curves of this chart is a locus of the maximum values of μ for the corresponding pressures. It is the 'ridge' of a semi-saddleback fold of the surface which the curves represent. All of the curves of Chart XIV are asymptotic to the pressure axis. Chart XV is an isenthalpic plot of $\log \mu$ against the corresponding pressures; the curves of Chart XIV transform into the straight lines of Chart XV.

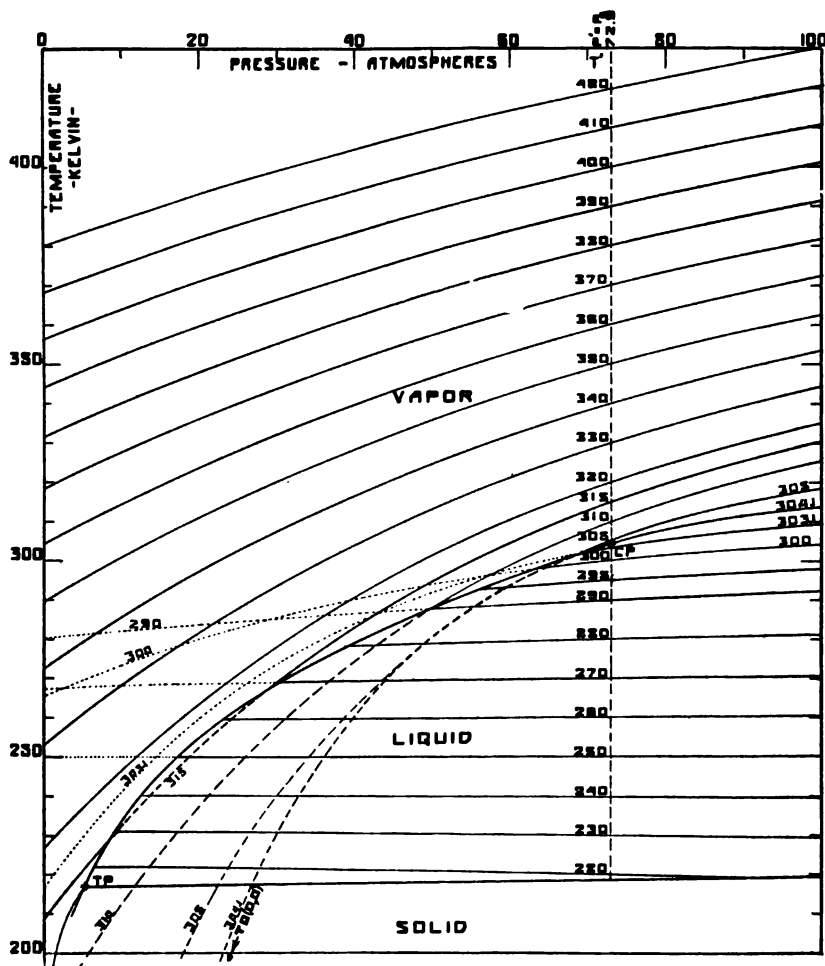


Chart XIII

Isenthalpic throttling curves uniformly spaced on the critical isobar; computed from 1922 formulation.

195. Chart XVI shows the isenthalpic and isobaric variations of μ with temperature, and Chart XVII the isothermal variation of μ with pressure. Chart XVIII shows on a greatly enlarged scale the isobaric variation of μ with temperature, in the liquid phase, from about 300°K down through the

thus the linear temperature variation of μ along the isenthalps according to Eq. (28), p. 69. The envelopes of Charts XIV and XVII are, in Chart XVI, the locus of the maximum points of its isobaric curves.

197. The dotted 50 atm. curve of Chart XVI is presented to show the character of the mathematical continuity connecting the limiting values of μ for the vapor and liquid respectively at the intersections of the corresponding isenthalps with the saturation curve at 50 atm. pressure.

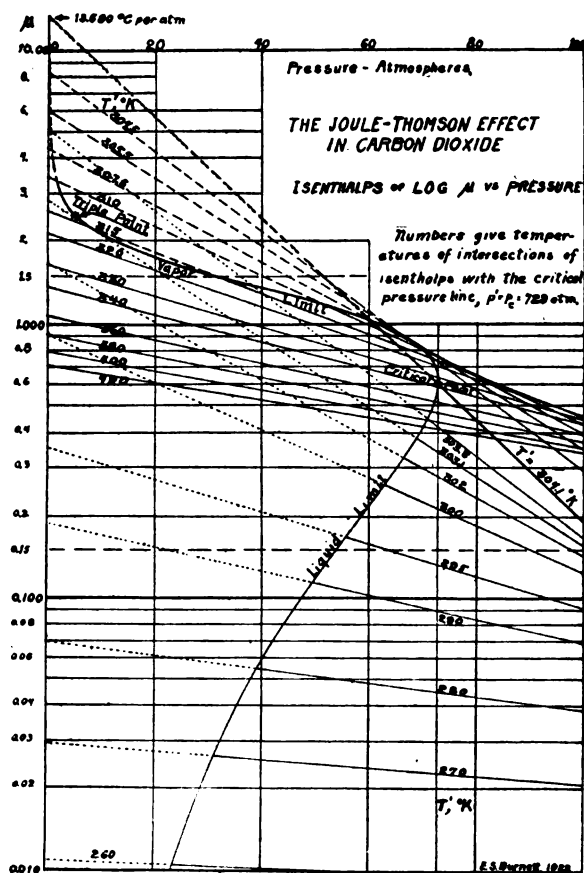


Chart XV

198. **Significance of computed values.**—Most of the values listed in Table X, computed from Eqs. (31) and (33) and their derivatives, have been calculated, in general, to four or five significant figures by the use of six place logarithms for the purpose of securing smoothness in the progressive variations of other magnitudes derived therefrom. A few were read from large scale plots of the others. Values of $(1/\mu')(d\mu'/dT')$ and $(1/b)(db/dT')$ as well as μ' and b as functions of T' are given since they are of value for future applications, particularly in connection with the determination of the ratio of constant-pressure specific heats along isenthalps. Values of T_a are also given as a matter of interest.

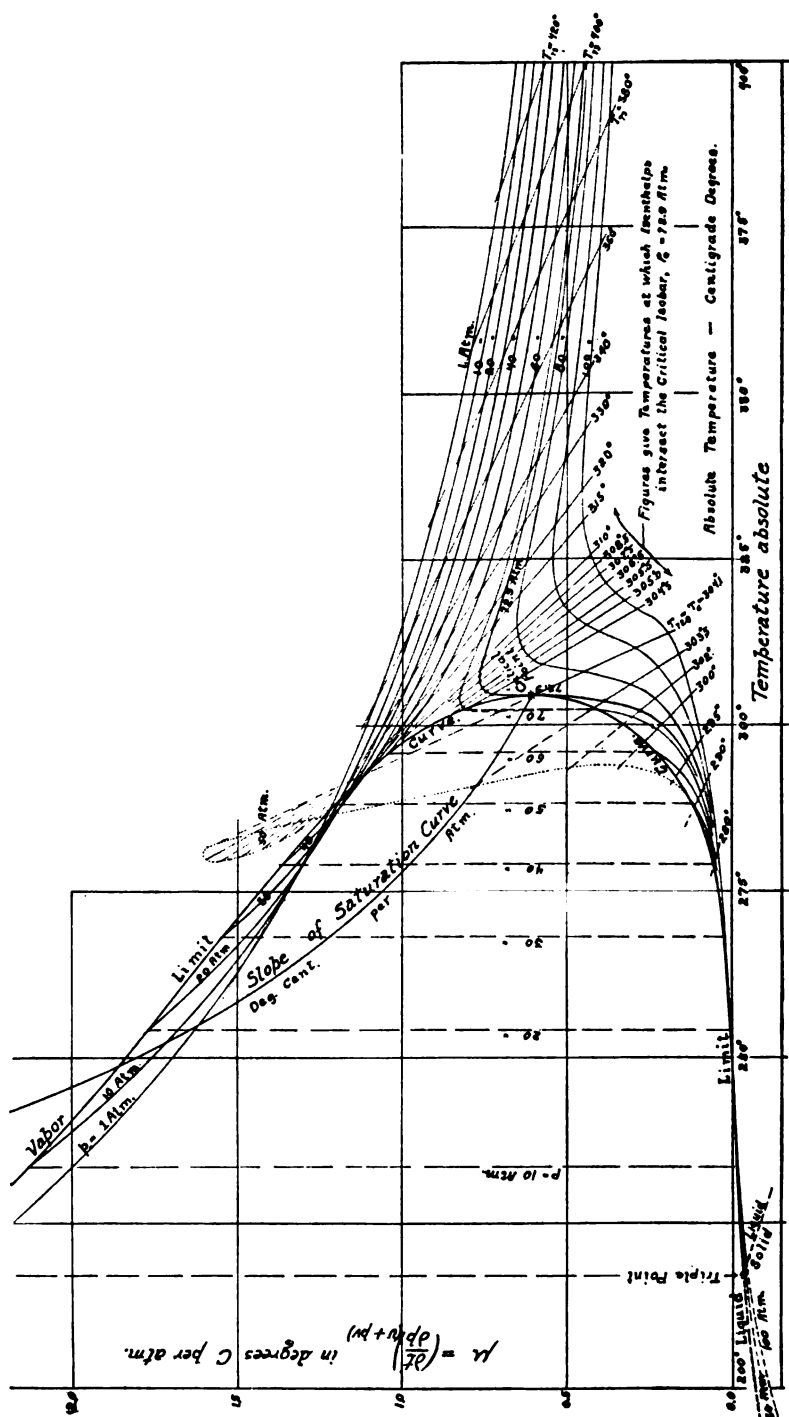


Chart XVI. Curves show μ as an isobaric function of temperature; straight lines show μ as an isenthalpic function of temperature.

199. Most of the computed values of Table XII were obtained by use of a circular slide-rule accurate to four significant figures and are so given for similar reasons. The values of Table XIII were obtained by cross-reading large-scaled plots (similar to Chart XVI) of the values of Table XII. The "supplementary" values of these tables were, in general, determined graphically. Some of them are supplied to assist in accurately locating the curves

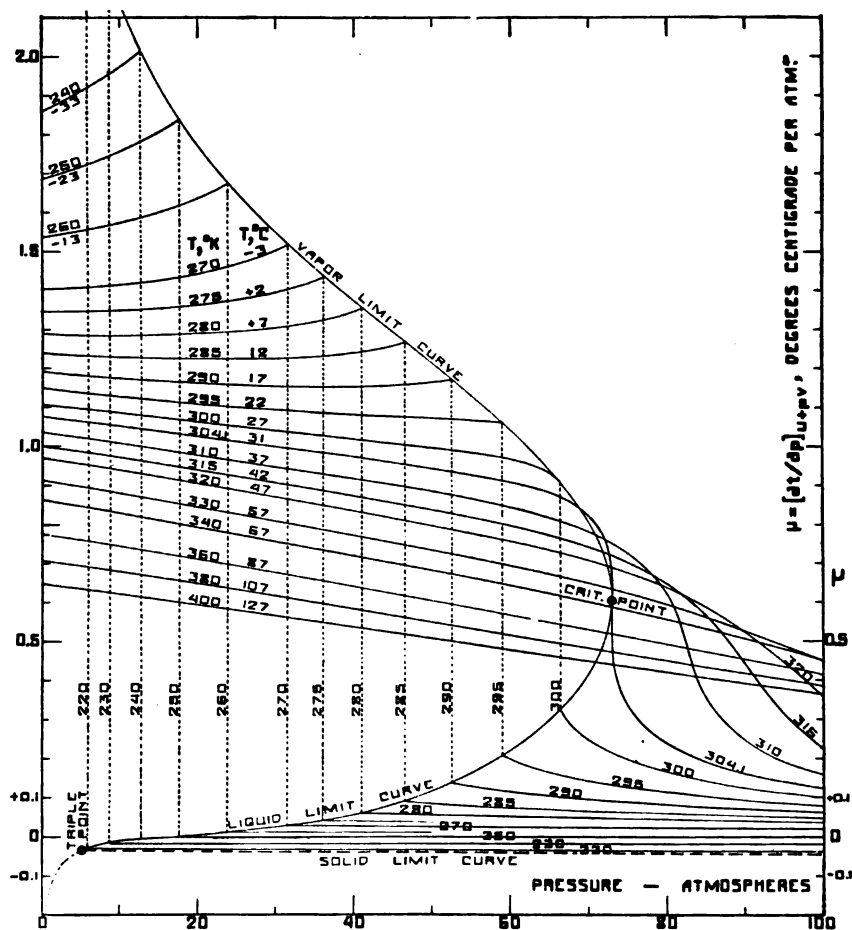


Chart XVII. Isothermal variations of μ with pressure.

in regions where their variation is rapid; the writer has found it very difficult or impossible to represent other data graphically, with certainty, because of the lack of such supplementary values in the published tables.

VAPOR PRESSURE DETERMINATION BY POROUS PLUG METHOD

200. The experimental points recorded in Table VI, A and VI, B which are obviously located along the saturation curve have been omitted from Chart IX

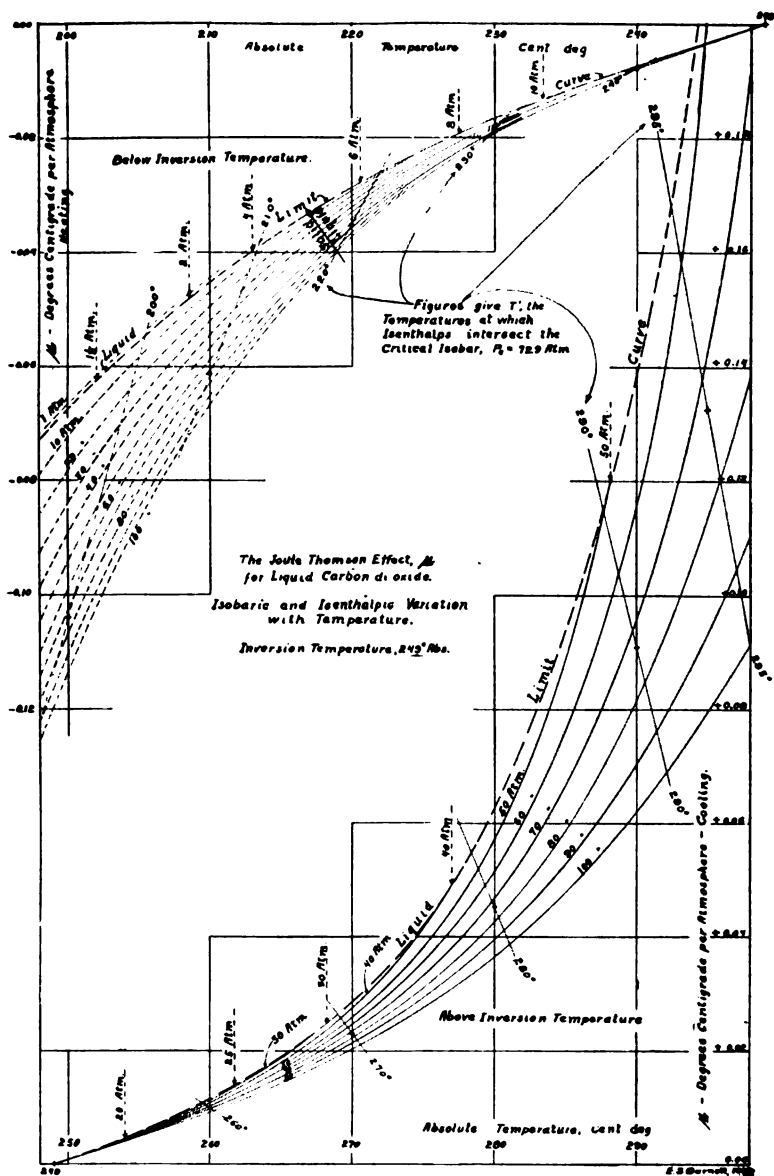


Chart XVIII

Enlargement of Chart XVI near the inversion temperature.

to avoid confusion. They have been assembled in Table IV, p. 33, and their indications discussed in paragraphs 118–121. It may be further noted, however, that for a pure fluid the saturation curve can be definitely located by observations on the flow of either its vapor or its liquid through a porous plug, provided that the vapor be initially in a state sufficiently close to the dry saturated condition; that is, but slightly superheated and at a sufficiently high pressure; and provided also that the expansion be carried sufficiently far with either vapor or liquid.

201. In the case of the vapor, partial condensation occurs, followed at later stages of the expansion by complete re-evaporation. Between the initial condensation and the ultimate re-evaporation, observations of pressure and temperature on the low pressure side of the plug are of those along the saturation curve. Breaks in the continuity of the curve representing all the observations of such a run occur at two points, namely, where the isenthalp first meets the saturation curve, and where it later leaves that curve at a lower pressure. The two extreme sections of such an experimental isenthalp lie in the vapor field and are undoubtedly sections of a curve continuous in its mathematical expression, as indicated by the broken curves of Chart XIII.

202. The pure liquid undergoing porous plug expansion is reduced in pressure and suffers accompanying temperature changes; for CO₂ these are downward if initially above -24°C , and upward if initially below -24°C ; but in either case the liquid eventually reaches a boiling point depending upon its initial energy content. During further expansion partial evaporation ensues and continues at the lowering saturation pressures and temperatures observed on the low pressure side of the plug. Here then are two distinct but essentially equivalent methods of determining the saturated vapor pressures and temperatures of pure fluids.

203. In this connection there is the possibility that in the partial condensation of the expanding vapor the resulting liquid may and probably does in large measure exist as a fog of exceedingly minute particles of finely divided liquid floating in an atmosphere of uncondensed vapor; unless the surface of the thermometer case, or the thermometer wires, if exposed, be wet by some of the condensate, yielding a liquid surface of a sufficiently small curvature, the observed temperatures may differ appreciably from the true boiling point temperatures for the observed pressures because of the effect of increased vapor tension associated with liquid surfaces of great curvature.

204. In the formulation of the coefficients μ' and b as functions of T' the critical temperature, the critical pressure and the slope of the saturation curve at its critical point limit are all factors as has previously been mentioned. There appears to be some question yet as to the true values of these critical constants for CO₂. The critical pressure seems to have been fairly definitely established as 72.9 atmospheres. The critical temperature seems to be uncertain to the extent of several tenths of a degree centigrade, the values given by various observers ranging from 31.0°C to 31.5°C clustering around 31.35°C . The evaluation of the slope of the saturation curve at the critical point, $(dt/dp)_{sat}$ is necessarily involved with the location and form of the upper end of that curve; there is considerable disagreement amongst the observed values all along its whole course as recorded by various investigators. The values variously obtained for $(dt/dp)_{sat}$ at the critical point range from 0.60 to 0.63 degrees centigrade per atmosphere; it has been shown in preceding paragraphs (174–177, p. 73) that $(dt/dp)_{sat} = (\partial t/\partial p)_g = \mu$ at the critical point.

205. The uncertainty in the values of these several critical constants for CO_2 must necessarily be reflected in the formulation herein derived, and in the distribution of the isenthalpic curves computed therefrom. It was necessary to decide on some definite values to use in the present formulation, and those given by Keyes¹⁶ were selected as previously stated (par. 173, p. 72) because in many respects they appeared to the writer to be the best selection based upon a critical study of all of the data then available, and because they are connected by smooth and definite though empirical relations with the saturation properties of CO_2 throughout the entire liquid range from the triple point to the critical point. In view of the uncertainties of the porous plug data in the vicinity of the saturation curve it did not appear advisable to undertake a formulation similar to Keyes' before all of these values are more definitely established.

206. Slight changes in these values would obviously affect the constants of the writer's formulation and would change the map of isenthalpic curves somewhat from that shown on Charts IX and XIII, especially in the region near the critical point. It would not to any large extent, however, affect the characteristic variation of the porous plug and allied effects as depicted on these charts from the writer's present formulation.

207. Variations of this sort do, however, seriously affect the exact determination of the location of the points of intersection of the isenthalps with the saturation curve, and of relations directly dependent thereon: for the difference of enthalpy between two isenthalps having a common intersection with the saturation curve, one from the liquid field and one from the vapor field, is the total heat of vaporization of the fluid at the saturation pressure and temperature of the intersection.

208. The absolute saturation pressures and temperatures adopted as standard for this paper are from Keyes' equation which follows:

$$\log_{10} p = 12.0722 - \frac{1496.7}{T} - \frac{2.8458}{10^2} \cdot T + \frac{2.64}{10^5} \cdot T^2 \quad (34)$$

in which p is computed in atmospheres as a function of T in degrees K. This equation yields $(dt/dp)_{\text{sat}} = 0.6049$ deg. per atm. at the critical point, $p = 7.29$ atm., $T = 304.14^\circ\text{K}$, (31.0°C) .

209. Various determinations of the triple point of CO_2 locate it at temperatures ranging from -54.6° to -57.0°C , averaging about -56.1° , and at pressures ranging from 5.1 to 5.3 atm.; Keyes takes -56.2° and 5.15 atm. Tamman²⁷ has traced the liquid-solid phase boundary curve for CO_2 from the triple point (which he takes from the work of P. Villard and R. Jerry²⁸ at -56.7° and 5 atm.) through a pressure range of nearly 5000 atm., along which the temperature is shown to vary according to his formula (transformed into degrees centigrade and atmospheres):

$$t = -56.8 + 0.02065 \cdot p - 0.00776 \cdot p^2 \quad (36)$$

210. This equation shows that within the limited pressure range of 100 atm. covered by the charts of this paper, its graph is practically a straight line with a positive slope of 2.065 degrees centigrade per 100 atmospheres. The triple point on the charts has been taken at -56.2°C and 5.15 atm., and the liquid-solid phase-boundary, the melting point curve for solid CO_2 , has been drawn from that point with the above slope.

²⁸ Villard and Jerry, *Comp. Rend.* 120, 1413, 1895.

211. On some of the charts some of the isenthalpic curves or other curves thereon presented, have been shown by dotted extension crossing this boundary into the solid phase field for the purpose of better exhibiting the character of their variation as they approach this boundary. It is obvious that their courses in the solid field of the charts can have little or no physical significance, for porous plug expansion of solids appears practically to be beyond the possible range of experimental investigation.

212. In practically all of the charts of this paper the various functions represented have been spread over a range of pressure extending from zero to 100 atm.; and over an absolute temperature range extending from 200° to 400°K. This field includes the full experimental range of pressure and temperature, extending that temperature range appreciably upwards by some 30° to 50° at pressures below 25 atm. In the vapor field the curves covering pressure ranges below about 10 to 15 atm. at temperatures from -25°C to +25°C, and below 30 atm. from +25°C upwards, and the curves covering pressure ranges from 70 or 75 atm. to 100 atm. at temperatures higher than the critical temperature, 31°C, are pure empirical extrapolations beyond the experimental range; their indications without that range are admittedly without direct experimental support. The same must be said for the curves of the liquid field covering the pressure range from 100 atm. to the higher pressure limits of the experimental chords shown on Chart IX, which limits vary from about 70 atm. in their upper temperature range just under 20°C to about 45 atm. in their lower temperature range at about -55°C.

213. The writer appreciates the uncertainty inherent in so wide an extrapolation, but he feels that his study of the experimental basis of his formulation—which includes a comparison of the relatively much wider pressure range of the corresponding approximate and indirect data on air from the investigations of Witkowski,¹ Bradley and Hale,¹² Dalton¹⁴ previously mentioned, as well as some of his own preliminary work on air, and the certain data on isenthalpic curves for air soon to be presented by his esteemed colleague, Dr. J. R. Roebuck,¹⁹ which it has been his privilege to examine,—much of which was obtained while the writer was developing the formulations and curves of this paper in Dr. Roebuck's laboratory in Sterling Hall, University of Wisconsin—warrants him in expressing the belief that the values computed from his formulation are not far from the true values in the extrapolated regions, as well as being within the limits of experimental uncertainty within the experimental range.

COMPARISONS

214. Previous experiments on the Joule-Thomson effect for CO₂ are very limited in scope compared to the range included by the data of this paper. Only three previous investigators have recorded results that are available for comparison. The determinations by Jenkin and Pye¹⁷ of the Joule-Thomson effect for CO₂ liquid and vapor were made subsequent to those of the writer: the agreement of their data with those of the writer in the sections of the field common to both, and of the respective indications of these data when projected over the combined field, were found to be excellent as has been hereinbefore discussed: their data have therefore been incorporated with those of the writer and are embodied in the formulation which this paper presents. [see par. 191.]

215. On the basis of the variation of μ with temperature there are available for comparison for the vapor phase only, the results of Joule and Thomson⁸

determined between 1854 and 1862, and those of Kester observed in 1904²⁹ and 1905³⁰ (Chart XIX). The former worked with very moderate pressures not exceeding six atmospheres, over a temperature range from 0°C to 100°C. The latter undertook to determine the variation of μ with both pressure and temperature, but up to 40 atmospheres could not in 1905 discern with certainty a pressure effect at any temperature of his investigations (from 0° to 100°C),

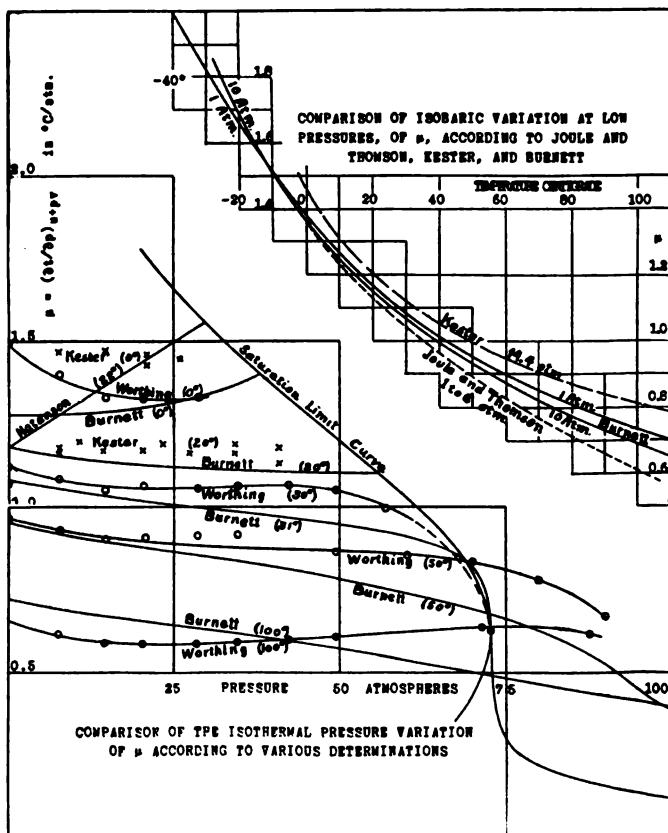


Chart XIX.

thus failing to substantiate his announcement²⁹ of the previous year expressing μ at 22°C as a linearly diminishing function of the pressure according to the relation,

$$\mu \text{ (in deg. C per atm.)} = 1.19 - 0.0015p \quad (37)$$

which is in entirely opposite accord with the relation given by Natanson³⁰ in 1887 for μ at 20°C up to 25 atm., namely:

$$\mu = 1.18 + 0.0126p_m \quad (38)$$

in which p_m is the mean pressure (in atm.) of a short range expansion.

²⁹ Kester, F. E., *Phys. Zeitschr.*, 6, 44, 1905.

³⁰ Natanson, E., *Wied. Ann.*, 31, 502, 1887.

216. The writer's 22° isotherm for μ , which rises to the value 1.15 at one atm. can, in the vapor field, be very closely represented between 10 atm. and the saturation pressure, 49 atm. by the linearly diminishing relation,

$$\mu = 1.135 - 0.0013p \quad (39)$$

in which the pressure coefficient is seen to agree in sign and very nearly in magnitude, with Kester's 1904 conclusion, [Eq. (37)].

217. Kester expressed his 1905 results in the form of a curve showing the variation of μ with temperature from 0°C to 100°C, and compared them with those of Joule and Thomson which he likewise expressed. Both curves are reproduced on Chart XIX for comparison with the writer's corresponding curves for one and for ten atm. there shown. Kester's values are distinctly higher than are those of Joule and Thomson over this temperature range, the ratios being about 4 to 3 at 100°, 10 to 9 at 50° and 11 to 10 at 0°. The writer's two curves fall between the others, averaging nearly midway above 40° but favoring Joule and Thomson below 40°.

218. In view of the fact that neither Kester's values nor those of Joule and Thomson were used by the writer as a guide in establishing his formulation, the location, between their curves, of the latter's one and ten atmosphere curves obtained by extrapolation of that formulation from 10 to 45 atm. beyond the data on which it is based, indicates rather remarkable agreement and may be taken as support of his belief that this formulation represents a fairly close approximation to the facts, within the limits of its application beyond the experimental range as defined in par. 212 and presented on the several charts of this paper. It would therefore by this comparison appear that the writer's extrapolated values are not less reliable than either Kester's or those of Joule and Thomson within the range of comparison.

219. Buckingham²⁸ in 1907 compared all of the observations on the temperature variation of μ for carbon dioxide, air, oxygen, nitrogen and hydrogen, at the low pressures used in gas thermometry. On plotting these observations expressed in "reduced units" according to the "corresponding state" law, he found them to "lie along one smooth curve, as nearly as we can tell" which he took as justification for the assumption that "this curve, based on all the observations, gives a better representation of the facts than can be obtained from the few observations on each gas separately." He favored Kester's more consistent though higher values for CO₂ to those of Joule and Thomson; so that values of μ "reduced back" from Buckingham's smooth curve (for which he supplied an equation), give a slightly flatter curve than that shown as Kester's in the upper figure of Chart XIX, lying above the latter between 20° and 80° and below it without that range of temperature.

220. Kester's 1905 results for μ for CO₂ at 0° and 20° at different pressures are shown by the scattered crosses (x) of the lower figure of Chart XIX; they cannot be said to disclose any variation of μ with pressure at either temperature. Natanson's 20° linear variation of μ , with a positive pressure coefficient is also shown; his zero pressure value is 1.18; that of the writer for 20° is 1.166, both of course being extrapolations. Natanson's experimental points, when plotted, indicate, however, slightly increasing pressure coefficient.

221. Attention is called to the crowding together, crossing over, and subsequent separation of the low pressure isobars of μ , on Chart XVI when viewed in the direction of decreasing temperature. This "mix-up" is most confusing in the range from about 275°K to 300°K, or from about 0°C to 25°C, at pressures up to 60 atm. It was to this particular portion of the field that

many of the earlier investigations were confined, especially those by Natanson and those by Kester in their efforts to determine the isothermal variation of μ with pressure. It is obvious that in this region impurities and experimental error are particularly disturbing, so much so that the writer found it impossible to untangle by graphical means alone the apparently conflicting evidence of his own data in this section of the field, and was therefore forced to seek a solution by the empirical formulation method described within this report. According to the writer's formulation the pressure variation of μ at 0° is slightly positive, and at 20° slightly negative; it is therefore not surprising in view of the rather large experimental uncertainties in Kester's 1905 values, that he was unable to discern a definite pressure variation of μ at these temperatures.

222. Worthing²⁴ computed μ as a pressure function at several temperatures on the basis of thermodynamic relations between μ and other more or less well determined properties of CO_2 , including his own determination of the ratio of the constant pressure and constant volume specific heats of that substance. His results, and those of the writer, as computed for several comparable temperatures in the vapor field, are shown by the remaining curves of Chart XIX. Considering the methods by which both sets of these curves were obtained, Worthing's computed values of μ are in fair agreement with those of the writer in certain sections of the field, as shown by both form and location.

223. It seems quite possible, if not probable, that an empirical formulation so involved as the one herein developed might introduce into the curves which portray it irregularities more or less slight, peculiar to the formulation, that do not exist in the phenomena which it is designed to represent. Indeed a contemplation of the variable curvature of the isotherms of μ , computed from this formulation, when plotted as a function of pressure on Chart XVII would seem at first to substantiate the suspicion that spurious, though perhaps minor irregularities of this character had been introduced. The fact that Worthing's curves, derived by a method so utterly different, do exhibit variations of curvature similar to, and in some cases almost identical with those of the writer's curves, serves to enhance confidence in the belief that the writer's formulation does very closely represent the true variation.

224. Davis,¹⁰ reviewing throttling experiments on steam only, has followed Buckingham's method²⁵ of correlating the available data by the corresponding-state relation, which he assumes to have been shown valid by his comparison. He ignores a slight though uncertainly defined pressure effect that can be detected from his data which is taken from the experiments of Peake,²⁰ Grindley,²¹ Griesman,²² and Dodge.²³ Later, Davis⁸ evidently experienced misgivings as to the validity of this application of a corresponding state law, for, in discussing the value of Joule-Thomson observations for computing steam tables he expressed some doubt that μ does follow such a law.

225. The comparison of throttling curves for steam and CO_2 shown on Chart XX tends to discredit the assumption that μ is amenable to treatment by a law of corresponding states. Onto this chart the observed absolute temperatures in degrees centigrade, and the observed pressures in atmospheres, along the throttling curves for steam transferred from the writer's smooth curves through data from Peake and Dodge, have been transformed into "equivalent absolute temperatures and pressures in centigrade degrees and atmospheres respectively of the CO_2 chart by the following ratios:

$$\frac{\text{Equivalent CO}_2 \text{ temperature}}{\text{Observed steam temperature}} = \frac{\text{Critical temperature of CO}_2}{\text{Critical temperature of steam}} = \frac{304}{638} \quad (40)$$

$$\frac{\text{Equivalent CO}_2 \text{ pressure}}{\text{Observed steam pressure}} = \frac{\text{Critical pressure of CO}_2}{\text{Critical pressure of steam}} = \frac{72.9}{194.6} \quad (41)$$

226. This comparison plainly shows that the *saturation curves* for steam and for CO₂ *do not follow* such a corresponding state law in the field covered by this chart. The throttling curves for steam are seen to be steeper than

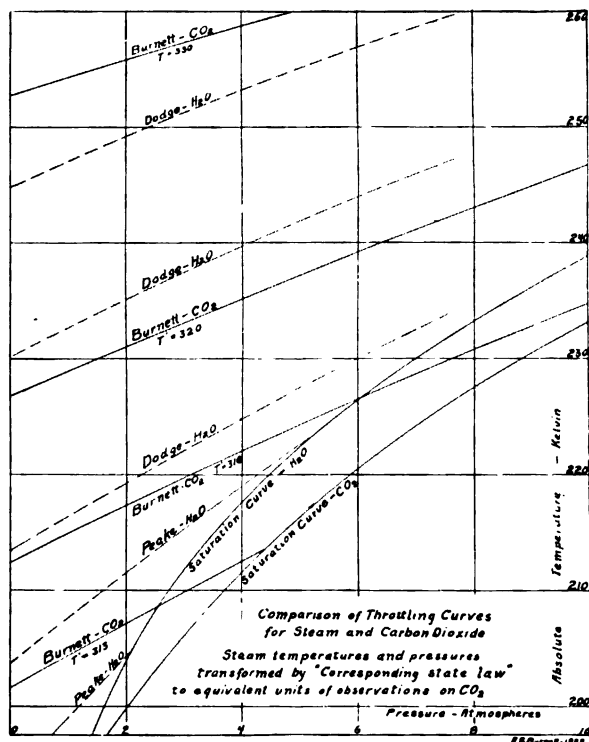


Chart XX

those for CO₂ especially at the lower temperatures. The latter comparison is not conclusive, however, for the CO₂ curves are extrapolations from the writer's formulation and are not yet supported by experimental data below the ten atmosphere line over this temperature range. It is quite possible to find forms of equations for the CO₂ isenthalps which would equally well represent the experimental curves as far down as the ten atmosphere line, and then continue them toward lower pressures with increasing steepness essentially equivalent to that shown by the steam curves in the field of this chart. The writer knows of no experimental data available for steam at pressures exceeding the range of this chart.³¹ More experimental data for both steam and CO₂ are needed to settle the validity of this application of the corresponding state law.

³¹Recent steam throttling curves obtained by integrating Joule-Thomson coefficients from the Harvard throttling experiments are reported in *Mechanical Engineering* 48, 144, 1926, nearly double this range, and confirm the above remarks.

227. **Limit or Boundary Curves.**—On several of the charts of this paper are shown boundary or limit curves, severally marked, Vapor Limit Curve, Liquid Limit Curve, Saturation Limit Curve, and Liquid-Solid Line, values for which are listed in Table XII, A, p. 101, and Table XIII, A, p. 103, the significance of these loci is obvious. On the charts that depict μ as isothermal, isobaric, or isenthalpic functions of pressure or temperature as the case may be, the vapor and liquid limit curves are seen to be divergent toward the μ axis, and of opposite slope. The only previous attempt at determining these loci for CO_2 , of which the writer is aware, was made by H. Lorenz²² in estimating the behavior of its superheated vapor and undercooled liquid on the basis of thermodynamic and other relations combined with a wide extrapolation of Joule and Thomson's empirical formula

$$\mu = a/T^2_{\text{obs}} = 0.276 \left(\frac{273}{T} \right)^2 \quad (42)$$

which was based upon observations at low pressure only, over the narrow temperature range from 0° to 100°C. A comparison of these limiting loci as pressure and temperature functions is presented on Charts XXI and XXII.

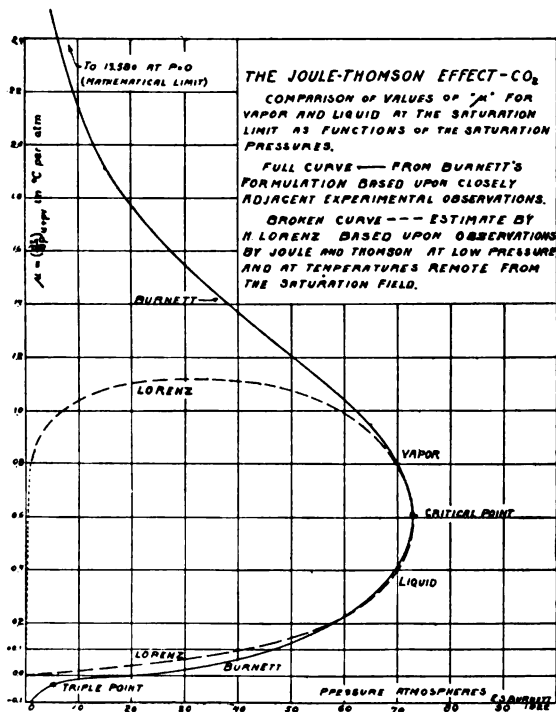


Chart XXI

228. On this basis Lorenz's curves for μ have approximately the same limiting critical-point values as the writer's, and are essentially agreeable therewith as they approach that point: his vapor limit curves show maximum values at about 0° and 34 atm. below which he assumes them to approach, like his liquid

²² Lorenz, Zeitschrift für die gesamte Kälte-Industrie, 3, 30, 1896.

limit curves, an ultimate zero value at zero pressure and zero absolute temperature, a relation that is not supported by the present evidence which seems instead to indicate widely divergent values at these respective limits, positive for the vapor, and negative for the liquid. There is no evidence of a maximum value on the vapor limit curve, nor of a finite upper limit, the empirical value of the writer's formulation being a purely arbitrary closure for the purpose of covering the vapor field as outlined in pars. 180 and 181. Neither is there evidence indicating a sudden change in the course of either liquid or vapor isenthalps as they approach closely to the saturation curve which it would be necessary to assume occurs to account for a limit curve like that of Lorenz.

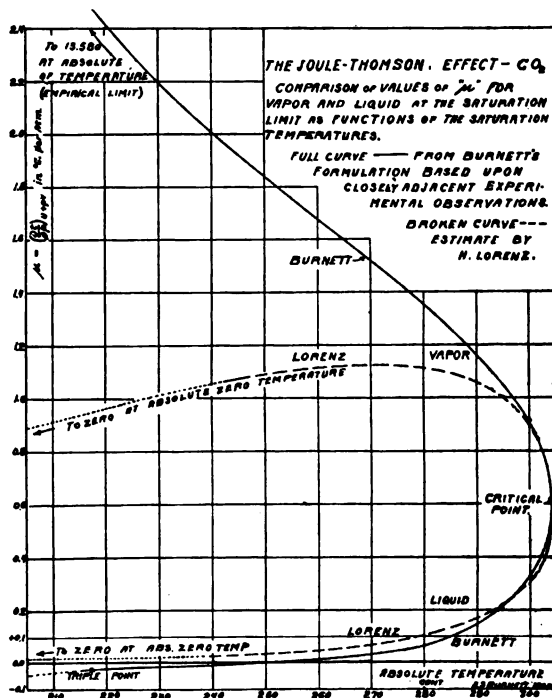


Chart XXII

If, however, future investigations should show that those liquid isenthalps, which, throughout their courses, lie entirely below some temperature slightly higher than -24°C , do exhibit an inversion temperature before they meet the saturation curve, then the Lorenz liquid curve will be the more probable. The Dieterici equation of inversion temperatures indicates such a possibility. Also if μ for vapor does approach a zero limit as the pressure (and temperature) approach zero values, then the vapor limit curve must eventually reach a maximum and retrograde therebelow to a zero limit as Lorenz's curve indicates. In any case, however, the agreement of his loci with those of the writer over the region contiguous to the critical point is truly remarkable, considering the data at Lorenz's disposal.

229. The presentations and discussions embodied in this report indicate very definitely that the Joule-Thomson Effect for a fluid is not a simple function of pressure and temperature as it has by some at times been assumed to be. The formulation herein evolved to represent that function for carbon dioxide liquid and vapor on the basis of experimental evidence has been used to present herewith in tabular and graphical form these interesting and important relations. By means of this formulation the writer has also determined (but not here reported) the variation along isenthalps of the constant-pressure specific heats of CO_2 , and by combination of this variation with absolute values of specific heats and other data for CO_2 determined from other sources, has prepared charts and tables showing the variation of its constant-pressure specific heat as a function of pressure and temperature, and also the variation of the total heat content of CO_2 liquid and vapor with pressure and temperature. Discussion of these and additional developments will be presented in subsequent papers.

Note on effects and probable cause of explosion, see illustrations on pp. 12 and 13.

The cause of this accident as determined from examination of the results of it and from the condition of the machinery immediately following its occurrence appeared to be an explosive combustion of the air-saturated lubricating oil deposited along the walls of the discharge pipe leading from the high pressure cylinder of the compressor. The first break was 16 ft. from the discharge valve, measured along the double-extra heavy 1 in. steel pipe that was coiled over the cylinders in the open cast iron water box surrounding them. The last break was 8 ft. farther along in a double-extra heavy $\frac{3}{4}$ in. steel pipe that led to a receiver drum serving also as an oil trap. Between these points the pipe and fittings including a pop safety valve that was set for 3300 lbs. per sq. in. and an emergency disk safety valve designed for 3500 lbs. per sq. in. These were blown into fragments as was also the whole rear end of the cast iron water box. Examination showed that the high pressure discharge valve of the compressor was in perfect condition. The pipe leading from it to the first break had a trace of fresh oil deposit, for the compressor continued to run for a moment or two after the explosion occurred, until stopped by the operator. The compressor proper was uninjured, its water jacketing box only being broken. Between the first and last breaks along the pipe the inner walls of the pipe were burned completely clean leaving not a trace of oil or carbon deposit. Beyond the last break and on to the oil trap the walls were coated with oil from the compressor cylinder lubrication.

The explanation most plausible to the writer is that some foreign particle became temporarily lodged on the discharge valve or its seat and prevented its complete closure which thus permitted some of the high pressure discharge, hot from the third stage compression, to leak back into the cylinder during each suction stroke. Such leakage, involving no cooling, or very little due to expansion, and perhaps even a Joule-Thomson heating effect, would result in an accumulative increment in the temperature of succeeding discharges until the flash point of the high grade cylinder lubricant in use was reached. The flame propagation thus started would increase rapidly in velocity as the products of the combustion augmented the pressure within the pipe until the heat of compression of the air in the pipe, due to this advancing pressure wave, raised its temperature for some distance ahead of the wave or flame front to the flash point, thus causing a general and explosive combustion within the pipe from the point to which the flame had burned, forward to a point where the inertia of the compressed air had prevented the wave compression temperature from reaching the flash point of the combustible mixture. From the nature of the breakage and the known or estimated strength of the materials it was evident that the explosion pressure over the portions that gave way must have been upwards of 50,000 lbs. per sq. in. or over three thousand atmospheres.

One of the two pictures on pp. 12 and 13 shows the appearance of the compressor following the accident. The terrific force is evident from the number and character of the fragments collected as shown by the other picture.

ASSEMBLED REFERENCES

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³¹ Recent steam throttling curves obtained by integrating Joule-Thomson coefficients from the Harvard throttling experiments are reported in *Mechanical Engineering* 48, 144, 1926, nearly double this range, and confirm the above remarks.

³² Lorenz, H., *Zeitschrift für die gesamte Kälte-Industrie*, 3, 30, 1896.

TABLE XII
 ISENTHALPIC VALUES FROM 1922 FORMULATION
Pressures in atmospheres; temperatures in degrees Kelvin; μ in degrees Kelvin per atm.

Note: Values between brackets [] in main part of this Table indicate "mathematical" courses of the curves when both liquid and vapor phases are present; these are shown by the broken lined portions of a few of the curves of the Chart XIV. Physically and experimentally the temperatures and slopes observed are those of the saturation curve for the pressures corresponding to the values within the brackets.

T'	p	0	1	10	20	30	40	50	60	70	72.9	80	90	100
Vapor Isenthalps														
420.00	T	379.94	390.63	386.77	393.12	399.04	404.56	409.65	414.39	418.78	420.00	422.88	426.69	430.22
	μ	0.7080	0.7029	0.6581	0.6118	0.5685	0.5284	0.4913	0.4564	0.4243	0.4154	0.3944	0.3665	0.3407
410.00	T	368.22	368.97	375.41	382.11	388.28	393.99	399.30	404.21	408.75	410.00	412.97	416.85	420.47
	μ	0.7488	0.7430	0.6935	0.6420	0.5945	0.5506	0.5098	0.4720	0.4371	0.4275	0.4047	0.3748	0.3470
400.00	T	356.30	357.10	363.91	370.92	377.42	383.40	388.92	394.00	398.71	400.00	403.05	406.98	410.74
	μ	0.7924	0.7860	0.7312	0.6748	0.6226	0.5745	0.5301	0.4892	0.4514	0.4410	0.4165	0.3843	0.3546
390.00	T	344.01	344.84	352.14	359.55	366.40	372.70	378.47	383.79	388.55	390.00	393.14	397.26	401.04
	μ	0.8454	0.8384	0.7766	0.7138	0.6558	0.6025	0.5536	0.5086	0.4683	0.4560	0.4294	0.3945	0.3625
380.00	T	331.40	332.30	340.05	347.99	355.25	361.90	367.96	373.52	378.60	380.00	383.27	387.53	391.43
	μ	0.9054	0.8974	0.8286	0.7582	0.6938	0.6348	0.5810	0.5317	0.4866	0.4742	0.4452	0.4074	0.3728
370.00	T	318.14	319.13	327.52	336.06	343.82	350.90	357.34	363.20	368.54	370.00	373.40	377.83	381.84
	μ	0.9826	0.9734	0.8944	0.8142	0.7412	0.6748	0.6142	0.5591	0.5089	0.4952	0.4632	0.4216	0.3839
360.00	T	304.30	305.35	314.50	323.77	332.17	339.76	346.63	352.85	358.48	360.00	363.57	368.18	372.35
	μ	1.0760	1.0655	0.9742	0.8816	0.7978	0.7220	0.6535	0.5914	0.5352	0.5200	0.4944	0.4594	0.3967
350.00	T	289.22	290.44	300.62	310.86	320.04	328.41	335.72	342.40	348.39	350.00	353.68	358.60	362.92
	μ	1.2000	1.1870	1.0782	0.9687	0.8705	0.7820	0.7027	0.6313	0.5672	0.5500	0.5096	0.4590	0.4115
340.00	T	272.42	273.88	285.43	297.03	307.33	316.46	324.60	331.83	338.22	340.00	344.00	349.09	353.64
	μ	1.3750	1.3590	1.2232	1.0885	0.9686	0.8620	0.7670	0.6826	0.6074	0.5672	0.5405	0.4810	0.4280
330.00	T	252.69	254.32	268.15	281.67	293.54	303.95	313.08	321.10	328.12	330.00	334.30	339.70	344.44
	μ	1.6462	1.6250	1.4438	1.2666	1.1110	0.9745	0.8549	0.7498	0.6577	0.6331	0.5768	0.5060	0.4438
320.00	T	226.91	229.09	246.67	263.56	278.04	290.42	301.05	310.15	317.97	320.00	324.63	330.37	335.27
	μ	2.1172	2.0948	1.8230	1.5620	1.3383	1.1470	0.9838	0.8422	0.7216	0.6900	0.6184	0.5299	0.4540

TABLE XII (Continued)

T'	P	0	1	10	20	30	40	50	60	70	72.9	80	90	100
315.00	T	208.69	211.26	[232.38	252.26]	268.91	282.91	294.65	304.56	312.85	315.00	319.82	325.67	330.59
	μ	2.5786	2.5338	[2.1650	1.8181]	1.5270	1.2826	1.0773	0.9046	0.7598	0.7223	0.6381	0.5360	0.4501
310.00	T	181.28	[184.70	212.28	237.43	257.87	274.42]	287.90	298.85	307.66	310.00	314.97	320.84	325.60
	μ	3.4205	[3.3588	2.7848	2.2620	1.8372	1.4928]	1.2128	0.9850	0.8020	0.7533	0.6500	0.5280	0.4290
308.50	T	168.50	[172.88	203.42	231.32	253.66	271.50	285.75]	297.10	306.22	308.50	313.49	319.30	323.94
	μ	3.901	[3.815	3.117	2.491	1.9903	1.5900	1.2707]	1.0154	0.8114	0.7603	0.6483	0.5180	0.4140
307.50	T	157.40	[161.24	196.02	226.49	250.47	269.35	284.24	295.97]	305.21	307.50	312.48	318.16	322.77
	μ	4.343	[4.2514	3.4215	2.6950	2.1230	1.6727	1.3178	1.0350]	0.8177	0.7630	0.6442	0.5088	0.3998
306.50	T	143.62	[148.48	187.10	220.84	246.92	267.10	282.72	294.83]	304.20	306.50	311.48	317.06	321.40
	μ	4.935	[4.811	3.820	2.957	2.289	1.7718	1.3718	1.0616]	0.8216	0.7628	0.6351	0.4923	0.3810
305.50	T	120.90	[126.80	173.13	212.44	242.07	264.32	281.11	293.71]	303.21	305.50	310.36	315.73	319.77
	μ	6.000	[5.832	4.516	3.400	2.558	1.926	1.449	1.0910]	0.8210	0.7560	0.6179	0.4651	0.3508
305.00	T	103.90	[110.68	163.10	207.45	239.00	262.76	280.29	293.20]	302.74	305.00	309.76	314.84	318.78
	μ	6.8705	[6.6645	5.0676	3.7375	2.7565	2.0330	1.4992	1.1060]	0.8156	0.7468	0.6017	0.4437	0.3273
304.50	T	76.00	[81.40	147.40	198.40	234.82	260.85	279.45	292.77	302.29]	304.50	309.09	313.99	317.42
	μ	8.396	[8.114	6.000	4.288	3.0655	2.1915	1.5675	1.1200	0.8007]	0.7264	0.5724	0.4078	0.2926
Critical Isenthalp														
304.10	T	00.00	[13.38	110.70	182.77	229.82	260.56	280.61	293.70	302.35	304.10]	307.80	311.44	313.82
(T _c)	μ	13.580	[13.01	8.858	5.783	3.774	2.463	1.6075	1.0490	0.6848	0.6050]	0.4470	0.2917	0.1906
Liquid Isenthalp														
303.80	T	[167.56	170.62	208.01	240.16	262.45	277.86	288.98	296.28]	302.50	303.80	306.49	309.32	311.34
	μ	[5.1535	4.9800	3.6640	2.6023	1.8403	1.3133	0.9331	0.6835]	0.4710	0.4265	0.3346	0.2377	0.1689
303.50	T	[195.21	198.95	227.82	251.57	268.88	281.51	290.72	297.44]	302.34	303.50	305.92	308.53	310.43
	μ	[3.7974	3.6800	2.7690	2.0194	1.4730	1.0744	0.7838	0.5716]	0.4145	0.3804	0.3041	0.2218	0.16175
303.10	T	[216.78	219.60	241.24	269.94	273.78	284.14	291.90	297.71]	302.05	303.10	305.37	307.75	309.58
	μ	[2.8150	2.7345	2.1140	1.5782	1.1818	0.8850	0.6626	0.4962]	0.3717	0.3417	0.2766	0.2084	0.1560
302.00	T	[245.42	247.07	260.24	271.83	280.86	287.94	293.47	297.79]	301.16	302.00	303.81	305.88	307.48
	μ	[1.6720	1.6312	1.3066	1.0210	0.7982	0.6239	0.4877	0.3812]	0.2980	0.2774	0.2329	0.1820	0.1423

TABLE XII (Continued)

T''	p	0	1	10	20	30	40	50	60	70	72.9	80	90	100
300.00	T	[265.03	265.92	273.28	280.04	285.57	290.11	293.82	296.87]	299.33	300.00	301.41	303.15	304.45
	μ	[0.9102	0.8924	0.7456	0.6108	0.5004	0.4099	0.3359	0.2751]	0.2253	0.2127	0.1846	0.1490	0.1239
295.00	T	[278.44	278.81	281.80	284.73	287.28	289.50	291.44]	293.13	294.60	295.00	295.89	297.00	298.00
	μ	[0.3584	0.3535	0.3127	0.2727	0.2379	0.2075	0.1810]	0.1579	0.1377	0.1324	0.1202	0.1048	0.0914
290.00	T	[280.18	280.37	281.99	283.64	285.12	286.45]	287.66	288.75	289.73	290.00	290.62	291.43	292.15
	μ	[0.1906	0.1886	0.1722	0.1555	0.1405	0.1269]	0.1147	0.1036	0.09362	0.09091	0.08451	0.07641	0.06901
280.00	T	[275.80	275.93	278.54	277.16	277.78]	278.34	278.88	279.39	279.87	280.00	280.32	280.74	281.14
	μ	[0.06978	0.06936	0.06579	0.06202	0.05848]	0.05514	0.05198	0.04901	0.04622	0.04543	0.04357	0.04108	0.03873
270.00	T	[268.10	268.13	268.34	268.67	268.94]	269.20	269.45	269.70	269.93	270.00	270.16	270.38	270.58
	μ	[0.02847	0.02937	0.02848	0.02753	0.02661]	0.02572	0.02486	0.02403	0.02323	0.02278	0.02245	0.02170	0.02078
260.00	T	[259.24	259.26	259.35	259.46]	259.56	259.67	259.77	259.88	259.97	260.00	260.07	260.17	260.26
	μ	[0.01096	0.01094	0.01078	0.01061]	0.01044	0.01027	0.01010	0.00993	0.00978	0.00973	0.00962	0.00946	0.00931
250.00	T	[249.946	249.947	249.954]	249.960	249.968	249.976	249.983	249.990	249.998	250.000	250.005	250.012	250.019
	μ	[0.07368	0.07367	0.07358]	0.07349	0.07339	0.07329	0.07319	0.07310	0.07300	0.07297	0.07291	0.07284	0.07272
240.00	T	[240.54	240.53	240.47]	240.40	240.32	240.25	240.18	240.10	240.02	240.00	239.94	239.87	239.79
	μ	[0.007042	0.007050	0.007125]	0.007225	0.007319	0.007416	0.007510	0.007608	0.007707	0.007736	0.007808	0.007908	0.008010
230.00	T	[231.23	231.22]	231.08	230.92	230.75	230.59	230.42	230.24	230.06	230.00	229.87	229.67	229.48
	μ	[0.01522	0.01527]	0.01565	0.01610	0.01656	0.01703	0.01751	0.01801	0.01852	0.01867	0.01905	0.01959	0.02015
220.00	T	[222.19	222.17]	221.93	221.66	221.39	221.08	220.77	220.44	220.13	220.00	219.74	219.37	218.98
	μ	[0.02537	0.02548]	0.02654	0.02778	0.02905	0.03040	0.03181	0.03329	0.03472	0.03528	0.03664	0.03812	0.03989

TABLE XII, A.
 SUPPLEMENTARY VALUES AT GRAPHICALLY DETERMINED INTERSECTIONS OF THE ISENTHALPS WITH THE SATURATION CURVE.

T'	μ'	b	p_1	T_1	μ_1	p_2	T_2	μ_2	Remarks
315.85	0.7166	-0.01705	17.1	249.1	1.855	17.1	249.1	1.855	Tangent or grating point
315.70	0.7175	-0.01713	20.65	255.1	1.763	13.0	240.85	2.002	
315.50	0.7190	-0.01722	22.95	258.7	1.699	11.05	236.2	2.086	Lower point is triple point
315.0	0.7223	-0.01746	27.15	264.6	1.606	8.4	228.7	2.226	
314.5	0.7255	-0.01770	30.25	268.45	1.544	6.65	222.95	2.344	Lower point on saturation curve
314.0	0.7288	-0.01795	33.15	271.85	1.488	5.5	218.05	2.444	
313.88	0.7296	-0.01803	33.75	272.55	1.478	5.15	216.9	2.475	Lower point on saturation curve extension below the triple point
312.75	0.7369	-0.01870	39.5	278.5	1.376	3.3	197.5	2.707	
310.0			50.45	288.35	1.2015	Values under above headings for this group are to be determined by intersections of isenthalps with the vapor-solid phase-boundary.			
308.5			55.60	292.5	1.1205	Intersections of liquid isenthalps with the saturation curve at points lower than those listed at left, when such occur, are without physical significance.			
307.5			59.30	295.25	1.0550				
306.5			63.1	297.9	0.9805				
305.5			66.5	300.15	0.9068				
305.0	0.7468	-0.03045	68.25	301.25	0.8605				
304.5			70.7	302.75	0.7820	Upper point is critical point, lower point is the null point (both arbitrary)			
304.1	0.6050	-0.04267	72.9	304.10	0.6050	0.0	0.00	13.580	
303.8			71.25	303.1	0.4512	Inversion temperature of the Joule-Thomson effect for liquid carbon dioxide; pressure influence not yet determined, but certainly small for these pressures.			
303.5			69.9	302.3	0.4182				
303.1			68.7	301.5	0.3860				
302.0			66.3	300.0	0.3280				
300.0			62.6	297.5	0.2610				
295.0	For values not listed in these two columns see Table X, p. 75.		55.35	292.3	0.1685				
290.0			49.5	287.55	0.1155				
280.0			39.25	278.25	0.0554				
270.0			30.6	268.9	0.0263				
260.0			23.50	259.5	0.0105				
250.0			17.6	249.95	+0.0007+	Triple point; (T' , p') for this isenthalp is a point in the solid phase.			
249.1	0.0	0.0	17.1	249.1	0.0				
240.0			12.8	240.4	-0.0072-				
230.0			9.2	231.1	-0.0156				
220.0			6.5	222.0	-0.0261				
(214.1)	-0.0496	+0.0568	5.15	216.9	-0.03375				

TABLE XIII
ISOTHERMAL AND ISOBARIC VALUES OF μ CORRESPONDING TO CHARTS XVI AND XVII; IN DEG. PER ATM.

Temp. °K	Pressure in atmospheres.												
	0	1	10	20	30	40	50	60	70	72.9	80	90	100
Vapor phase above sigsag line													
400.0	0.6475	0.6440	0.6210	0.5950	0.5655	0.5375	0.5080	0.4790	0.4500	0.4410	0.4225	0.3915	0.3635
390.0	0.6755	0.6725	0.6485	0.6200	0.5900	0.5595	0.5290	0.4965	0.4550	0.4560	0.3850	0.3530	0.3235
380.0	0.7080	0.7045	0.6780	0.6475	0.6155	0.5835	0.5500	0.5165	0.4840	0.4742	0.4505	0.4180	0.3855
370.0	0.7415	0.7335	0.7100	0.6775	0.6450	0.6160	0.5755	0.5405	0.5050	0.4952	0.4705	0.4350	0.3995
360.0	0.7790	0.7750	0.7455	0.7110	0.6770	0.6420	0.6050	0.5685	0.5310	0.5200	0.4930	0.4545	0.4155
350.0	0.8195	0.8150	0.7850	0.7500	0.7150	0.6780	0.6465	0.6020	0.5615	0.5500	0.5210	0.4780	0.4340
340.0	0.8640	0.8595	0.8290	0.7950	0.7580	0.7205	0.6825	0.6425	0.6000	0.5872	0.5550	0.5055	0.4580
330.0	0.9140	0.9095	0.8795	0.8450	0.8095	0.7720	0.7345	0.6925	0.6480	0.6331	0.5945	0.5255	0.4480
325.0	0.9425	0.9375	0.9075	0.8745	0.8400	0.8025	0.7650	0.7230	0.6755	0.6605	0.6165	0.5370	0.4520
320.0	0.9710	0.9665	0.9380	0.9050	0.8710	0.8360	0.8000	0.7570	0.7075	0.6900	0.6380	0.5540	0.4570
315.0	1.0020	0.9985	0.9705	0.9395	0.9065	0.8735	0.8395	0.7970	0.7440	0.7223	0.6500	0.4430	0.2210
310.0	1.0360	1.0320	1.0055	0.9765	0.9465	0.9160	0.8845	0.8435	0.7840	0.7534	0.6100	0.2505	0.1585
305.0	1.0710	1.0675	1.0445	1.0155	0.9900	0.9640	0.9365	0.9000	0.8195	0.7468	0.2690	0.1700	0.1270
304.1	1.0775	1.0740	1.0505	1.0240	0.9985	0.9735	0.9470	0.9100	0.8220	0.6050	0.2420	0.1590	0.1215
300.0	1.1070	1.1045	1.0840	1.0600	1.0385	1.0175	0.9955	0.9675	0.2460	0.2147	0.1650	0.1220	0.1005
295.0	1.1480	1.1455	1.1270	1.1090	1.0980	1.0805	1.0715	0.1990	0.1425	0.1324	0.1134	0.0931	0.0794
290.0	1.1920	1.1900	1.1750	1.1635	1.1530	1.1525	1.1560	0.1156	0.0957	0.0909	0.0815	0.0704	0.0619
285.0	1.2395	1.2385	1.2280	1.2245	1.2260	1.2400	0.0887	0.0761	0.0666	0.0635	0.0586	0.0527	0.0478
280.0	1.2900	1.2900	1.2845	1.2915	1.3045	1.3470	0.0571	0.0515	0.0467	0.0454	0.0425	0.0395	0.0364
275.0	1.3455	1.3455	1.3470	1.3645	1.3960	0.0414	0.0380	0.0355	0.0330	0.0324	0.0309	0.0292	0.0275
270.0	1.4050	1.4060	1.4155	1.4455	1.5010	0.0274	0.0258	0.0246	0.0233	0.0228	0.0221	0.0212	0.0202
260.0	1.5375	1.5405	1.5735	1.6375	0.0111	0.0106	0.01035	0.0101	0.00885	0.00973	0.0096	0.0093	0.0090
250.0	1.6885	1.6945	1.7570	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
240.0	1.860	1.870	1.9740	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
230.0	2.060	2.072	-0.0168	-0.0171	-0.0174	-0.0177	-0.0180	-0.0183	-0.01865	-0.0187	-0.0190	-0.01925	-0.0195
220.0	2.2865	2.3035	-0.0204	-0.0204	-0.0314	-0.0323	-0.0332	-0.0341	-0.0350	-0.0353	-0.0359	-0.0367	-0.0375
Liquid phase below sigsag line													
				723	732	742	751	761	771	774	781	791	801

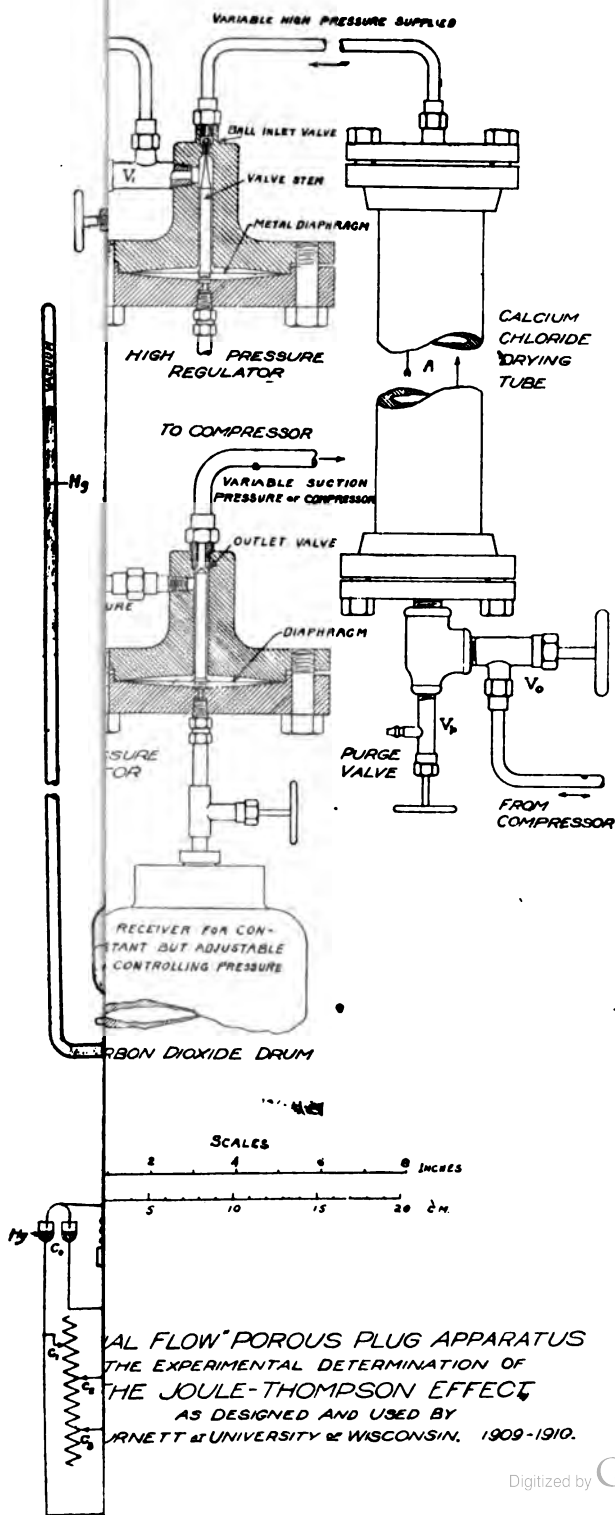
TABLE XIII. A.

SUPPLEMENTARY VALUES AT GRAPHICALLY DETERMINED INTERSECTIONS OF THE ISOTHERMS AND ISOBARS WITH THE SATURATION CURVE.													
<i>p</i>	0	1	5	10	15	20	30	40	50	60	65	70	72.9
<i>T</i>	0.0	(193.5) [†]	216.2	233.4	245.05	254.1	268.15	279.0	288.0	295.7	299.15	302.35	304.1
μ_v	+13.580	(3.035)	2.51	2.135	1.922	1.773	1.545	1.371	1.211	1.043	0.937	0.801	0.605
μ_l	(Solid)	Above values at vapor-solid intersection.	-0.0346	-0.0083	-0.0031	+0.0046	0.0245	0.0575	0.1182	0.2225	+0.2985	0.418	0.605

	<i>T</i> = 304.1													
	<i>p</i>						μ							
<i>T</i>	220.0	230.0	240.0	250.0	260.0	270.0	275.0	280.0	285.0	290.0	295.0	300.0	72.0	0.767
<i>p</i>	5.95	8.8	12.65	17.6	23.9	31.55	36.05	41.0	46.55	52.55	59.0	66.3	74.0	0.382
μ_v	2.410	2.200	2.020	1.841	1.675	1.515	1.436	1.354	1.269	1.174	0.961	0.914	76.0	0.310
μ_l	-0.029	-0.0167	-0.0074	+0.0007	+0.0115	0.0285	0.0419	0.0625	0.0938	0.1386	0.2095	0.3235	85.0	0.192

VALUES BELOW, AND THOSE TO THE RIGHT ABOVE UNDER <i>T</i> = 304.1 CORRESPOND TO REGIONS OF RAPID RATES OF CHANGE OF μ													
<i>T</i>	210.0	210.0	210.0	220.0	230.0	240.0	250.0	250.0	310.0	310.0	310.0	310.0	315.0
<i>p</i>	0.0	1.0	(5.0)	5.0	5.0	5.0	5.0	15.0	76.0	78.0	82.0	85.0	95.0
μ	2.543	2.570	2.690	2.387	2.132	1.910	1.720	1.806	0.710	0.665	0.520	0.360	0.312

LOCUS OF POINTS OF CLOSEST CONSTANT-PRESSURE APPROACH OF ISENTHALPS.													
<i>p</i>	66.8 (sat.)	70.0	72.9	75.0	80.0	85.0	90.0	95.0	100.0	See Chart IX, p. 53 and par. 193, p. 80.			
<i>T</i>	300.3 (sat.)	304.0	307.1	309.5	314.8	320.0	325.0	329.9	334.5				



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